The Hand Rule and *United States v. Carroll Towing Co.* Reconsidered

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Judge Learned Hand's opinion in *United States v. Carroll Towing Co.* (1947) is canonized in the law-and-economics literature as the first use of cost-benefit analysis for determining negligence and assigning liability. This article revisits the case in which the Hand formula was born and examines whether Judge Hand’s ruling in that case would provide correct incentives for efficient levels of precaution. We argue that the negligence test as used by Judge Hand is somewhat different from the Hand test as used by modern law-and-economics theorists. With a game theoretic analysis of the case, we show that Judge Hand’s negligence test could in fact produce games with inefficient equilibria, or with liability determinations opposite Judge Hand’s.

1. Introduction

Judge Learned Hand’s opinion in *United States v. Carroll Towing Co.*¹ is canonized in the law-and-economics literature. It is like the opening.

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¹ Note that this case stems in part from an appeal of *Connors Marine Co. v. Penn. R. Co.*, decided by Judge Moscowitz. We will rely on Judge Moscowitz’s opinion for some factual information.
measure of Beethoven’s Fifth Symphony, or the third line of the Bible: “And God said, Let there be light.” What is mainly remembered, of course, is the judge’s formula (p.173):

Since there are occasions when every vessel will break from her moorings, and since, if she does, she becomes a menace to those about her; the owner’s duty, as in other similar situations, to provide against resulting injuries is a function of three variables: (1) The probability that she will break away; (2) the gravity of the resulting injury, if she does; (3) the burden of adequate precautions. Possibly it serves to bring this notion into relief to state it in algebraic terms: if the probability be called \( P \); the injury, \( L \); and the burden, \( B \); liability depends upon whether \( B \) is less than \( L \) multiplied by \( P \): i.e., whether \( B < PL \).

Thus the judge throws mathematics and rational cost-benefit analysis into the law, and the theme has continued to play for 58 years in the literature of economic analysis of liability rules.

As the economic analysis now stands, it is widely believed that a determination of each party’s negligence should be based on some cost-benefit test like Judge Hand’s: in essence, if a party’s cost of preventing accidents (what the judge calls the burden) is less than the expected losses from the accidents, that party should be found negligent; and it is also widely believed that, under certain general conditions, a negligence-based liability rule (like simple negligence, negligence with a defense of contributory negligence, or comparative negligence) will lead rational victims and injurers to choose efficient levels of care.²

The purpose of this article is to revisit the United States v. Carroll Towing Co. case, and Judge Hand’s opinion. We will examine the Hand rule in that particular case. We will argue that there are three different ways to apply Judge Hand’s test, and that the facts of United States v. Carroll Towing Co. suggest Judge Hand must have been assuming one of these three ways, which we call the Hand rule contingent on the other party’s actual behavior. We will show that, under this interpretation of the Hand rule, the circumstances described in the United States v. Carroll Towing Co. opinion might produce games with efficient equilibria, as

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² See Brown (1973), Cooter and Ulen (2000), Landes and Posner (1987), Rea (1987), and Shavell (1987). We will use Hand test and Hand rule interchangeably, to denote Judge Hand’s algebraic method for determining negligence. A liability rule is a rule for allocating losses between parties, and may or may not be based on the parties’ negligence.
law-and-economics theorists would want, but they might also produce games with inefficient equilibria. The conclusion is that, contrary to the usual claim, the Hand rule might not be a key to efficiency, even in circumstances like those described by Judge Hand in *United States v. Carroll Towing Co.*

In section 2 we describe the *United States v. Carroll Towing Co.* case. In section 3 we discuss three ways to approach the Hand test. In section 4 we analyze the *United States v. Carroll Towing Co.* game model. In section 5 we make concluding remarks.

2. Structure of the Case

*United States v. Carroll Towing Co.* grew out of an accident that took place in New York Harbor on January 4, 1944. The parties involved were as follows:

- Conners Marine Company, Inc., owner of the covered barge *Anna C.*, and employer of the barge attendant, the “bargee.”
- Pennsylvania Railroad Company, charterer of the barge *Anna C.*
- The United States, owner of a cargo of flour aboard the *Anna C.*
- Carroll Towing Company, Inc., owner of the steam tug *Joseph F. Carroll*, and employer of the tug master and the tug deckhand.
- Grace Line, Inc., charterer of the tug *Carroll*, and employer of the harbormaster.

Litigation resulting from the accident was appealed from the District Court (Eastern District, New York, Judge Moscowitz), to the Federal Second Circuit Court of Appeals, Judges L. Hand, Chase, and Frank presiding. Judge Hand wrote the appeals court opinion.

Some relevant facts of the case, recounted in Judge Hand’s opinion, or in Judge Moscowitz’s earlier *Connors Marine Co. v. Penn. R. Co.* opinion, are as follows: The Pennsylvania Railroad Co. had chartered the covered barge *Anna C.* from Conners Marine Co., and the charter arrangement included the services of a bargee attendant, provided by Conners Marine Co., between the hours of 8 a.m. and 4 p.m. On or before January 2, 1944, the barge was loaded with a cargo of flour owned by the United States. Sometime on or before January 4, 1944, the barge was moored to the end of pier 52 on the North River. At some time after *Anna C.* was moored,
five other barges were moored outside her—that is, on the river side—making a tier of six barges at the end of pier 52. The next pier north of pier 52 was the public pier. On January 4, 1944, the tug *Carroll* was sent to the public pier to “drill out” a barge; that is, to remove one barge from the tier of four barges at the end of that pier. On board the *Carroll* were her master and a deckhand, employed by Carroll Towing Co., and also a harbormaster, employed by the chartering company, Grace Line, Inc. There was a line connecting the two tiers of barges on the two piers, and the *Carroll* had to throw off this line in order to “drill out” the barge it was after on the public pier. The master of the *Carroll* put his deckhand and the harbormaster on the pier 52 tier of barges (including the *Anna C.*), to ensure that these barges were safely moored, before throwing off the connecting line. The deckhand and the harbormaster went aboard these barges, readjusted their fasts, threw off the line connecting the two tiers of barges, and then reboarded the tug *Carroll*. Shortly thereafter, the tier of barges on pier 52 broke adrift. Around 2:15 p.m. the *Anna C.* struck a tanker, sprang a leak, careened, dumped her cargo, and sank. At the time this occurred, the bargee who was supposed to be on board the *Anna C.* was absent.

The bargee’s absence was crucial and deserves additional comment. Judge Hand is clear that if he had been on board, “and had done his duty to his employer,” the bargee would have seen that the *Anna C.* was leaking after she collided with the tanker; he would have been able to get help from the tugs in the vicinity, and he would have been able to prevent her sinking and the loss of her cargo. On the other hand, Judge Hand is doubtful that the bargee’s absence contributed to the pier 52 barge flotilla’s breaking away in the first place, because if he had been on board “there is not the slightest ground for saying that the deckhand and the harbormaster would have paid any attention to any protest [regarding the fasts] which he might have made.” Judge Hand separates the damages into two types: “collision damages” sustained by the *Anna C.* when she broke her fasts, drifted, and struck the tanker;

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3. Oddly, although central in Judge Hand’s opinion, the bargee’s absence is not noted in Judge Moscowitz’s opinion. Also oddly, the captain of the *Anna C.* may have been absent during some of these events, which was remarked upon by Judge Moscowitz, but not by Judge Hand.
and “sinking damages,” to the Anna C. and the owner of her cargo, when she careened, dumped the cargo and sank. He found that the bargee’s absence contributed to sinking damages but not to collision damages. Judge Hand is also clear that the bargee’s absence was inexcusable: the bargee had been away since 5 p.m. of the previous day, “and we hold that his fabricated story was affirmative evidence that he had no excuse for his absence.”

This complex accident gave rise to several issues of negligence and liability. Was Carroll Towing Company, owner of the tug and employer of her master and deckhand, negligent because of inadequate readjustment of the fasts connecting the Anna C. and the five other barges to pier 52? Was Grace Line, charterer of the Carroll and employer of the harbormaster, negligent because of inadequate readjustment of the fasts? And was Conners Marine Company, owner of the Anna C., negligent because its employee the bargee was absent from his post? There was no claim that the United States, owner of the cargo, was negligent. And there was the additional complication of the two types of damages already mentioned. So the questions became as follows: who was negligent in the creation of what damages? And how should the damages be parcelled out among the various parties, under the governing liability rule? The liability rule was the “equal division” rule, from pre-1975 U.S. Admiralty law. In a simple single-defendant and single-plaintiff case, this would put all damages on the negligent party if one party was negligent and the other was not, and would split damages equally if both were negligent.

Ultimately the appeals court apportioned the damages as follows: Of the “collision damages,” one-half was put on Grace Line, the charterer of the tug and employer of the harbormaster, and one half was put on Carroll Towing Company, the owner of the tug and employer of the master and the deckhand. None was put on Connors Marine Company. Of the “sinking damages,” one-third was put on Grace Line, one-third on Carroll Towing Company, and one-third on Conners Marine Company, the owner of the barge Anna C. and employer of the bargee. But why?

3. Three Ways to Apply Judge Hand’s Test

Judge Hand’s test is $B < PL$: the burden of adequate precaution is less than the product of the accident probability and the loss in case of an
accident. If the inequality holds, the party to whom the test is being applied is negligent. But this is really a much more complicated issue than the simple inequality suggests. To explain the complications, let us start by changing Judge Hand’s notation in a minor way. Instead of using $B$ for burden we will, from this point onward, use $c$ for cost. And we observe immediately that most litigated accidents involve at least two parties, an injurer (defendant) and a victim (plaintiff).

However, note that United States v. Carroll Towing Co. involves five parties, multiple plaintiffs and defendants. Like Judge Hand, we will focus on the plaintiff who had a choice to make, namely Connors Marine, owner of the barge and employer of the bargee. We will not discuss the United States, the owner of the cargo, whose name is on the case, because its role was purely passive. And we will focus on one defendant, namely Carroll Towing, employer of the tug master, and employer of the deckhand who (along with Grace Line’s harbormaster) adjusted lines and fasts without sufficient care. We are in effect lumping together the two defendants Carroll Towing and Grace Line. We could treat them separately, but this would greatly complicate the exposition, with no gain in insight.

In many accidents (including this one), preventive actions might have been taken by the defendant or by the plaintiff. In the abstract, we use $c_I$ to represent the cost to an injurer of taking a preventive action, and $c_V$ to represent the cost to a victim of taking a preventive action. Applying this abstract notation to this case, we say that $c_I$ is a cost Carroll Towing might incur to prevent accidents, and $c_V$ is a cost Connors Marine might incur. The Hand test can be applied to the injurer: $c_I < PL$; or to the victim: $c_V < PL$.

Several theoretical questions come up about the variables $c_I$, $c_V$, and $P$, and the connections among them. These questions might or might not be relevant to facts of any particular case, including this one. The first is whether the preventive actions, and the costs $c_I$ and $c_V$ of taking such actions, are continuously variable choices, or discrete choices. Is there an infinite range of possible preventive actions, or are there only one, or two, or a few actions? To put it another way, can the party at hand take any degree of care (or spend any amount of money) from $0$ to $100,000, say, or is it a matter of taking one particular action or not? In the case of United States v. Carroll Towing Co., Judge Hand applies his formula to Connors Marine and its employee, the disappearing bargee of the Anna C. The bargee’s presence or absence appears to us to be a clearly discrete and
dichotomous choice. The bargee could have been on board, or not, and he was not. In his opinion Judge Hand does not discuss whether Connors Marine might have spent a little more or a little less to employ slightly more responsible or slightly less responsible bargees. We think it would be artificial to assume that \( c_V \) is anything other than dichotomous. Our approach here is somewhat similar to that of Grady (1989), who argues that “untaken precautions,” such as absent bargees, are central in negligence law.

The nature of \( c_I \) is not quite so clear. There may well have been several degrees of care that might have been taken by Carroll Towing’s (and Grace Line’s) employees. However, Judge Hand clearly finds that Carroll Towing (and Grace Line) were in fact negligent (as had Judge Moscowitz), and so, whatever the nature of the \( c_I \) variable, we can be sure that Judge Hand thought the chosen \( c_I \) was too low. It is also clear that some greater degree of care on the part of defendants would have prevented the accident. Judge Moscowitz had opined (p. 398): that “if the lines of the Anna C had been properly adjusted by the harbormaster and the deckhand or if another tug had been used, the accident would have been avoided.” Consistent with this fact, and to allow for as simple a mathematical model as possible, we will assume in what follows that Carroll Towing’s \( c_I \) was also dichotomous, and that Carroll Towing chose the lower value for \( c_I \), thereby creating the possibility of barges breaking away.

The second theoretical question is whether or not there is some meaningful interaction between actions that might be taken, or, in the abstract model, the amounts \( c_I \) and \( c_V \) that are expended. In the standard modern liability rule model, the three variables are explicitly interconnected, with \( P \) a function of \( c_I \) and \( c_V \), often a continuous and differentiable function. Judge Hand’s opinion allows us to draw limited inferences about what the connections might have been among \( P, c_I \) and \( c_V \). We will lay these out in section 4.

The third question is related to the first two, and has to do with how the Hand inequalities (for the injurer or for the victim) are applied. As we see it, there are three different ways to apply the Hand tests. Note that these three ways are described here in the abstract; we will refocus on \textit{United States v. Carroll Towing Co.} in section 4. Also note that, no matter how it is applied, a Hand test is a test only for negligence; liability is a separate issue that depends on the legal rule in place.
3.1. Independent Application

Sometimes it is appropriate to apply the two tests \( c_I < PL \) and \( c_V < PL \) independently. In particular, suppose \( c_I \) and \( c_V \) are dichotomous variables. That is, the injurer can either take some specific care (and spend \( c_I > 0 \)), or take no care (and spend, say, \( 0 \)); the victim can either take some specific care (and spend \( c_V > 0 \)), or take no care (and spend, say, \( 0 \)). Suppose that if neither takes care, the accident probability is \( P \). Suppose that if either the injurer takes care (and spends \( c_I \)), or the victim takes care (and spends \( c_V \)), the accident probability is reduced to zero. This is the perfect prevention, alternative care case.

Under these circumstances, the Hand test can be applied to the injurer—in essence, \( c_I < PL \)—independently of the behavior of the victim, or to the victim, independently of the behavior of the injurer. That is, the question of party A’s negligence or nonnegligence can be determined with no reference to party B’s behavior.4

3.2. Conditional Application, Contingent on the Other Party’s Actual Behavior

Sometimes it is natural to make the injurer’s test \( c_I < PL \) contingent on the victim’s actual action or inaction, and the victim’s test \( c_V < PL \) contingent on the injurer’s actual action or inaction. For a numerical example, assume that if neither party acts, that is, if \( c_I = c_V = 0 \), then \( P = .1 \), \( L = 1,000 \), and \( PL = 100 \). Suppose that if the injurer acts to prevent an accident, at a cost of \( c_I = 60 \), the accident probability drops to 0. Suppose that if the victim acts, at a cost of \( c_V = 40 \), the effect is to reduce the accident probability by half. (We are again assuming the victim’s and the injurer’s choices are both dichotomous.)

Suppose that when the court does a Hand test calculation for one party, say A, it can take into account what both parties have actually spent, and what would have happened if party A had spent more. But it cannot determine what would have happened if A had spent more and B were doing something other than what he was actually doing.

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The logic of the Hand test is now as follows. First, assume both parties take no care; that is, \( c_I = c_V = 0 \), and an accident occurs. Apply the Hand test to the injurer: he could have spent $60 instead of $0 to prevent accidents, \( 60 < PL = 100 \), and therefore he is negligent. Now apply the test to the victim: he could have spent $40 instead of $0; this would have reduced expected accident costs by half, \( 40 < DPL = 50 \), and therefore the victim is also negligent. (Note that the right-hand side of the inequality has the change in the accident probability \( \Delta P \), times the loss \( L \), rather than \( P \) times \( L \).) Second, suppose \( c_I = 60 \) and \( c_V = 0 \). Then the accident probability is zero, there are no accidents and no lawsuits, and further action by the victim is redundant. Third, suppose \( c_I = 0 \) and \( c_V = 40 \), and an accident occurs. The accident probability was already reduced to .05; and expected accident costs were .05 \( \times \) \$1,000 = \$50. The question becomes as follows: given that the victim already spent \$40, should the injurer be obliged to spend \$60 to further reduce expected costs, by \$50, to zero? The economic answer is “no”; it makes no sense to spend \$60 to reduce accident costs by \$50. The Hand test for the injurer is now \$60 versus \$50, and he is nonnegligent even though he “spent” only \$0.

In a general model with \( c_I \) and \( c_V \) possibly multivalued, and possibly interactive in their effects on \( P \), the Hand test contingent on the other party’s actual behavior would operate as follows: The injurer is negligent if, given the victim’s actual choice of \( c_V \), an incremental expenditure on precaution by the injurer would reduce the expected accident losses by more than that incremental expenditure. (In a discrete model the increment would be discrete and in a continuous model the increment would be infinitesimal.) The definition of the victim’s negligence is similar.

### 3.3. Conditional Application, Contingent on the Other Party’s Efficient Behavior

Following is the usual approach of the standard modern liability rule model, since Brown (1973). Assume that the injurer, the victim, and the

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5. Brown (1973) also examines the conditional-based-on-actual-behavior approach, which he calls the limited information approach. But his principal results are based on the conditional-efficient approach, and the limited information approach has long been ignored in the literature. In his recent article on the Learned Hand rule (Brown, 1998), he does not mention the limited information approach. For an extensive formal analysis of the conditional-based-on-actual-behavior approach and its implications, see Kim (2003).
court are wise enough to know exactly how the accident probability $P$ (and possibly the extent of the loss $L$) depends on the choices $c_I$ and $c_V$. Assume that all are able to recognize that total expected costs to society from accidents of the type being litigated are given by the formula $c_I + c_V + PL$. Assume that there is a unique pair of values $c^*_I$ and $c^*_V$, the efficient care levels, that minimize total expected costs to society from these accidents. Finally, assume that everyone knows these efficient care levels.

Now it is natural to make the Hand test for injurer’s negligence, $c_I < PL$, contingent on $c^*_V$, and the test for victim’s negligence, $c_V < PL$, contingent on $c^*_I$. That is, the court can make the test for each party contingent on the efficient behavior of the other party. For instance, when the court is determining whether or not the injurer is negligent, it can first determine the efficient level of care for the victim, and it can then judge the injurer’s care as if the victim were choosing his efficient level.

This would work as follows in the numerical example from the preceding subsection. Note that the efficient choices in the example are for injurer to spend $c^*_I = $60, and for victim to spend $c^*_V = $0. First, assume both parties take no care, that is, $c_I = c_V = $0, and an accident occurs. The Hand test is applied to the injurer: since $c_V = c^*_V = $0, the test works exactly as in the previous subsection, and the injurer is found negligent. Next, apply the Hand test to the victim. But this test is made contingent not on the actual expenditure $c_I = $0 of the injurer, but on the efficient $c^*_I = $60 of the injurer. The court ignores the fact that the injurer is spending nothing. It calculates what would have happened if the injurer had been spending his efficient $60. And here we have a result that is different from that of the previous subsection, because if the injurer were spending the efficient $60 for care, then it would make no sense for the victim to spend $40, because that $40 would buy only a $0 reduction in expected accident losses. Therefore, the victim is nonnegligent by the Hand test. Second, suppose $c_I = $60, $c_V = $0. Then there are no accidents and no lawsuits. Third, suppose $c_I = $0, $c_V = $40, and an accident occurs. Apply the Hand test to the injurer. Again we have a result contrary to the previous subsection.

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6. To see how the case where $c^*_I$ and $c^*_V$ are not unique might be treated, see Jain and Singh (2002).
because now the court judges the injurer’s behavior contingent on the efficient behavior of the victim, namely $c_v = 0$. The court ignores the fact that the victim is already spending $40. Instead, it calculates the hypothetical benefit that would occur if the injurer increased his spending from $0$ to $60, \text{ and if the victim were spending the efficient } 0 \text{ instead of the actual } 40. \text{ This hypothetical benefit is } 100. \text{ Therefore the injurer is negligent by the Hand test.}

Before leaving this section we will note the following: in the standard tort liability theory, where everything is known to everybody, and where the benevolent judicial system is calculating a unique efficient pair $c_i$ and $c_v$ to minimize total expected social costs $c_i + c_v + PL$, the Hand test contingent on efficient behavior is equivalent to the following: injurer is negligent if and only if $c_i < c_i^*$; and victim is negligent if and only if $c_v < c_v^*$. That is, each party’s standard of care is set at the efficient level.


4.1. Modeling the Case with a Two-Player Dichotomous Choice Game

We now return to United States v. Carroll Towing Co. As already noted, we are focusing on defendant Carroll Towing as the injurer; we are abstracting from the similar role of Grace Line. Carroll Towing was not sufficiently careful in adjusting the fasts mooring the Anna C. and the five other barges to the end of pier 52. We are focusing on plaintiff Connors Marine as the victim. Connors Marine was negligent, in Judge Hand’s opinion, because its employee the bargee was absent.

Note that in this model we assume that only Connors Marine could be the victim and only Carroll Towing could be the injurer. That is, there is no uncertainty as to what role each party would play in an accident. This pair of assumptions is reasonably consistent with the circumstances of this case. Carroll Towing owned the tug; tugs move vessels around and might well cause harm. Connors Marine owned the barge; barges do not have engines, are more passive, and are less likely to precipitate accidents. In Kim and Feldman (2003), we analyze a model where the central question is, “who is injurer / who is victim?”
We now flesh out our care model and modify our care cost notation as follows. We assume Carroll Towing could have done one of two things: it might have been sufficiently careful in adjusting the fasts mooring the Anna C. and the other barges to pier 52. We call this choice Careful. We let $c_I$ represent the cost of this choice. If Carroll Towing had been Careful, we assume the breakaway probability would have been zero. That is, Carroll Towing could have perfectly prevented the accident, if it had spent $c_I$. Judge Hand was not absolutely clear about this, but Judge Moscowitz was (see his quotation in section 3). Alternatively, Carroll Towing could have been (and in fact was) insufficiently careful, a choice we call Careless. We assume that the cost of this choice is $0$. (In reality, Carroll Towing’s employees did exercise some care, but it was not enough. We could assume a positive cost for this insufficient care, but it complicates matters and makes no difference to the results.) We assume Connors Marine could have done one of two things. It could have had a bargee aboard the Anna C. We call this choice Bargee. Alternatively, it could have had no bargee aboard, a choice we call No Bargee. We let $c_V$ represent the cost of having the bargee aboard, and we assume the cost of No Bargee is $0$. (Unlucky Connors Marine of course paid $c_V$ but ended up with no bargee, because of the dereliction of its employee.)

When Carroll Towing is being Careless, there is some given positive probability $P$ that the tier of barges will break away from pier 52. We assume that if the tier of barges breaks away there will at least be a collision, and, if no bargee is around, a sinking. As in the abstract model, an $L$ stands for a loss, but we must complicate this because of Judge Hand’s distinction between collision damages—that is, cost of repairs to the Anna C. had she collided with the tanker but been prevented from sinking, and sinking damages, the cost of salving the Anna C., and the value of the lost cargo. We denote the dollar amount of the collision damages $L_C$, and the dollar amount of the sinking damages $L_S$.

We find two basic inequalities in Judge Hand’s opinion, one explicit and the other implicit: First, if the victim Connors Marine had had its bargee

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7. It is also possible to assume that the accident probability would be reduced by the injurer’s careful mooring job, but not to zero. This would be “imperfect prevention.” This assumption would change the game’s payoff structure and make the analysis significantly more complicated, but the main conclusion of this section—that the game may lead to inefficient equilibria—would remain intact.
on board the *Anna C.* there would still have been collision damages, but sinking damages would have been prevented. Judge Hand applies his formula to the victim’s behavior, and concludes that Conners Marine was negligent with regard to *sinking* damages because it failed his test. To translate Judge Hand’s notation to ours, he asserts in his famous inequality that \( c_V < P L_S \). Therefore, we assume this inequality in what follows. Second, if Carroll Towing had chosen *Careful*, the probability of an accident would have dropped to zero, and neither type of damage would have occurred. But it chose *Careless*, and an accident happened. In Judge Hand’s opinion, Carroll Towing’s negligence is taken for granted. The judge does not bother to direct his test toward the injurer, but we can be sure that if he had he would have found \( c_I < P (L_C + L_S) \). We assume this inequality must also hold in what follows.

In Table 1 we show total social costs, defined as the sum of caretaking costs of each party plus the expected accident losses, for each of the four possible scenarios, in this interaction between the victim and the injurer. The four scenarios are the four combinations of actions: *(Bargee, Careful), (Bargee, Careless), (No Bargee, Careful), and (No Bargee, Careless).* Note that Table 1 is not a payoff matrix for the liability game, which will follow in Tables 2 and 3. We have not yet incorporated the liability rule. Table 1 merely allows us to determine what is efficient, and what is not.

Table 1 shows, for each combination of actions, the resulting social costs, composed of the sum of prevention costs and expected accident costs. For instance, at *(Bargee, Careless)*, the victim spends \( c_V \) to ensure the bargee is aboard; the injurer spends 0 on adjustment and inspection; accidents occur with probability \( P \), but when they do occur there are no sinking damages; expected accident costs are therefore \( P L_C \), and total social costs are therefore \( c_V + P L_C \).

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**Table 1.** Total Social Cost Matrix

<table>
<thead>
<tr>
<th></th>
<th>Injurer (Carroll Towing)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Victim (Conners Marine)</strong></td>
<td><strong>Careful</strong></td>
</tr>
<tr>
<td>Bargee</td>
<td>Social cost = ( c_V + c_I )</td>
</tr>
<tr>
<td>No Bargee</td>
<td>Social cost = ( c_I )</td>
</tr>
</tbody>
</table>
Efficiency in the model is simply a matter of total social cost minimization. The pair \((\text{Bargee}, \text{Careful})\) cannot be efficient, since the victim’s having a bargee aboard is wasteful, given that the accident probability is already 0 when the injurer chooses \text{Careful}. The pair \((\text{No Bargee}, \text{Careless})\) is not efficient, since care-taking behavior by either party would reduce total social costs, given our assumption that the inequalities \(c_V < P L_S\) and \(c_I < P(L_C + L_S)\) both hold. So the only possible candidates for efficiency are \((\text{Bargee}, \text{Careless})\) and \((\text{No Bargee}, \text{Careful})\). A quick examination of Table 1 leads to the conclusion that if \(c_I < c_V + P L_C\), then \((\text{No Bargee}, \text{Careful})\) is efficient; and if \(c_I > c_V + P L_C\), then \((\text{Bargee}, \text{Careless})\) is efficient.\(^8\)

4.2. Is the Conditional/Efficient Application or the Independent Application of the Hand Rule Consistent with Judge Hand’s Opinion in \textit{United States v. Carroll Towing Co.?}\(^9\)

If the Hand formula were applied conditionally in this case, contingent on the other party’s \textit{efficient} behavior, each party’s negligence or nonnegligence would depend on a hypothetical calculation that assumes the other party’s efficient behavior. For example, suppose \(c_I < c_V + P L_C\), and \((\text{No Bargee}, \text{Careful})\) is efficient. Suppose the injurer is careless and an accident occurs. Suppose no bargee was aboard. When the court is determining the victim’s negligence or nonnegligence, it would ignore the fact that the injurer was careless. It would reason that if the injurer had been careful (the efficient choice), there would have been no gain from the victim’s putting a bargee aboard. So the victim would be found nonnegligent. Next, suppose \(c_I > c_V + P L_C\), and \((\text{Bargee}, \text{Careless})\) is efficient. Suppose the victim has no bargee aboard and an accident occurs. Suppose the injurer was careless. When the court is determining the injurer’s negligence or nonnegligence, it would reason that the victim should have had a bargee aboard, and, had he done so, expected accident losses would have been limited to \(P L_C\). Under these circumstances it would have made no sense for the injurer to be careful, since \(c_I > c_V + P L_C > P L_C\). Therefore, the injurer would be found nonnegligent.

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\(^8\) Note that throughout this article we assume all inequalities are strict; that is, for the sake of simplicity, and for the sake of unique equilibria, we assume away the equality cases.
The implication of this is that, if the Hand test were contingent on the other party’s efficient behavior, it would be impossible to find both parties negligent.

But Judge Hand did find both Carroll Towing and Connors Marine negligent, and therefore equally liable for the sinking damages under the equal division Admiralty rule. A careful reading of his opinion reveals that he had no interest in finding which Table 1 cell, (No Bargee, Careful) or (Bargee, Careless), was the efficient (social cost minimizing) cell. He had no interest in comparing \( c_I \) and \( c_V + PLC \). He did not attempt to discover the lower-cost avoider of accidents and put the liability on that party. The words from his opinion clearly show that he thought a barge owner should anticipate occasional actual careless behavior of tug owners and should take steps to mitigate the resulting accident risks: “At the locus in quo—especially during the short January days and in the full tide of war activity—barges were being constantly drilled in and out. Certainly it was not beyond reasonable expectation that, with the inevitable haste and bustle, the work might not be done with adequate care. In such circumstances we hold—and it is all that we do hold—that it was a fair requirement that the Conners Company should have a bargee aboard... during the working hours of daylight” (p.174): So we are led to the conclusion that Judge Hand, in United States v. Carroll Towing Co., did not apply the Hand formula in the conditional-based-on-efficient-behavior sense.

Also, we believe an independent application of the Hand formula is inconsistent with the structure of this case: because the expected benefit from careful adjustment and inspection of the Anna C.’s moorings varies depending on whether the victim has a bargee aboard or not, a reasonable Hand test applied to the injurer would have to be contingent on the victim’s behavior. If a bargee had been aboard the Anna C., preventing sinking damages, and a lawsuit had followed because of the collision damages, we think that, in order to test the injurer’s negligence, Judge Hand would have compared \( c_I \) with \( PLC \), rather than with \( P(L_C + L_S) \). That is, the Hand test applied to the injurer would have depended on what the victim had done.

4.3. Equilibria of the Game

We have argued that Judge Hand’s negligence test, as applied to the case of United States v. Carroll Towing Co., was not an independent application, or a conditional-based-on-efficient-behavior application. We
believe that his negligence test was a conditional test contingent on the actual behavior of the other party. In this section we will carefully analyze the equilibria of the United States v. Carroll Towing Co. game, with this understanding of Judge Hand’s method. The game analyzed here depends first on the Hand test determination of negligence, and second on the liability rule. As noted already, the liability rule is the Admiralty equal division rule, which puts all of the damages on the negligent party if one party is negligent and the other is not, and splits the damages equally if both are negligent.

We need to make a few more comments on the game structure: This is a two-player noncooperative game in which each player has two strategies. Both players, and the court, know the prevention costs and the expected accident costs. Both players know how the Hand test for negligence is applied (viz., contingent on the actual behavior of the other party). Both players know the liability rule. The players are moving simultaneously.9 We are interested in characterizing the Nash equilibria.

As we have indicated, we are making two assumptions about parameter values in this study: first, that \(c_V < PL_S\), and second that \(c_I < P(L_C + L_S)\). In order to proceed with the analysis, we will distinguish between two

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9. In United States v. Carroll Towing Co., the bargee’s choice was made before the accident, possibly the day before, so it might reasonably be argued that this was a sequential-move game: The bargee chose to be “AWOL,” which put Conners Marine in the No Bargee position, and then, after observing that the bargee was missing, Carroll Towing acted. A sequential-move game suggests use of sequential-move tort principles, namely the “last clear chance” doctrine (used when the injurer is the second mover), or the doctrine of avoidance (when the victim is the second mover). These doctrines require the second mover to compensate for the first mover’s prior negligence, by taking extra precaution. See Keeton et al. (1984), Shavell (1983), Landes and Posner (1987), and Grady (1988). We feel that these doctrines are similar in spirit to the conditional-based-on-actual-behavior version of the Hand test, in the sense that the second mover’s negligence is judged contingent on the first mover’s actual, rather than efficient, behavior. However, a sequential-move tort doctrine can be applied only to a party who can observe the other party’s care level before choosing his own. The doctrine could be applied to Carroll Towing, which could see the bargee was absent. But Judge Hand was directing his formula at Connors Marine, employer of the disappearing bargee. When the bargee went AWOL, he did not know that Carroll Towing was going to choose Careless. In terms of the game, Connors Marine made its choice without knowing what Carroll Towing’s choice would be. So the doctrine of avoidance seems irrelevant to this case, and a simultaneous-move game model appears appropriate.
alternative cases consistent with the second inequality. The two cases are: case 1, \( c_I < P_L C \), and case 2, \( P_L C < c_I < P(L_C + L_S) \). These two cases will produce different outcomes in the game.\textsuperscript{10}

4.3.1. Case 1: \( c_I < P_L C \). Table 2 represents the payoff structure of the game, distributed between victim Conners Marine and injurer Carroll Towing, when \( c_I < P_L C \) holds; the Hand test, contingent on actual behavior of the other party, is used to determine negligence; and the liability rule is the equal division rule. Note that Table 2 and Table 3, unlike Table 1, are game payoff matrices.

Since \( c_I < P_L C \) holds by assumption in case 1, if an accident occurs and the Hand rule is applied to the injurer, he will be found negligent both when the victim is cautiously choosing \textit{Bargee} (and expected damages are \( P_L C \)), and when the victim is carelessly choosing \textit{No Bargee} (and expected damages are greater than \( P_L C \)). Therefore, the injurer is negligent in the entire \textit{Careless} column. In the (\textit{Bargee, Careless}) instance, the injurer is negligent and the victim, with his bargee aboard, is nonnegligent. Therefore, expected accident costs \( P_L C \) fall entirely on the injurer. In the (\textit{No Bargee, Careless}) instance, the injurer is negligent, and, under the Hand test contingent on actual behavior of the other party, the victim is also negligent since \( c_V < P_L S \). That is, the victim is negligent because of a cost-

<table>
<thead>
<tr>
<th>Victim (Conners Marine)</th>
<th>Injurer (Carroll Towing)</th>
<th>Careful</th>
<th>Careless</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>((c_V, c_I))</td>
<td>((c_V, P_L C))</td>
</tr>
<tr>
<td>Bargee</td>
<td>No accidents</td>
<td>Not neg., Negligent</td>
<td></td>
</tr>
<tr>
<td></td>
<td>((0, c_I))</td>
<td>(\frac{1}{2} P_L S, P_L C + \frac{1}{2} P_L S)</td>
<td></td>
</tr>
<tr>
<td>No Bargee</td>
<td>No accidents</td>
<td>Negligent, Negligent</td>
<td></td>
</tr>
</tbody>
</table>

Notes: We show whether accidents occur and, if they do occur, which party (parties) is (are) negligent. In parentheses we show the costs ultimately falling on the respective parties. For each pair of entries, the first of the pair refers to the victim, and the second refers to the injurer. For instance, for (\textit{Bargee, Careless}), the victim is not negligent, the injurer is negligent, with costs falling on the victim equal to \( c_V \), and costs falling on the injurer equal to \( P_L C \).

\textsuperscript{10} The likelihood of \( c_I < P_L C \) is not clear. We know that \( L_C \) was small compared to \( L_S \), that the cost of raising the \textit{Anna C.} was $2,000, and that the lost cargo of flour was valuable. Otherwise, we must guess.
benefit test applied to sinking damages only. Therefore, under the Admiralty rule, the sinking damages are split equally between the two negligent parties. The collision damages, on the other hand, fall entirely on the injurer, since the victim could not have prevented such damages.

Based on the payoff structure of this game, careful adjustment and inspection of the moorings is a dominant strategy for the injurer. Given this, the victim’s best choice is to have no bargee. So we have a unique Nash equilibrium for the game at \((\text{No Bargee, Careful})\).

Is the \((\text{No Bargee, Careful})\) combination efficient? It is if \(c_I < c_V + PLC\) holds. And in case 1, since \(c_I < PLC\) holds, \(c_I < c_V + PLC\) must also hold. Therefore, in case 1, the Hand rule, applied conditionally according to actual behavior of the other party, provides incentives for a game whose sole equilibrium is efficient. Surely that is a good result.

But note that in case 1, at the efficient game equilibrium \((\text{No Bargee, Careful})\), there should be no bargee aboard the \textit{Anna C}. And this would fly in the face of Judge Hand’s famous statement on the bargee.

4.3.2. Case 2: \(PL_C < c_I < P(L_C + L_S)\). Now we consider the other possible case, where \(PL_C < c_I < P(L_C + L_S)\) holds. Table 3 represents the payoff matrix in this case; note that it is constructed in the same manner as Table 2; that is, it is based on the Hand test, contingent on the actual behavior of the other party, and on the Admiralty equal division rule.

Now, whether the injurer’s cautious inspection of moorings is cost-justified or not depends on whether there is a bargee aboard or not. Given that the victim chooses \textit{Bargee}, the injurer’s choice of \textit{Careless} is not negligent by the Hand rule, because the additional expected benefit from careful

| Table 3. Game Payoff Matrix When \(PL_C < c_I < P(L_C + L_S)\) |
|-----------------------------|-----------------------------|
| ** Victim (Conners Marine)** | ** Injurer (Carroll Towing)** |
| | Careful | Careless |
| Bargee | No accidents \((c_V, c_I)\) | Not neg., Not neg. \((c_V + PLC, 0)\) |
| No Bargee | No accidents \((0, c_I)\) | Negligent, Negligent \(\left(\frac{1}{2}PL_S, PLC + \frac{1}{2}PL_S\right)\) |

Notes: We show whether accidents occur and, if they do occur, which party (parties) is (are) negligent. In parentheses we show the costs ultimately falling on the respective parties. For each pair of entries, the first of the pair refers to the victim, and the second refers to the injurer.
inspection, $P_L$, is less than the care-taking cost $c_I$. Therefore, in the 
(Bargee, Careless) instance, the injurer is not negligent. On the other hand, 
in the (No Bargee, Careless) instance, the injurer is negligent, because the 
benefit from careful inspection, now $P(L_C + L_S)$, is greater than the cost $c_I$.

With respect to liability in the (Bargee, Careless) instance, under the Admir-
alty rule the nonnegligent injurer is not liable, whereas in the (No Bargee, 
Careless) instance the injurer is negligent, and liable (for the collision 
damages plus one half the sinking damages.) Note that the only difference 
between Table 2 and Table 3 arises in the (Bargee, Careless) instance.

In case 1, shown in Table 2, the injurer had a dominant strategy: to 
adjust and inspect the moorings carefully. In case 2 this is no longer 
generally true. As for Nash equilibria for the game, we have several 
different possibilities, depending on the relative magnitudes of the para-
eters. Listed below are four subcases, all consistent with the two original 
inequalities we assumed for our various parameters, and with the inequal-
ities defining case 2. In the following we very briefly describe each subcase. 
The reader can refer to Table 3 to ascertain the accuracy of our description 
of the game’s equilibria and their efficiency or lack thereof:

- If $c_V + PL_C > \frac{1}{2} PL_S$ and $c_I > PL_C + \frac{1}{2} PL_S$, then (No Bargee, Careless) 
is the unique Nash equilibrium. However, by the remarks following Table 1, 
it is inefficient.
- If $c_V + PL_C < \frac{1}{2} PL_S$ and $c_I < PL_C + \frac{1}{2} PL_S$, then there exist two pure-
strategy Nash equilibria, (Bargee, Careless) and (No Bargee, Careful). By 
the remarks following Table 1, one is efficient and the other is inefficient. 
There is also a mixed strategy equilibrium, which is inefficient.
- If $c_V + PL_C > \frac{1}{2} PL_S$ and $c_I < PL_C + \frac{1}{2} PL_S$, then (No Bargee, Careful) is 
the unique Nash equilibrium. This is efficient if $c_I < c_V + PL_C$ is also 
satisfied, but if $c_I > c_V + PL_C$, it is inefficient.
- If $c_V + PL_C < \frac{1}{2} PL_S$ and $c_I > PL_C + \frac{1}{2} PL_S$, then (Bargee, Careless) is 
the unique Nash equilibrium, and it is efficient, because the two above 
inequalities together imply $c_I > c_V + PL_C$.

In sum, the United States v. Carroll Towing Co. game must have a 
unique efficient Nash equilibrium in case 1. However, in case 2, it might 
have a unique efficient Nash equilibrium, but it might also have multiple
equilibria, some of which are inefficient, or it might have a unique inefficient equilibrium.

We find it surprising to see that the Hand formula itself, as we believe Judge Hand was applying it, does not guarantee efficient outcomes in a model specifically based on United States v. Carroll Towing Co. Moreover, the possibility of inefficiency that our analysis reveals does not depend on the Admiralty law equal division rule that governed United States v. Carroll Towing Co. We can show that every kind of negligence-based rule fails to guarantee efficiency,\(^\text{11}\) so long as both parties are negligent in (No Bargee, Careless), which a careful reading of Judge Hand’s opinion seems to demand.

5. Concluding Remarks

The Learned Hand rule for determining negligence has fascinated students of law and economics for many decades, partly because Judge Hand wrote an algebraic expression—an inequality with an expected value on one side—that is intuitive and attractive to economists. Although Judge Hand did not use the words “efficient,” “efficiency,” “game,” or “equilibrium” in his United States v. Carroll Towing Co. opinion, the law-and-economics literature has come to the belief that the great jurist was groping toward a basis for efficient liability rules, and that, when analyzed in a modern formal game-theoretic framework, the Hand rule can provide injurers and victims incentives that lead to efficient equilibria.

In this study we have argued that there are three distinct ways to interpret Judge Hand’s test, and that one of them, which we call the conditional-based-on-actual-behavior approach, best represents Judge Hand’s meaning. We have shown that, based on this understanding of the Hand rule, a careful model of the facts of United States v. Carroll Towing Co. can produce (1) inefficient equilibria, and (2) odd paradoxes, such as the possibility that an efficient equilibrium might require that no bargee be aboard the Anna C. We conclude that the Hand rule, as Judge

\(^{11}\) For example, if the liability rule is negligence with contributory negligence as a defense, the payoff structure in (No Bargee, Careless) in Tables 2 and 3 changes to (\(PL_S\), \(PL_C\)). The game’s equilibrium is efficient only when one of the following two conditions holds: (1) \(c_I < PL_C\), or (2) \(c_V + PL_C < PL_S\) and \(c_V + PL_C < c_I\). Otherwise, the game has a unique inefficient equilibrium.
Hand used it, would not necessarily produce the economic results Judge Hand and his followers hoped for, even in the case of *United States v. Carroll Towing Co.*

**References**


*United States v. Carroll Towing Co.*, 159 F.2d 169 (2d Cir. 1947).