Towards Convergence to a Rational Expectations Equilibrium: An Axiomatic Approach*

Matthew Schiffman
Brown University Department of Economics
2528 Sheridan Road
Evanston IL, 60201
847 567 4697
Matthew_Schiffman@brown.edu

April 14, 2009

*I would like to thank my advisor Professor Roberto Serrano

1 Introduction

Matching and bargaining games with common values can converge to a rational expectations equilibrium with sufficiently patient players. This paper conjectures conditions under which information becomes fully revealed in a class of games with incomplete information. I do this in order to provide a dynamic and decentralized foundation for a rational expectations equilibrium.

Matching and bargaining games are models of trades in markets. Players search for trading partners. When they find one (the matching), they decide whether they want to make a deal and on what terms (the bargaining). This basic framework allows for a broad range of possible situations. For example, how big is the market? Who gets matched up with whom? How does the bargaining proceed? What do players know at the start of the game? This paper conjectures that an equilibrium in which information becomes fully revealed exists in many of these games despite specifying neither the matching nor the bargaining processes.

These games are useful because they provide dynamic examples of decentralized markets with incomplete information. *Dynamic* means that the game has a time element and play takes place in stages. A *decentralized* market has no central authority running the trading and players know only the details of their own trades. These markets limit the information that agents can use to make decisions. The models in this paper assume a matching mechanism that leads to decentralized trading. Thus trading takes place in meetings of small groups behind closed doors instead of on an open trading floor. This assumption is unlike many classical economic models that are premised on a centralized trading mechanism that acts like an auctioneer. It calls out possible deals to all the players who then decide if they would like to participate.

Incomplete information can exist in two forms: goods can have either common or private values. *Private values* occur when a trader's ex-post valuation is independent of information held by others. Otherwise, it is the case of interdependent values. An extreme case of interdependent values is *common values*, in which the ex-post value is the same for everyone. Markets with common values are the focus of this paper.

The difference between these two types of information can be illustrated by considering two example of auctions. Common values with incomplete information would occur if land was being auctioned for oil drilling. Everyone knows what oil is worth and how much it would cost to drill a well, but people may have different ideas about how likely it is that the well will find an oil deposit. An example of private values would be an art auction. People know the details of the painting being sold but are willing to pay different amounts for the art. The information is incomplete because people do not know what each other's values are.

One of the important solution concepts used in matching and bargaining games with common values is the Rational Expectations Equilibrium (REE). This equilibrium is one in which players' actions are the same as they would be in the same game with perfect information. An REE only occur when the system is fully revealing (i.e., any missing information can be deduced as the game progresses). In other words, market forces alone are sufficient to convey information through the system. An example is a market for parcels of land that may contain oil, with a bargaining procedure in which the only discussion that can occur between players is quoting prices. A rational expectations equilibrium occurs if the amount paid for the land exactly matched the quantity of oil under it.

The classic example of a dynamic decentralized matching and bargaining game was created by Wolinsky (1990)[9]. This paper developed a model in which large numbers of buyers and sellers of a single good were randomly paired every round and played a bargaining game. If a pair of players reached a deal they would exit the market; otherwise, they would be matched with different people in the next round (the dynamic element). The game involved a decentralized market: the only information a player had was the result of his bargaining in previous rounds. The bargaining subgame assumes that only two values for the good exist and that trading occurs at one of three possible prices. The restricted nature of the bargaining game was a key idea introduced in the paper. It allowed the paper to prove that a non-neglible fraction of players would transact at a different price then they would have arrived at with perfect information.

Variations of this basic model were examined in later work on the subject. Refinements led to papers that proved convergence to a REE, such as, Gottardi and Serrano (2005) [3]. This paper examined what happened when a small number of sellers dealt with a large number of buyers in a similiar price model. It concluded that with enough competition between sellers, information was fully revealed. Another paper by Serrano and Yosha (1993)[7] shows that if one side of the market is completely informed then a fully informing equilibrium exists. Work has also been done on creating static games that converge to a REE. Reny and Perry have designed a double auction with private information and common values that converges to a rational expectation equilibrium as the number of buyers and sellers goes to infinity [6].

Convergence of classes of matching and bargaining games has been examined in the private values case by Lauermann (2006)[5]. This paper gives conditions where games converge to a Walrasian Price Equilibrium where only people whose value was above or below a certain threshhold (for buyers and sellers respectively) would trade.

In section 2, I present a pairwise matching and bargaining game. Section 3 generalizes such games to describe any model that utilizes a mixture of common and private values. I introduce definitions and conditions for the main conjecture, then present it. Section 4 applies the main conjecture to the model introduced in section 2. Section 5 concludes.

2 An Illustrative Model

This section presents a basic model of dynamic matching and bargaining games in markets with incomplete information. It is based on Serrano and Yosha's (1993)[7], which is in turn based on Wolinsky's (1990) model[9]. Its purpose is to illustrate the use of the general framework.

The market consists of two equal continuous populations: buyers and sellers. Sellers each possess one unit of an indivisible good and buyers are interested in purchasing exactly one unit of this good. Rounds occur at discrete time intervals that can be indexed from $-\infty$ to ∞. In each period every buyer is matched randomly with a seller. If the pair reach an agreement then they transact and exit the market. At the end of each round, a mass of M new buyers and M new sellers enter the market.

The incomplete information in the world is represented by having two different states in which the good has a different common value (called the high and low value states). Upon entering the market, each buyer recieves a signal about the state of nature, $\sigma \in \{H, L\}$, while sellers know the true state of nature, so they receive a perfectly informative signal. Let the true state of nature be $\omega \in \{H, L\}$. The signals are informative, that is, for buyers $P(\omega = H|\sigma = H) > P(\omega = H|\sigma = L)$ and $P(\omega = L|\sigma = L) > P(\omega = L|\sigma = H)$. To continue the potential oil field example, oil only exists under the land in the high state of nature. The seller knows whether or not his land contains oil while buyers receive a signal (the results of their research). If they receive the high signal they think it is likely that there is oil, while if they receive the low signal they think an oil strike is unlikely.[1]

The model contains elements of common values with incomplete information and private values. A player's payoff after a deal is reached is a function of the transaction price, p, as well as ω and σ. A seller's utility is $p - c(\omega, \omega)$ and buyer's utility is $v(\sigma, \omega) - p$ where $c(\omega, \omega)$ and $v(\sigma, \omega)$ are utility functions (the state term is repeated twice in the seller's utility function because they receive a signal telling them the exact state of nature). Denote $c(H)$ as c_H and $c(L)$ as c_L. Let the buyer's utlity function take the values below:

	L, L	H, L	L, H	H, H
$v(\sigma, \omega)$	v_L	$v_L + \epsilon_b$	v_H	$v_H + \epsilon_b$

The ϵ_b terms represent a bonus that buyers recieve when they get the high signal. This is added to allow for a measure of private values that can be made arbitrarily small (which is relevant for the general result). Let $v_H > c_H > v_L > c_L$ and $v_L + \epsilon_b < c_H$. This ensures that when both players agree upon the state a mutually beneficial transaction can be reached, but when the state is H but the buyer thinks that the state is L no possible

[1] It is important to note that the state of nature is the same for everyone in this example. This means that either all of the land has oil under it or none of it does.

sale can occur.

All agents are impatient with a discount factor of $\delta < 1$. Thus the ulity of a player who transacts after t rounds and gets a payoff of y is $\delta^t y$. A player who never transacts receive a utility of zero.

The final determinant of a player's utility is the transaction price, which is determined by a bargaining game. When two agents meet, they simultaneously announce either high or low - represented as h and l. The following matrix shows the possible outcomes of the game with the seller on the left and the buyer on the top:

	h	l
h	p^{hh}	Disagree
l	p^{lh}	p^{ll}

with $v_H > p^{hh} > c_H > v_L > p^{ll} > c_L$ and $p^{hh} > p^{lh} > p^{ll}$.

This means that when both agents agree on a state they trade at the corresponding price. When they disagree but both are "soft" (the seller agreeing to l and the buyer to h) they trade at p^{lh} and when both are "tough" (the seller insisting on h and the buyer on l) no agreement can be made. Notice that the parameters are such that when they agree on the true state they can obtain a positive surplus, but when they disagree yet still transact one of the players will suffer a loss.

I now turn to characterizing possible strategies of the players. Since an agent who plays "soft" will always transact, a strategy is definined as the number of rounds that an agent is willing to play tough before switching to soft. The market can then be defined by the number of sellers (which is assumed equal to the number of buyers), $K(t)$, as well as the fraction of buyers and sellers who are trading at the tough and soft prices.

Fraction adopting positon

	High	Low
sellers	$S^h(t)$	$S^l(t) = 1 - S^h(t)$
buyers	$B^h(t)$	$B^l(t) = 1 - B^h(t)$

Using these functions it becomes possible to characterize the probibility that an agent will transact in the next period. A "soft" agent will transact while a "tough" agent will transact with probability $B^h(t)$ for sellers and $S^l(t)$ for buyers. The same analysis can be made in aggregate: the number of meetings between position i buyers and position j sellers is $K(t)B^i(t)S^j(t)$. These functions can also be uses to calculate $K(t+1)$, as an agent will stay in the market if he and his opponent both act tough, so $K(t+1) = K(t)S^h(t)B^l(t)+M$.

I can now turn to an analysis of the equilbria of this model. A steady state will occur when $K(t)$, $S(t)$, and $B(t)$ are constant for all t. Since the following analysis is only concerned with the steady state the t argument will be dropped. A steady state equilibrium can be characterized by the state contingent constants: the size of the market K_ω, the distribution of sellers $S_\omega = (S^h_\omega, S^l_\omega)$, the distributions of buyers $B_\omega = (B^h_\omega, B^l_\omega)$ and an assignment of utility maximizing buyer and seller strategies based on the signals they recieve.

The strategies that the players utilize are utility maximizations dependent on their belief of the true state and their assumed knowledge of the distribution of positions H and L amongst agents of the opposite type. Since a player who acts soft leaves the market, a strategy for an agent is determined by the number of rounds that a player acts tough before switching to soft. Thus for a seller, the strategies are to insist on H for n, rounds then switch to L on round $n+1$, where $n \in [0, \infty]$. When $n = \infty$ an agent never switches.

Say the market is in a steady state. All buyers and sellers know that if the true state is ω then the distribution of positions is B_ω, S_ω. Fixing these values, let $U_s(n, \sigma)$ be the expected value of a strategy n to a seller whose signal was $\sigma \in \{1, H, L, 0\}$ with H, L being the normal signals and $1, 0$ being the perfect signals received by the seller telling the state with 1 being high and 0 being low. Let

$$U^S(\sigma, \omega) = \arg\max_n U_S(n, \sigma),$$

where n can also be ∞. An optimum strategy then belongs to the set of values of $U^S(P(\omega = H|\sigma), \sigma)$. The value of $U_S(n, \sigma)$ is:

$$U_S(n, \sigma) = P(\omega = H|\sigma) U_S(n, \sigma|\omega = H) + P(\omega = L|\sigma) U_S(n, \sigma|\omega = L).$$

The expanded conditional value is:

$$\begin{aligned} U_S(n, \sigma|\omega) = &\sum_{t=0}^{n-1} \delta^t (B^l_\omega)^t B^h_\omega [p^{hh} - c_\omega + 1_{\sigma=h}] \\ &+ \delta^{n+1} (B^l_\omega)^n \left(B^l_\omega [p^{ll} - c_\omega + 1_{\sigma=h}] \right). \end{aligned}$$

Each of these sums is the discounted value of encountering a tough player for the first t rounds while playing tough plus the discounted expected value of the last round when the player is soft.

Another way of looking at $U_S(n,\sigma)$ is recursively. Thus

$$U_S(n,\sigma) = U_S(0,\sigma) + \sum_{t=0}^{n-1} \delta^t (B_H^l)^t [U_S(1,\sigma|\omega = H)) - U_S(0,\sigma|\omega = H)] P(\omega = H|\sigma) + \sum_{t=0}^{n-1} \delta^t (B_L^l)^t [U_S(1,\sigma|\omega = L)) - U_S(0,\sigma|\omega = L)] P(\omega = L|\sigma). \quad (1)$$

In other words, the value of a strategy of playing tough for n periods and then playing soft is equal to the value of switching immediatly plus the sum of the values of postponing the switch for t periods.

This gives us two cases for optimal strategies. $U_S(1,\sigma|\omega = H)) - U_S(0,\sigma|\omega = H)$ will always be a positive quantity. If $U_S(1,\sigma|\omega = L)) - U_S(0,\sigma|\omega = L) > 0$ then $U_S(n,\sigma) < U_S(n+1,\sigma)$, so $n = \infty$. Otherwise, since $B_H^l < B_L^l$ there must be some finite n such that

$$0 \geq (B_H^l)^t [U_S(1,\sigma|\omega = H)) - U_S(0,\sigma|\omega = H)] P(\omega = H|\sigma) + (B_L^l)^t [U_S(1,\sigma|\omega = L)) - U_S(0,\sigma|\omega = L)] P(\omega = L|\sigma) \quad (2)$$

Let n_σ denote that n. This will be the optimal strategy. Note that if equality holds in (2) then both n and $n+1$ are optimal.

The next step is unwrapping this condition. B_H^l and B_L^l are both endogenously specified in this case. It would also be possible to endogenously specify optimal strategies and look at the population's evolution to determine these two, but that approach is not as analytically tractable. $U_S(n,\sigma)$ is the expected value of a strategy of playing tough n times then switching to soft, so the explicit expressions is:

$$U_S(0,\sigma|\omega=H) = B_H^l(p^{ll} - c_h + 1_{\sigma=h}) + B_H^h(p^{hl} - c_h + 1_{\sigma=h}).$$

$$U_S(1,\sigma|\omega=H) = B_H^h(p^{hh} - c_h + 1_{\sigma=h}) + \delta U_S(0,\sigma|\omega=H).$$

$$U_S(0,\sigma|\omega=L) = B_L^l(p^{ll} - c_l + 1_{\sigma=h}) + B_L^h(p^{hl} - c_l + 1_{\sigma=h}).$$

$$U_S(1,\sigma|\omega=L) = B_L^h(p^{hh} - c_l + 1_{\sigma=h}) + \delta U_S(0,\sigma|\omega=L).$$

The gain for delaying a round is:

$$D_S(\sigma,\omega) = U_S(1,\sigma|\omega) - U_S(0,\sigma|\omega)$$

$$= B_\omega^h(p^{hh} - c_h + 1_{\sigma=h}) - (1-\delta)\left(B_\omega^l p^{ll} + B_\omega^h p^{hl} - c_h + 1_{\sigma=h}\right)$$

Finally, denote $\nu_B(\sigma)$ as the optimal waiting time of a player who receives the signal σ.

Symmetrical values of $U_B(\sigma,\omega)$ and $V^B(\sigma,\omega)$ and $D_B(\sigma,\omega)$ can be likewise defined and expressed analogously.

Theorem. *An equilibrium exists with finite trading time.*

See proposition 1 in Serrano and Yosha (1993) [7] for the proof.

Now that an equilibrium has been shown to exist, the next step is to look at the level of information revelation in the market. Since buyers do not know the true state of nature, the obvious question is what fraction of them end up transacting at the correct price (p^{hh} when $\omega = H$ or p^{ll} when $\omega = L$). At any given point in time a player only knows that he has encountered a tough opponent in all of his previous rounds of trading. Since searching is costly due to the discount factor, a non-neglible fraction will transact at the wrong price. The question then becomes what happens as the frictions disappear. To model a market with arbitrarily small frictions, δ is sent to one. This allows for participants to search arbitrarily long. It does not guarantee complete information revelation; only a majority of players are given a signal that tells them the correct state is most likely. Also, in each round more uninformed players enter the market to further dilute the information present. To model this information revelation, let f_B be the fraction of buyers who in state L transact at p^{lh} or p^{hh}. To compute this quantity, notice that in any period the buyers who transact at the incorrect prices are the $K_L B_L^h$ buyers who play soft that period. This yields the equation:

$$f_B = \frac{K_L B_L^h}{M}$$

$$= \begin{cases} P(\sigma = H|\omega = H)(S_H^l)^{V^S(h)} + P(\sigma = L|\omega = H)(S_H^l)^{V^S(l)} & \text{if } \nu^B(H) + \nu^B(L) \in (0, \infty), \\ 1 & \text{if } \nu^B(H) + \nu^B(L) = 0, \\ 0 & \text{if } \nu^B(H) + \nu^B(L) = \infty. \end{cases}$$

A rational expectations equilibrium can only occur when f_B goes to zero as frictions become negligible. Conditions required for this to occur will be discussed and analyzed later.

3 The General Framework

Definitions

The basic setup is a market with two groups: buyers and sellers. The buyers are interested in purchasing exactly one unit of an indivisible good that the sellers all possess. The value of this good, denoted $\omega \in [0, 1]$, has a cumulative density function $F(\omega)$. Every buyer and seller recieves a signal $\sigma \in [0, 1]$ about the value of the objects distributed with cumulative density functions $G^S(\sigma|\omega)$, for sellers and $G^B(\sigma|\omega)$ for buyers. Without loss of generality, let $E[G^S(\sigma|\omega)] = E[G^B(\sigma|\omega)] = \omega$. This implies that any player who receives a signal will assume that the state is most likely equal to that signal. The marginal density functions $G^B(\sigma)$ and $G^S(\sigma)$ are defininen in the usual way, $G^i(\sigma) = \int G^i(\sigma|\omega)dF(\omega)$.

A player who transacts immediately exits the market. The utility of a buyer after exiting the market is $v(\sigma, \omega) - p$, while a seller obtains a utility of $p - c(\sigma, \omega)$. Also, let $v(\sigma) = \int v(\sigma, \omega)dF(\omega|\sigma)$ and $c(\sigma) = \int c(\sigma, \omega)dG(\omega|\sigma)$. Let $F, G^B, G^S, c, v \in C^2$. Let $v_\sigma(\cdot, \cdot) > 0$, $c_\sigma(\cdot, \cdot) > 0$, $v_\omega(\cdot, \cdot) \geq 0$ and $c_\omega(\cdot, \cdot) \geq 0$. This means that a player must have some private valued component coming from the private signal received and may have a common valued component, given the true state. Later this privately valued component will be made arbitrarily small to analyze the effect of a purely common valued problem.

Trading occurs in rounds $t \in \mathbb{Z}$. Each round, some mass of new players enter the market, then all players are matched up and bargain in an unspecified way. Those that choose to transact exit the market, along with some portion $(1 - \delta)$ with $\delta \in [0, 1)$ of untransacted players (the discount factor). Given some trading mechanism, the economy can be described by the distributions of F, G^B, G^S and the value of δ. The outcome of one of these economies is specified by $A = \{V^S(\sigma), V^B(\sigma), Q^S(\sigma), Q^B(\sigma)\}$ where $V^S(\sigma)$ is the expected payoff of a seller who receives signal σ, $V^B(\sigma)$ is the same for buyers, $Q^S(\sigma)$ is the probability that a seller with signal σ trades in his lifetime, and $Q^B(\sigma)$ the same for buyers. These are not state-dependent quantities; instead, they are expectations over the states of nature and the possible outcomes of the game in each state.

Notice that a player does not know the true state of nature. This means their strategy will not depend on what the state is, so a player will have the same strategy in two different states in which he receives the same signal. On the other hand, the final outcome will be different because there will be a different distrubtion of beliefs amongst the players. This means that while a player may have the same strategy in two different states, that strategy can lead to different outcomes.

Conditions

Definition (Mass Balance). *An outcome $A = \{V^S(\sigma), V^B(\sigma), Q^S(\sigma), Q^B(\sigma)\}$ satisfies mass balance if two things hold: the total mass of sellers who trade equals the total mass of buyers who trade, and the total transfers made by sellers equals the total transfers made by buyers. The first condition can be written as*

$$\int_0^1 Q^S(\sigma) dG^S(\sigma) = \int_0^1 Q^B(\sigma) dG^B(\sigma). \tag{3}$$

The second condition can be written as

$$\int_0^1 (v(\sigma)Q^B(\sigma) - V^B(\sigma))dG^B(\sigma) = \int_0^1 (V^S(\sigma) + c(\sigma)Q^S(\sigma))dG^B(\sigma). \tag{4}$$

A sequence $\{A_k\}_{k=1}^{\infty}$ *is said to satisfy mass balance if* A_k *satisfies it for all* k*.*

We can also define the ex ante trading surplus of some outcome A as

$$S(A) = \int_0^1 V^B(\sigma)dG^B(\sigma) + \int_0^1 V^S(\sigma)dG^S(\sigma).$$

Achieving efficiency is equivalent to maximizing surplus given the mass balance constraint. Let S^* denote the maximum surplus. This can be achieved with the Walrasian Allocation and is formalized below.

Definition (Monotonicity)**.** *A sequence* $\{A_k\}_{k=1}^{\infty}$ *is said to satisfy monotonicity if for all* k

$$Q_k^S(\cdot) \text{ is monotonically decreasing and } Q_k^B(\cdot) \text{ is monotonically increasing} \tag{5}$$

and

$$V_k^S(\cdot) \text{ is monotonically decreasing and } V_k^B(\cdot) \text{ is monotonically increasing} \tag{6}$$

I am going to denote pointwise limits with a bar above the function, so for example

$$\lim_{k\to\infty} Y_k^i(\sigma) = \bar{Y}^i(\sigma).$$

Definition (No Rent Extraction). *A sequence $\{A_k\}_{k=1}^{\infty}$ is said to satisfy no rent extraction if for all $\sigma_x \in [0,1]$ such that $\bar{Q}^S(\sigma_x)$ and $\bar{V}^S(\sigma_x)$ both exist and $\bar{Q}^S(\sigma_x) = 1$, then*

$$\liminf V_k^S(\sigma) \geq \bar{V}^S(\sigma_x) + c(\sigma_x) - c(\sigma)$$

for all σ in $[0,1]$. Also, if $\bar{Q}^B(\sigma_x)$ and $\bar{V}^B(\sigma_x)$ both exist and $\bar{Q}^B(\sigma_x) = 1$ then

$$\liminf V_k^B(\sigma) \geq \bar{V}^B(\sigma_x) + v(\sigma) - v(\sigma_x)$$

for all σ in $[0,1]$.

This statement can be interpreted as a bound on how different the expected payoff functions can be relative to an individual's utility function. This means that small differences in signals equate to small differences in expected utility. The last part refers to agents who always trade in the limit. This covers a wide range of people - sellers with zero cost, buyers with valuation 1, players whose optimal strategy is to offer the soft price immediately in the original example, etc.

Definition (Availability and Weak Pairwise Efficiency). *A sequence A_k satisfies availability and weak pairwise efficiency if whenever some pair of signals σ_x^B and σ_x^S satisfy*

$$\bar{Q}^B(\sigma_x^B) < 1 \text{ and } \bar{Q}^S(\sigma_x^S) < 1,$$

then for all pairs of signals σ^B, σ^S with $\sigma_x^B > \sigma^B$ and $\sigma_x^S < \sigma^S$, it is the case that

$$\bar{V}^S(\sigma^S) + \bar{V}^B(\sigma^B) \geq v(\sigma^B) - c(\sigma^S). \tag{7}$$

This concept entails two economically distinct conditions that are mathematically linked. Availibility requires that a player be frequently matched with traders who are *available*. These are players that do not trade with certainly and thus remain in the market for multiple periods. Weak Pairwise Efficiency requires that pairs of availalable traders combined expected payoffs are at least their surplus in the most-likely state.

The Main Conjecture

Theorem. *Any sequence of outcomes $\{A_k\}_{k,n=1}^{\infty}$ that satisfies mass balance, monotonicity, no rent extraction, availability, and weak pairwise efficiency converges to a Walrasian equilibrium. A sequence of these outcomes $\{A_{k,n}\}_{k,n=1}^{\infty}$ with payoff functions $v_n(\sigma, \omega)$ and $c_n(\sigma, \omega)$ converges to a fully revealing rational expectations equilibrium as values become common in the sense that*

$$\lim_{n \to \infty} \sup_{\omega_1, \sigma_1, \sigma_2} v_n(\sigma_1, \omega) - v_n(\sigma_2, \omega) + c_n(\sigma_1, \omega) - c_n(\sigma_2, \omega) = 0.$$

An outline of the proof is presented in the appendix. It relies on several lemmas. The first one shows that the assumptions lead to an interim efficiency result. The next lemma shows that this efficiency result is also ex-post efficient. The final lemma shows that an efficent allocation leads to all trades occuring at the Walrasian price. The main proof then concludes that when the private valued components are removed from the payoff functions, this becomes a rational expectations equilibrium. I start with the lemmas.

Lemma. *If some sequence $\{A_k\}_{k=1}^{\infty}$ satisfies mass balance, monotonicity, no rent extraction, availability, and weak pairwise efficiency, then*

$$\bar{V}^S(\sigma) + \bar{V}^B(\sigma) \geq v(\sigma) - c(\sigma).$$

Lemma. *If*

$$\bar{V}^S(\sigma) + \bar{V}^B(\sigma) \geq v(\sigma) - c(\sigma))$$

for all signals σ then

$$\lim_{k \to \infty} S(A_k) = S^*.$$

Lemma. *If $S(A) = S^*$ then all trading occurs at the Walrasian equilibrium price.*

4 Applications

The next step in the analysis is to look at the application of the main conjecture to the illustrative model introduced earlier. Consider a sequence of equilibria A_k with $\delta \to 1$ as $k \to \infty$. The next sections will show what conditions are needed for the simple model to follow the axioms and thus converge to a rational expectations equilibrium.

Mass Balance

Mass balance is satisfied in a sequence of equilibria whenever (3) and (4) hold. The first is trivially true. To show the second condition holds, first notice that everyone eventually plays soft in an equilibrium. Thus $Q^i(\sigma) = 1$. The condition can then be expressed as:

$$(v(H) - V^B(H))P(\sigma = H) + (v(L) - V^B(L))P(\sigma = L)$$

$$= (c(H) + V^S(H))P(\sigma = H) + (c(L) + V^S(L))P(\sigma = L).$$

This expression can be rewritten as

$$\begin{aligned} 0 = & \left([v(H) - c(H)] - [V^B(H) + V^S(H)]\right) P(\sigma = H) \\ & + \left([v(L) - c(L)] - [V^B(L) + V^S(L)]\right) P(\sigma = L) \end{aligned} \tag{8}$$

Thus mass balance is satisfied whenever (8) - an equation based on exogenously determined variables - is satisfied.

Monotonicity

Since buyers and sellers will always eventually trade in both states, $Q^S_k = Q^B_k = 1$ for all k, giving us a trivially monotonic function.

No Rent Extraction

To show that A_k satisfies no rent extraction, it is necessary to show that whenever $\bar{Q}^B(\sigma_x)$ and $\bar{V}^B(\sigma_x)$ both exist and $\bar{Q}^B(\sigma_x) = 1$, then

$$\liminf V^S_k(\sigma) \geq \bar{V}^S(\sigma_x) + c(\sigma_x) - c(\sigma)$$

for all σ. Understanding when this holds first requires knowing if either signal would have people always trade in the limit. Say buyers with both signals trade in the limit. The condition trivially holds true for $\sigma_x = \sigma$ (which is why the seller case is trivial) so the only relevant case is when $\sigma_x = H$ and $\sigma = L$. Thus I need to show that

$$\liminf V_k^B(L) - \bar{V}^B(H) - [v(H) - v(L)] \geq 0$$

First,

$$\liminf V_k^B(L) - \bar{V}^B(H) - [v(H) - v(L)]$$

$$\geq P(\omega = H|\sigma = H)(v_h - p^{hh}) + P(\omega = H|\sigma = H)(v_l - p^{hh}(1 - \liminf f_k^B) + p^{hl} f_b)$$

$$-P(\omega = H|\sigma = L)(v_h - p^{hh}) - P(\omega = L|\sigma = L)(v_l - p^{ll}(1 - \liminf f_k^B) + p^{hl} f_b)$$

$$-\left[v_H P(\omega = H|\sigma = H) + v_L P(\omega = L|\sigma = H) - v_H P(\omega = H|\sigma = L) - v_L P(\omega = L|\sigma = L)\right].$$

Simplifying this expression leads to a condition:

$$p^{hh}\left(1 - \liminf f_k^B\left(P(\omega = L|\sigma = L) - P(\omega = L|\sigma = H)\right)\right)$$

$$+\liminf f_k^B\left(p^{ll}P(\omega = L|\sigma = L) - p^{lh}P(\omega = L|\sigma = H)\right)$$

$$-p^{hh}\left(P(\omega = H|\sigma = H) - P(\omega = H|\sigma = L)\right) \geq 0.$$

Thus no rent extraction is satisfied whenever the coefficents (all of which are exogenously chosen) satisfy (4) for buyers.

Avalability and Weak Pairwise Efficiency

This is trvially true because there exists only one signal for sellers, so even if sellers would not always trade in the limit, no better signal exists to satisfy the conditions.

Convergence to REE

The main conjecture applies to the illustrative model as long as (8) and (4) are satisfied with the exogenous parameters. The convergence to common values comes from a sequence of ϵ_B that goes to zero. In the limit the game consists of entirely common values with a Walrasian Allocation. This implies the model is at a rational expectations equilibrium.

5 Concluding Remarks

This paper has examined a broad generalization of matching and bargaining games. I have conjectured a set of sufficient conditions for these games to converge to a rational expectations equilibrium. If this can be proven it will show a remarkable fact: a very diverse set of models all result in information being fully revealed through market forces.

There are numerous opportunities for future work to be done in this area. Obviously the main conjecture needs to be proven. The conditions required for convergence are also quite technical: a more economically based set would be a great help in explaining the results. Also, once the conjecture is proven an investigation on information revelation would be enlightening. A rational expecations equilibrium is fully revealing, but the conjecture gives no intution as to how information is propogated.

Dynamic matching and bargaining games are excellent tools to model efficiency decentralized markets. This conjecture would lead to an invaluable understanding of when efficiency is achieved and how much information is spread.

Appendix: Proof Ideas

This appendix sketches out the proofs of the main theorem and its lemmas. Note that they are not entirely correct but known flaws and gaps will be pointed out.

Lemma 1

Proof. The monotonicity condition and the no rent extraction condition require that all components of A_k are monotone. Thus by Helly's selection principle there exists a pointwise convergent subsequence $\{A_{k'}\}_{k=1}^{\infty}$ that converges to $\{\bar{V}^S, \bar{V}^B, \bar{Q}^S, \bar{Q}^B\}$.

For the sequence A_k define the signals σ^B and σ^S as the two signals such that

$$\sigma^S = \inf\{1, (\sigma^S | \bar{Q}^S(\sigma^S) < 1)\} \text{ and } \sigma^B = \sup\{0, (\sigma^B | \bar{Q}^B(\sigma^B) < 1)\}.$$

It is the case that for all σ

$$\bar{V}^S(\sigma) \geq \bar{V}^S(\sigma^S) + (c(\sigma^S) - c(\sigma)),$$

and

$$\bar{V}^B(\sigma) \geq \bar{V}^B(\sigma^B) + (v(\sigma, \sigma) - v(\sigma^B, \sigma^B)).$$

For the first inequality there are three cases. The trivial case is when $\sigma = 0$. The second case is if $\sigma \in [\sigma^S, 1]$. In this case $(c(\sigma^S) - c(\sigma))$ is negative, so it follows directly from the no rent extraction condition. Finally, if $\sigma \in (0, \sigma^S)$ then choose some $\epsilon \in (0, \sigma^S)$. From the definition of σ^S it is the case that $\bar{Q}^S(\sigma^S - \epsilon) = 1$. Then for all $\sigma \leq \sigma^S - \epsilon$ the no rent extraction condition implies that $\bar{V}^S(\sigma) \geq \bar{V}^S(\sigma^S) + (c(\sigma^S) - c(\sigma - \epsilon,))$. By continuity, $\bar{V}^S(\sigma) \geq \bar{V}^S(\sigma^S) - (c(\sigma^S) - c(\sigma))$. The second inequality follows by symmetry.

Adding the two probabilities together yields:

$$\bar{V}^S(\sigma) + \bar{V}^B(\sigma) \geq \bar{V}^S(\sigma^S) + \bar{V}^B(\sigma^B) + (v(\sigma) - c(\sigma)) - (v(\sigma^B) - c(\sigma^S)).$$

The next step is to show that the right hand side is greater then or equal to $v(\sigma) - c(\sigma)$. Consider the case where $\sigma^S < \sigma^B$ and let $0 < \epsilon < \sigma^B - \sigma^S$. By monotonicity $\bar{Q}^S(\sigma^S + .5\epsilon) < 1$ and $\bar{Q}^B(\sigma^B + .5\epsilon) < 1$. By availability and weak pairwise efficiency

$$\bar{V}^S(\sigma^S + \epsilon) + \bar{V}^B(\sigma^B + \epsilon) \geq v(\sigma^B + \epsilon) - c(\sigma^S + \epsilon).$$

By continuity

$$\bar{V}^S(\sigma^S) + \bar{V}^B(\sigma^B) \geq v(\sigma^B) - c(\sigma^S).$$

If $v(\sigma^B) \leq c(\sigma^S)$. Then $v(\sigma^B) - c(\sigma^S) \leq 0$ and since utility is non-negative the same inequality holds. Thus

$$\bar{V}^S(\sigma) + \bar{V}^B(\sigma) \geq v(\sigma) - c(\sigma).$$

□

Lemma 2

Proof. Let P^w be the Walrasion price. $\bar{p} \equiv \inf_{\sigma<p^w}(\bar{V}^S(\sigma)+c(\sigma,\sigma))$. Since $\bar{V}^S(\sigma)+\bar{V}^B(\sigma) \geq v(\sigma,\sigma)-c(\sigma,\sigma)$ for all σ it follows that $V^B(\sigma) \geq v(\sigma,\sigma)-\bar{p}$. Thus:

$$S(A) \geq \int_{p^w}^{1} V^B(\sigma)dG^B(\sigma) + \int_{0}^{p^w} V^S(\sigma)dG^S(\sigma)$$

$$\geq \int_{p^w}^{1} (v(\sigma)-\bar{p})dG^B(\sigma) + \int_{0}^{p^w} (\bar{p}-c(\sigma))dG^S(\sigma).$$

By the definition of the Walrasian Price, maximum surplus is achieved so

$$= S^* + \bar{p}(G^S(p^w) - (1-G^B(p^w))) = S^*.$$

The other inequality is trivial, giving us equality. □

This proof suffers from two difficutlies. First of all, the surplus and efficiency are defined at an interim state as expectations over all possible states. This leads to the difficulty of finding a solution concept that makes sense in such a setting. Since the efficiency is only shown in expectations, there could exist states in that supply exceeds demand which are compensated for by other states in which demand exceeds supply. Such an "equilibrium" is awkward at best.

Lemma 3

This lemma is basically the second welfare theorem. A modified version of the proof which allows for incomplete information should be sufficient for this lemma.

Main Theorem

Proof. According to the lemmas, $\{A_{k,n}\}_{k=1}^{\infty}$ converge to a Walrasian equilibrium for any choice of n. As $n \to \infty$ the situation approximates a common values setup. This means that the critical signal for trading goes to 0 and 1 for sellers and buyers respectively. Thus everyone in the market will be trading at the Walrasian price. By the setup of the model, this will also be the true price of the object giving a fully revealing rational expectations equilibrium. $\square$

References

[1] Max Blouin and Roberto Serrano. A decentralized market with common values uncertainty: Non-steady states. *The Review of Economic Studies*, 68(2):323–346, 2001.

[2] Drew Fudenberg and Jean Tirole. *Game Theory*. The MIT Press, 1991.

[3] Piero Gottardi and Roberto Serrano. Market power and information revelation in dynamic trading. *Journal of the European Economic Association*, 3(6):1279–1317, 2005.

[4] Takashi Kunimoto and Roberto Serrano. Bargaining and competition revisited. *Journal of Economic Theory*, 115(1):78–88, 2004.

[5] Stephan Lauermann. Dynamic matching and bargaining games: A general approach. 2006.

[6] Philip Reny and Motty Perry. Toward a strategic foundation for rational expectations equilibrium. *Econometrica*, 74(5):1231–1269, 2006.

[7] Roberto Serrano and Oved Yosha. Information revelation in a market with pairwise meetings: The one sided information case. *Economic Theory*, 3(3):481–499, 1993.

[8] Roberto Serrano and Oved Yosha. Welfare analysis of a market with pairwise meetings and asymetric information. *Economic Theory*, 8(1):167–175, 1996.

[9] Asher Wolinsky. Information revalation in a market with pairwise meetings. *Econometrica*, 58(1):1–23, 1990.