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The type-agent core for exchange economies with asymmetric information

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Abstract

The type-agent core is a new solution concept for exchange economies with asymmetric information. It coincides with the set of subgame-perfect equilibrium outcomes of a simple competitive screening game. Uninformed intermediaries help the agents to cooperate in an attempt to make some profit. The paper extends the work of Perez-Castrillo [Cooperative outcomes through non-cooperative games, Games Econ. Behav. 7 (1994) 428–440] to exchange economies with non-transferable utility and asymmetric information. The type-agent core is a subset of Wilson's coarse core [Wilson, Information, efficiency, and the core of an economy, Econometrica 46 (1978) 807–816]. It is never empty, even though it may be a strict subset of Wilson's fine core. In addition, it converges towards the set of constrained market equilibria as the economy is replicated.

JEL classification: C71; C72; D51; D82

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1. Introduction

The objective of this paper is to study the allocation of scarce resources between agents that are asymmetrically informed about the fundamentals of the economy at the time of contracting.

First defined in the absence of uncertainty, the core specifies the set of contracts that are immune to coalitional objections. Particularly, the core of an exchange economy with complete information

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is the set of feasible allocations such that no group of agents can improve their satisfaction by re-allocating their initial endowments. Besides its many economic applications, the core also allows to justify the price-taking assumption underlying the concept of competitive equilibrium. Debreu and Scarf [2] indeed prove that the core shrinks towards the set of competitive equilibria when the economies are replicated.

The concept of contingent good introduced by Arrow and Debreu allows to extend the previous analysis (i.e. core, competitive equilibrium and convergence result) to exchange economies with uncertainty, as long as the agents have the same information at their disposal.

Important conceptual issues arise when the agents are asymmetrically informed. Two different aspects must be distinguished: (1) If the agents are asymmetrically informed at the time of contracting, then we have to describe how their bargaining strategies depend on their information; (2) If the agents are asymmetrically informed at the time of implementing the contracts, then we need to discuss which contracts are feasible (a contract may depend on the private information of the agents only if it gives the right incentives to the agents to reveal their information truthfully). Forges et al. [4] propose an excellent survey of the literature. More recent papers on the topic include McLean and Postelwaite [9], Myerson [11], and Serrano and Vohra [18].

I focus on situations where the true state of the economy is commonly known when the contracts are implemented. Incentive and measurability constraints are therefore irrelevant. Many economic examples fit in this category. The payment of an insurance contract for instance depends on the observable losses incurred. The payment of a financial asset (e.g. equities or options) is contingent on the realization of some observable events. In addition, the model could serve as a benchmark to understand more complex problems where the agents are still asymmetrically informed at the time of implementing the contracts.

The main reference for core concepts in this framework is Wilson [21]. An agreement specifies a way to split the endowment of the economy among the agents in each state. Such a function is called a feasible allocation rule. Wilson discusses various notions of objection against given feasible allocation rules. They differ by the amount of communication that is permitted between the agents. Two polar notions emerge: coarse objections are based on events that are common knowledge among the members of the coalition; fine objections are based on events that can be discerned by pooling the information of the members of the coalition. Every coarse objection is a fine objection. Hence, the fine core is a subset of the coarse core. Serrano et al. [19] show that neither the coarse nor the fine cores converge towards any reasonable notion of price equilibrium when the economies are replicated.

The agents are cooperating on their own in Wilson's theory. Objections emerge from coalitions. I study an alternative approach where uninformed intermediaries help the agents to coordinate in an attempt to make some profit. They compete a la Bertrand. The intermediaries correctly anticipate the set of agents that are going to buy the contracts (net trade vectors) they offer. Even without communication, this set may vary with the future state of the economy as each agent's decision is based on his own private information. This leads to an endogenous determination of the coalition that is going to form as a function of the state. In other words, I extend the competitive screening argument of Rothschild and Stiglitz [14] to coalition formation.

The very presence of asymmetric information implies that the agents will have difficulty to perform mutually beneficial exchange and insurance without the help of outside institutions. Perez-Castrillo [12] observes

> It is easy to find many situations in which, in order to achieve cooperation, coalitions are formed by the initiative of its own members. (...) On many other occasions, however, it is

difficult for the agents to join a coalition. Information problems between agents, absence of communication mechanisms, or just the incentive problems that arise when people try to work cooperatively can make the formation of coalitions difficult. In these cases, 'exogenous' agents or institutions may be interested in achieving the emergence of these groups. Given that a coalition can generate profits, it seems that it should be possible to find a payment scheme that will make it worthwhile for the agents to join the group. When there are several exogenous agents that may be interested in creating such groups, they will compete in order to attract agents to their coalitions. Solution concepts from non-cooperative game theory are then appropriate for the analysis of this type of competition.

Perez-Castrillo [12] studies cooperative games with transferable utility under complete information, even though he justifies his approach by referring mainly to informational constraints. Theorem 1 hereafter extends his main result to exchange economies with non-transferable utility and asymmetric information.

The set of subgame-perfect equilibrium outcomes associated with the competitive screening game constitutes an interesting new solution concept. I prove that it coincides with the core (as usually defined thanks to the Arrow–Debreu contingent goods) of a fictitious exchange economy with uncertainty and symmetric information. The fictitious agents are defined as in the type-agent representation of Bayesian games suggested by Harsanyi [6] in order to define the concept of Bayesian equilibrium. My new solution is therefore called the type-agent core.

The type-agent core is a subset of the coarse core. I propose an example where it is even a strict subset of the fine core. Although the fine core may be empty, the type-agent core is never empty. Wilson [21, footnote 6], defines a notion of constrained market equilibrium as a technical tool to prove the non-emptiness of the coarse core. I show that, under mild conditions, the set of constrained market equilibria is a subset of the type-agent core and that the type-agent core shrinks towards the set of constrained market equilibria as the economy is replicated. Such a convergence result is rather unexpected as Serrano et al. [19] show that neither the coarse nor the fine cores converge towards the set of constrained market equilibria when the economy is replicated. More than that, they show that the negative result is robust against many alternative definitions of both the core and the price equilibria.

2. Framework

The model is the same as in Wilson [21] and Serrano et al. [19]. Let N be the finite set of agents. Let L be the finite set of goods. The future state of the economy is uncertain. Let Ω be the finite set of possible states. Let π be the common prior that describes the relative probability of those states. I assume without loss of generality that $\pi(\omega) > 0$ for each $\omega \in \Omega$. The agents may have some private information. The information of agent i is summarized by a partition $\mathcal{P}_i$ of the set Ω. For each $\omega \in \Omega$, let $P_i(\omega)$ be the atom of the partition that contains ω. The interpretation goes as follows. When the future state of the economy is ω, agent i knows and only knows that it will be an element of $P_i(\omega)$. His beliefs are derived from π by Bayesian updating. *Events* are subsets of Ω. The probability $\pi(\omega|\mathcal{E})$ of any state ω given an event $\mathcal{E}$ equals 0 if $\omega \in \Omega \setminus \mathcal{E}$ and equals $\pi(\omega)/\pi(\mathcal{E})$ if $\omega \in \mathcal{E}$. The true state of the economy is common knowledge among the agents at some future date. It determines their preferences and endowments. Let $e_i : \Omega \to \mathbb{R}^L_+$ be the function that specifies the initial endowment of agent i and let $u_i : \mathbb{R}^L_+ \times \Omega \to \mathbb{R}$ be the function that specifies his preferences. The agents maximize their expected utilities when facing some uncertainty. The utility function of each agent is strongly increasing ($x' \gneq x$ implies $u_i(x', \omega) > u_i(x, \omega)$ for each $\omega \in \Omega$), continuous and concave in each state of the economy. Decisions are taken today about the way to redistribute the endowments when the state will be common knowledge. An *allocation rule* is a function $a: \Omega \to \mathbb{R}_+^{L \times N}$. It is *feasible* if $\sum_{i \in N} a_i(\omega) \leqslant \sum_{i \in N} e_i(\omega)$ for each $\omega \in \Omega$. There are typically opportunities for insurance, even if the agents are asymmetrically informed. The expected utility of agent i for some allocation rule a conditional on some event $\mathcal{E}$ is $E(u_i(a_i)|\mathcal{E}) = \sum_{\omega \in \Omega} \pi(\omega|\mathcal{E}) u_i(a_i(\omega), \omega)$. The model boils down to a classical exchange economy when there is only one possible state. It coincides with the traditional model of exchange economies with uncertainty and symmetric information when $\mathcal{P}_i = \{\Omega\}$ for each $i \in N$.

3. Definition

Let a be an allocation rule that one sees as a potential outcome. I test it against some alternative allocation rule a'. Let $D(a, a', \omega)$ denote the set of *deviators* should ω be the future state of the economy. It is the set of agents that prefer (given their private information) to receive a' instead of keeping a:

$$D(a, a', \omega) := \{i \in N | E(u_i(a'_i)|P_i(\omega)) > E(u_i(a_i)|P_i(\omega))\}$$

for each $\omega \in \Omega$.

The allocation rule a' is *strictly feasible* when proposed against a if

$$\sum_{i \in D(a,a',\omega)} a'_i(\omega) \leqslant \sum_{i \in D(a,a',\omega)} e_i(\omega)$$

for each $\omega \in \Omega$, the inequality being strict for some $\omega \in \Omega$.

An allocation rule a is *blocked* if there exists an allocation rule a' that is strictly feasible when proposed against a. A dummy player with no endowment and no information can obtain a strictly positive amount of some good in some state of the economy by challenging a and proposing a'.

The *type-agent core* is the set of feasible allocation rules that are not blocked. Observe that the type-agent core coincides with the usual notion of core when there is no uncertainty (i.e. $\#\Omega = 1$).

I will show in the next section how the type-agent core emerges from a simple competitive screening game. Before doing that, I justify the terminology. I adapt the idea of Harsanyi [6], who defines a Bayesian equilibrium to be any Nash equilibrium of the type-agent representation of the original Bayesian game. I suggest the following type-agent representation of the economy described in Section 2. It is a (fictitious) exchange economy with uncertainty and symmetric information. Let $\mathcal{N}$ be the set of type-agents. A *type-agent* is a couple $(i, \mathcal{E})$ where i is an agent and $\mathcal{E}$ is an atom of his information partition. The set of goods is L. The future state of the fictitious economy is uncertain. The set of possible states is Ω. The type-agents have no private information. They all hold the same belief π regarding the relative probability of the different states. The endowment of type-agent $(i, \mathcal{E})$ in state ω equals $e_i(\omega)$ if $\omega \in \mathcal{E}$ and equals zero otherwise. The utility of type-agent $(i, \mathcal{E})$ for bundle $x \in \mathbb{R}_+^L$ at ω equals $u_i(x, \omega)$ if $\omega \in \mathcal{E}$ and equals zero otherwise. The type-agents decide today about how to redistribute their endowments when the state will be common knowledge. An allocation rule in the fictitious economy is a function $\mathfrak{a}: \Omega \to \mathbb{R}_+^{L \times \mathcal{N}}$.

The presence of symmetric uncertainty in the fictitious economy, as well as the possibility to write contracts that are contingent on the future state of the economy, allow to apply the standard notion of the core (see the discussion about the ex ante core of an Arrow–Debreu economy in Forges et al. [4, Section 1.1.1]). It is the set of feasible allocation rules such that no coalition has a feasible allocation rule which increases the expected utility of each of its members. A formal definition is given in the Appendix.

There exists a natural isomorphism between the original economy and its type-agent representation. The allocation rule a in the original economy associated to an allocation rule $\mathfrak{a}$ in its type-agent representation is defined as follows: $a_i(\omega) = \mathfrak{a}_{(i,P_i(\omega))}(\omega)$ for each $i \in N$ and each $\omega \in \Omega$. Similarly, the allocation rule $\mathfrak{a}$ in the type-agent representation of the economy associated to an allocation rule a in the original economy is defined as follows: $\mathfrak{a}_{(i,\mathcal{E})}(\omega) = a_i(\omega)$ if $\omega \in \mathcal{E}$ and $\mathfrak{a}_{(i,\mathcal{E})}(\omega) = 0$ if $\omega \in \Omega \setminus \mathcal{E}$, for each $(i, \mathcal{E}) \in \mathcal{N}$. With this interpretation in mind, it is possible to prove that the type-agent core of the original economy coincides with the core of its type-agent representation (see the Appendix for the formal statement and its proof).

Wilson [21, p. 814] alludes to the type-agent representation of the economy, in order to prove the non-emptiness of the coarse core. The main difference is that he restricts the set of admissible coalitions, i.e. the set of coalitions that are allowed to object against given allocation rules.

4. Competitive screening

My objective is to adapt the competitive screening game of Rothschild and Stiglitz [14] (see [8, Section 13.D] for a modern exposition) in order to study coalition formation. The argument extends the result of Perez-Castrillo [12] by allowing for asymmetric information and non-transferable utility. Uninformed intermediaries (any number greater or equal to two) simultaneously offer contracts to the agents. I assume for simplicity that each intermediary proposes exactly one contract to each agent. A *contract* for agent i is a function $c_i : \Omega \to \mathbb{R}^L$. It specifies net trades. The agents simultaneously choose at most one contract among those offered. Time goes by, uncertainty is resolved and agreed-upon contracts are realized. Suppose that agent i chose some contract c_i offered by some intermediary j. Let $l \in L$ and let $\omega \in \Omega$ be the state of the economy. Then agent i is entitled to receive $c_i^l(\omega)$ units of good l from intermediary j when $c_i^l(\omega)$ is positive and intermediary j is entitled to receive $-c_i^l(\omega)$ units of good l from agent i when $c_i^l(\omega)$ is negative.

Notice that bankruptcy is possible a priori. An intermediary could promise to deliver more goods than what he will receive from his clients. An agent could promise to deliver more goods than what he owns. I want to keep the analysis as simple as possible and be consistent with the idea underlying the core that the status quo payoffs are guaranteed when challenged. Hence, I focus on markets that are regulated as follows. The regulating authority covers the deficits. At the same time, it prevents fraudulent promises by inflicting a high punishment (e.g. jail sentence or interdiction to participate to the market in the future) on the agents and the intermediaries that bankrupt. I further discuss this assumption and its consequences at the end of the section.

The preferences of the intermediaries are assumed to be continuous and strongly increasing on $\mathbb{R}_+^{\Omega \times L}$. For instance, the preference of an intermediary could be represented by some expected utility, $\sum_{\omega \in \Omega} \pi(\omega) u(., \omega)$, where the functions $(u(., \omega))_{\omega \in \Omega}$ are continuous and strongly increasing on $\mathbb{R}_+^L$. In addition, each intermediary prefers having no good in each state rather than bankrupting in some state, whatever the amount of goods he receives in the other states. Similarly, each agent prefers to sign no contract rather than bankrupting in some state, whatever the amount of goods he receives in the other states.

A strategy for an intermediary consists in proposing a contract to each agent. A strategy for an agent is the choice of whether to sign a contract and, if so, which one, as a function of the offers made by the intermediaries. A *subgame-perfect equilibrium* is a profile of strategies, one for each agent and each intermediary, such that (1) the choice of each agent is optimal[2] in each contract signing game (even those out of the equilibrium path) and (2) the strategy of each intermediary is optimal against the strategies of the other intermediaries and the strategies of the agents. Each strategy profile generates an *outcome* which specifies an allocation rule for the agents, the amount of goods kept by the intermediaries, as well as the set of agents and the set of intermediaries that bankrupt, in each state of the economy.

Theorem 1. *An outcome is supported by some subgame-perfect equilibrium if and only if there is no bankruptcy in any state of the economy, the intermediaries exactly break even in each good in each state of the economy, and the allocation rule for the agents belongs to the type-agent core.*

Proof. I consider the case where only two intermediaries are competing. The argument can easily be adapted to show that the theorem remains valid for any number of intermediaries greater or equal to two.

($\Rightarrow$) It is impossible that some agent bankrupts in some good in some state of the economy, as choosing to sign no contract would then be a profitable deviation. It is also impossible that some intermediary bankrupts in some good in some state of the economy as offering the null contract would then be a profitable deviation. Let $c = (c_i)_{i \in N}$ be the profile of contracts that is supported by the subgame-perfect equilibrium. It is not necessarily directly offered by one of the two intermediaries. It is the outcome of the game following the equilibrium contract offers and selections. Let a be the allocation rule associated to c, i.e. $a = c + e$. I prove that a belongs to the type-agent core in three steps. First, *a is feasible.* Indeed, $\sum_{i \in N} c_i(\omega) \leqslant 0$ for each $\omega \in \Omega$ as otherwise at least one of the two intermediaries bankrupts for some good in some state of the economy. Second, *both intermediaries exactly break even in each good in each state of the economy.* As argued before, none of the two intermediaries bankrupt in any good in any state of the economy. Hence, it is sufficient to prove that $\sum_{i \in N} c_i(\omega) \geqslant 0$ for each $\omega \in \Omega$, as a positive net quantity of some good in some state of the economy for some intermediary then means a negative net quantity (bankruptcy) of that good in that state of the economy for the other intermediary. Suppose on the contrary that $\sum_{i \in N} c_i^l(\omega) < 0$ for some $\omega \in \Omega$ and some $l \in L$. Then, one may construct for each $\varepsilon > 0$ a profile of contracts d such that the continuation payoff in good l for the intermediary proposing d is at least 0 when $\sum_{i \in N} c_i^l(\omega) = 0$ and equals $-\sum_{i \in N} c_i^l(\omega) - \varepsilon$ when $\sum_{i \in N} c_i^l(\omega) < 0$, for each $\omega \in \Omega$. For sure one of the two intermediaries strictly prefers to get the aggregate equilibrium payoff rather than his own equilibrium payoff. By continuity of the preferences, proposing d is a profitable deviation for at least one of the two intermediaries for each $\varepsilon > 0$ small enough. I now sketch how d may be defined. The idea is to equally distribute to the agents in each state of the economy ε additional units of each good in excess supply. Formally, $d_i^l(\omega) := c_i^l(\omega) + \varepsilon/n$ for each $(i, l, \omega) \in N \times L \times \Omega$ such that $\sum_{j \in N} c_j^l(\omega) < 0$ and $d_i^l(\omega) := c_i^l(\omega)$ for each other triple (i, l, ω). The payoff in each good l of an intermediary offering such a deviating contract indeed equals $-\sum_{i \in N} c_i^l(\omega) - \varepsilon$ when $\sum_{i \in N} c_i^l(\omega) < 0$, for each $\omega \in \Omega$. Nevertheless, he could possibly bankrupt in states ω such that $\sum_{i \in N} c_i^l(\omega) = 0$ for each $l \in L$ as indifferent agents do not necessarily choose the deviating offer. I slightly modify the definition of d at those states by making transfers between the agents. I focus on states ω such that $\sum_{i \in D(c+e, d+e, \omega)} d_i^l(\omega) > 0$ for some $l \in L$. Let $i \in D(c + e, d + e, \omega)$ be such that $d_i^l(\omega) > 0$.

[2] As the regulating authority covers the deficits, the contract signing game is a juxtaposition of individual decision problems. The choice of an optimal contract for an agent is independent of the strategies played by the other agents.

A small amount of good l is transferred from agent i to the other agents. Hence, every agent chooses the deviating contract should it be proposed and should the future state of the economy be ω. The deviating intermediary now exactly breaks even in that state as well. There could be new states where the intermediary bankrupts. Fortunately, applying the procedure recursively (using the fact that Ω is finite), I find a contract d such that $\sum_{i\in D(c+e,d+e,\omega)} d_i^l(\omega) \leqslant 0$ for each $(l,\omega) \in L \times \Omega$. Notice though that this non-positive number is not necessarily the continuation payoff of the deviating intermediary, as some agents not in $D(c+e, d+e, \omega)$ could also choose his contract should the future state be ω. Therefore, I modify one more time the definition of d by imposing that $d_i(\omega) = 0$ for each $\omega \in \Omega$ and each $i \in N$ such that $i \notin D(c+e, d+e, \omega)$. The above modifications do not affect the definition of d at states ω such that $D(c+e, d+e, \omega) = N$. So, the payoff in each good l of an intermediary offering such a deviating contract remains equal to $-\sum_{i\in N} c_i^l(\omega) - \varepsilon$ when $\sum_{i\in N} c_i^l(\omega) < 0$, for each $\omega \in \Omega$. Third, *a is not blocked.* Suppose on the contrary that a is blocked by some allocation rule a'. Let c' be the profile of contracts defined as follows: $c'_i(\omega) := a'_i(\omega) - e_i(\omega)$ when $i \in D(a, a', \omega)$ and $c'_i(\omega) := 0$ when $i \in N \setminus D(a, a', \omega)$, for each $\omega \in \Omega$. The intermediary proposing c' breaks even in each state of the economy and keeps some strictly positive amount of some goods in some states of the economy. Hence, c' is a profitable deviation for both intermediaries, given the second step of the proof.

($\Leftarrow$) Let a be a feasible allocation rule that cannot be blocked by any allocation rule. I consider the following strategies. Both intermediaries propose $c = a - e$. All the agents go to the first intermediary. If the first intermediary proposes something different from c, then each agent chooses to stay with him if and only if he strictly prefers his proposal to c. Otherwise the agents go to the second intermediary. If the second intermediary proposes something different from c, then each agent chooses to follow him if and only if he strictly prefers his proposal to c. Otherwise the agents stay with the first intermediary. These strategies are clearly part of a subgame-perfect equilibrium. □

There is no communication and no information transmission. First, the agents do not deduce any information from observing the alternative contracts as the intermediaries are uninformed. Second, the agents do not learn any information at the contract signing stage as they sign the contracts simultaneously. Nevertheless, I obtain a refinement of the coarse core (see Theorem 2 and Example 1 hereafter) because I apply a screening argument to coalition formation.

Some outcomes of the game are not feasible if the regulating authority has no goods at its disposal. Even if the intermediaries break even in each good in each state of the economy at equilibrium and even if the intermediaries break even in each good in each state of the economy when proposing a deviating contract, it may be the case that some of the active intermediaries facing an objection bankrupt. This is the usual feature of the competitive screening games. In Rothschild and Stiglitz [14], the pooling contract is subject to a separating objection attracting only the low risk agents. The insurance company offering the pooling contract bankrupts as it remains with the high risk agents.

If the regulating authority does not have goods at its disposal for covering the deficits, and if the agents are fully rational in the sense that they anticipate the possibility of bankruptcy, then one faces an important conceptual difficulty. Suppose that each intermediary has to specify in each state of the economy how he will re-allocate the endowment of its clients as a function of the set of agents that choose him. Then the contract signing stage is a Bayesian game instead of a juxtaposition of independent individual decision problems. Any allocation rule that is interim individually rational is supported by some subgame-perfect Bayesian equilibrium of the competitive screening game.

Indeed, choosing an alternative contract is never profitable if nobody else is signing it. There is a problem of coordination and of equilibrium selection.

I conclude the section by pointing out some differences with respect to the model of Rothschild and Stiglitz. First, I assume that the information of the agents is commonly known at the time of implementing the contracts, although the risk profile of the agents is not observable when the contract is realized in their model. Second, my argument is not restricted to one good (money). Third, feasibility is not expressed in expected terms. The intermediaries break even in each good in *each* state of the economy at equilibrium.

5. Properties

Let S be a coalition. An event $\mathcal{E} \subseteq \Omega$ is *common knowledge* among the members of S if it can be written as a union of elements of $\mathcal{P}_i$ for each $i \in S$. An allocation rule a is *feasible for* S if $\sum_{i\in S} a_i(\omega) \leqslant \sum_{i\in S} e_i(\omega)$ for each $\omega \in \Omega$. Coalition S has a *coarse objection* against an allocation rule a if there exist an allocation rule a' feasible for S and an event $\mathcal{E}$ that is common knowledge among the members of S such that $E(u_i(a_i')|P_i(\omega)) > E(u_i(a_i)|P_i(\omega))$ for each $i \in S$ and each $\omega \in \mathcal{E}$. The *coarse core* is the set of feasible allocation rules against which no coalition has a coarse objection [21].

Theorem 2. *The type-agent core is a subset of the coarse core.*

Proof. Let a be an allocation rule and let $(S, a', \mathcal{E})$ be a coarse objection against a. Then a is blocked by the allocation rule a'' where $a_i''(\omega) := (1-\varepsilon)a_i'(\omega)$ if $(i,\omega) \in S \times \mathcal{E}$ and $a_i''(\omega) := e_i(\omega)$ otherwise ($\varepsilon > 0$ is very small). $\square$

Let a be an allocation rule. A coalition S has a *fine objection* against a if there exist an event $\mathcal{E}$ and an allocation rule a' feasible for S such that the two following properties are true at each $\omega \in \mathcal{E}$: (1) $\bigcap_{i\in S} P_i(\omega) \subseteq \mathcal{E}$; (2) $E(u_i(a_i')|\mathcal{E} \cap P_i(\omega)) > E(u_i(a_i)|\mathcal{E} \cap P_i(\omega))$ for each $i \in S$. The *fine core* is the set of feasible allocation rules against which no coalition has a fine objection [21]. The fine core is a subset of the coarse core. In fact, the fine core is the smallest conceivable core according to Wilson, as fine objections allow for any kind of information sharing. This is wrong once we try to understand how the agreements emerge instead of testing *given* allocation rules. Example 11 in de Clippel and Minelli [3] illustrates this point when the tentative agreements are proposed by the agents themselves. I adapt the example in order to show that some fine core allocations may be blocked.

Example 1. I consider a sunspot economy with asymmetric information. There are two agents, two goods and two equiprobable states for the economy. Agent 1 knows the future state while agent 2 does not: $\mathcal{P}_1 = \{\{\omega_1\}, \{\omega_2\}\}$ and $\mathcal{P}_2 = \{\{\omega_1, \omega_2\}\}$. The endowments are defined as follows: $e_1(\omega) = (0, 100)$ and $e_2(\omega) = (1, 100)$ for each $\omega \in \Omega$. The utility functions are defined as follows: $u_1(x,\omega) = 100x^1 + x^2 - 100$ and $u_2(x,\omega) = x^1 + x^2 - 101$ for each $x \in \mathbb{R}^2_+$ and each $\omega \in \{\omega_1, \omega_2\}$. It is mutually beneficial to exchange good 1. Good 2 is money. Consider the two following feasible allocation rules:

	a_1	a_2	a_1'	a_2'
ω_1	(1, 198)	(0, 2)	(0, 100)	(1, 100)
ω_2	(1, 0)	(0, 200)	(1, 97)	(0, 102)

The allocation rule a belongs to the fine core but not to the type-agent core. Indeed, it favors too much agent 1 in ω_1 and hence is blocked, by a' for instance.

There is no general inclusion relation between the type-agent core and the fine core. Here is an example where the fine core is a subset of the type-agent core.

Example 2. I adapt Example 2 of Wilson [21]. There are three agents, one good (money) and two equiprobable state for the economy. Agent 3 knows the future state while agents 1 and 2 do not: $\mathcal{P}_1 = \mathcal{P}_2 = \{\omega_1, \omega_2\}$ and $\mathcal{P}_3 = \{\{\omega_1\}, \{\omega_2\}\}$. The endowments are defined as follows: $e(\omega_1) = (100, 0, 0)$ and $e(\omega_2) = (0, 100, 0)$. The utility functions are defined as follows: $u_i(x, \omega) = \sqrt{x}$ for each $i \in \{1, 2, 3\}$, each $x \in \mathbb{R}_+$ and each $\omega \in \{\omega_1, \omega_2\}$. The full-insurance allocation rule giving 50 dollars to each of the two first agents in each state of the economy belongs to both the type-agent core and the coarse core but not to the fine core. If the agents can communicate, then agent 3 will meet agent 1 when the state is ω_1, convince him that the future state is favorable to him, and agree with him to implement a different allocation, for instance $(75, 0, 25)$.

Notice that the fine core may be empty, as in the previous example. Hence, it is remarkable that the type-agent core is never empty even if it is included in the coarse core and may sometimes be a subset of the fine core.

Theorem 3. *The type-agent core is not empty.*

The result is a consequence of Scarf [15], as the type-agent core coincides with the core of the type-agent representation of the economy.

6. Convergence

An allocation rule a is a *constrained market equilibrium* if it is feasible and there exists a price system $p: \Omega \to \mathbb{R}^L_+$ such that

$$a_i \in \arg \max_{a'_i \in B_i(p,\omega)} E(u_i(a'_i)|P_i(\omega)),$$

where

$$B_i(p, \omega) := \left\{ a'_i \in \mathbb{R}^{L\times\Omega}_+ \,\middle|\, \sum_{\omega' \in P_i(\omega)} p(\omega') \cdot a_i(\omega') \leqslant \sum_{\omega' \in P_i(\omega)} p(\omega') \cdot e_i(\omega') \right\}$$

is the budget set of agent i at ω, for each $i \in N$ and each $\omega \in \Omega$ (Wilson [21, footnote 6]). It is a natural generalization of the Arrow–Debreu equilibrium in markets with contingent commodities to economies with asymmetric information when inside trading is prohibited. Indeed, in a world with contingent commodities, the uninformed 'invisible hand' specifies a price for each commodity in each state of the economy in order to clear all the markets. The agents do not learn anything by observing the price vector, as it does not depend on the future state of the economy. They maximize their expected utilities under the additional constraint that they may not sell contingent commodities associated to states that they know are not going to occur. I further analyze Example 2 of Wilson [21] in order to illustrate the concept.

Example 3. There are three agents, one good (money) and three equiprobable states for the economy. The following table specifies the information and the endowments of the agents:

Agent (i)	$\mathcal{P}_i$	$e_i(\omega_1)$	$e_i(\omega_2)$	$e_i(\omega_3)$
1	$\{\{\omega_1\}, \{\omega_2, \omega_3\}\}$	5	1	3
2	$\{\{\omega_2\}, \{\omega_1, \omega_3\}\}$	3	5	1
3	$\{\{\omega_3\}, \{\omega_1, \omega_2\}\}$	1	3	5

The following table specifies an allocation rule a that is a constrained market equilibrium for the price vector $(1, 1, 1)$. It also specifies the associated net-trades:

Agent (i)	$a_i(\omega_1)$	$a_i(\omega_1)$	$a_i(\omega_1)$	$z_i(\omega_1)$	$z_i(\omega_2)$	$z_i(\omega_3)$
1	5	2	2	0	1	-1
2	2	5	2	-1	0	1
3	2	2	5	1	-1	0

The equilibrium concept allows for some insurance between the two agents that are not fully informed. This would be impossible if the prices of the three contingent commodities were varying with the future state of the economy, much as in rational expectations equilibria. Markets have to clear ex post with Wilson's concept. If for instance the future state is ω_1, there is excess supply (resp., demand) of money in state 2 (resp., 3), as agent 1 is prohibited to buy or sell these contingent commodities. This is irrelevant as these claims will not have to be satisfied. What matters on the other hand is the fact that the demand for money from agent 3 is met by the supply of money by agent 2 in state 1. Varying the states, there are three equilibrium equations to be satisfied in total, not nine. The market is not fully decentralized in that sense. I think instead of an uninformed trading organization proposing the prices in order to match demand with supply ex post.

It is easy to check that the set of constrained market equilibria coincides with the set of Arrow–Debreu equilibria of the type-agent representation of the economy suggested in Section 3. Hence, the next three theorems are corollaries[3] of Debreu and Scarf [2] (see the Appendix).

Theorem 4. *Suppose that each agent is endowed with a strictly positive amount of each good in each state of the economy. Then the set of constrained market equilibria is not empty.*

Theorem 5. *The set of constrained market equilibria is a subset of the type-agent core.*

I replicate the agents of the economy described in Section 2 as in Serrano et al. [19]. Let $\bar{k}$ be the number of replicas. Each agent of the original economy now appears $\bar{k}$ times. Hence, there are $\bar{k}N$ agents in the replicated economy. Let $i \in N$ and let $k \in \{1, \dots, \bar{k}\}$. Copy k of agent i is denoted $i.k$. Agent $i.k$ in the replicated economy has the same endowment, the same information and the same utility function as agent i in the original economy.

[3] The assumptions of Debreu and Scarf [2, Section 5] are satisfied if the consumption set of type-agent $(i, \mathcal{E})$ is the set of contingent goods that are compatible with his information, i.e. $\mathbb{R}_+^{L \times \mathcal{E}}$, for each $(i, \mathcal{E}) \in \mathcal{N}$.

Theorem 6. *Suppose that each agent's utility function is strictly concave in each state of the economy and that each agent is endowed with a strictly positive amount of each good in each state of the economy. Then, the type-agent core shrinks to the set of constrained market equilibria as the number of replicas $\overline{k}$ tends to infinity. Observe that the type-agent representation of the replicated economy coincides with the Debreu–Scarf replication of the type-agent representation of the original economy.*

A similar result is obtained by Goenka and Shell [5] for sunspot economies with restricted market participation. There is uncertainty but no asymmetric information. The uncertainty does not affect the fundamentals of the economy. The agents are not necessarily allowed to trade on each market for contingent goods. Even if the interpretation of the models differ, the notion of sunspot equilibrium formally coincides with the notion of constrained market equilibrium and the idea of quasi-Walrasian economy [1] is equivalent to the type-agent representation I suggest for exchange economies with asymmetric information. Goenka and Shell define the core of the sunspot economy with restricted market participation as the core of the quasi-Walrasian economy. Without justifying the definition, they prove a similar convergence result [5, Theorem 7.3; 19, Section 4.1]. It is not clear whether there is anything more than a formal relation between sunspot economies with restricted market participation and economies with asymmetric information. In any case, the former would constitute only a small subclass of the latter.

The fine core may be empty for well-behaved exchange economies and all their replicas. Theorem 4 implies that the fine core does not converge towards the set of constrained market equilibria when the economy is replicated. Even though the coarse core always contains the set of constrained market equilibria, Serrano et al. [19] show on a simple example that there may be no convergence towards the set of constrained market equilibria when the economy is replicated. They even observe in Section 4 that the conclusion of their example is robust with respect to alternative definitions of the core [4] and alternative definitions of price equilibria. [5] I now illustrate why their argument does not apply to the type-agent core.

Example 4. Consider a sunspot economy with two agents, two goods and two equiprobable states for the economy. Agent 1 knows the future state while agent 2 does not: $\mathcal{P}_1 = \{\{\omega_1\}, \{\omega_2\}\}$ and $\mathcal{P}_2 = \{\{\omega_1, \omega_2\}\}$. The endowments are defined as follows: $e_1(\omega) = (24, 0)$ and $e_2(\omega) = (0, 24)$ for each $\omega \in \Omega$. The utility functions are defined as follows: $u_1(x, \omega) = u_2(x, \omega) = \sqrt{x^1 x^2}$ for each $x \in \mathbb{R}^2_+$ and each $\omega \in \{\omega_1, \omega_2\}$. Let a be the feasible allocation rule defined as follows: $a_1(\omega_1) := (15, 15)$, $a_1(\omega_2) := (8, 8)$, $a_2(\omega_1) := (9, 9)$ and $a_2(\omega_2) := (16, 16)$. Serrano et al. show that the $\overline{k}$-replication of a belongs to the coarse core (as well as other core concepts), but is not a constrained market equilibrium (nor an alternative price equilibrium in a broad sense) of the $\overline{k}$-replicated economy, for each $\overline{k} \in \mathbb{N}$. It is already blocked in the second replica. The second replication of a is given by

a'	1.1	1.2	2.1	2.2
ω_1	(15, 15)	(15, 15)	(9, 9)	(9, 9)
ω_2	(8, 8)	(8, 8)	(16, 16)	(16, 16)

[4] Such as the core with endogenous communication [20] and the coarse + core [7].

[5] Such as sunspot equilibria, Radner equilibria and rational expectations equilibria.

It is blocked by the following allocation rule:

a''	1.1	1.2	2.1	2.2
ω_1	(31/2, 31/2)	(0, 0)	(17/2, 17/2)	(0, 0)
ω_2	(10, 13/2)	(10, 13/2)	(28, 11)	(0, 0)

The coalitions are forming as follows: {1.1, 2.1} if the future state is ω_1 and {1.1, 1.2, 2.1} if the future state is ω_2.

7. Conclusion

I studied exchange economies with asymmetric information, assuming that cooperation is achieved through the help of uninformed intermediaries that compete in an attempt to make some profit. I defined a new notion of core that coincides with the subgame-perfect equilibrium outcomes of some competitive screening game. Contrarily to Wilson's [21] analysis, objections are not bound to emerge from state-independent coalitions because the intermediaries anticipate each agent's participation decision as a function of its private information. The coalitions that form are endogenously determined by comparing different allocation rules. This explains why the type-agent core may be a strict subset of the coarse core even though it involves no communication and no information transmission. I proved that the type-agent core converges towards the set of constrained market equilibria when the economy is replicated.

I suggest some directions for further research. Communication and information transmission could be discussed by modifying the competitive screening game. The intermediaries could be partially informed and learn additional information by observing the choices of the agents. The agents themselves could learn some information before choosing a contract if they observe the choice of other agents. More generally, there is a need for a better strategic foundation of core concepts in exchange economies with asymmetric information.[6] I analyzed a variant of the procedure suggested by Perez-Castrillo [12]. Other procedures supporting the core under complete information (see e.g. [13,16,17]) should be studied as well. Finally, the type-agent core and its non-cooperative justification should be extended to situations where the information is not commonly known at the time of implementing the agreements. Myerson [10,11] considers entrepreneurs devising alternative market organizations and mediators helping the agents to coordinate. It would be illuminating if his core concepts could be characterized as the set of equilibrium outcomes associated with some explicit competitive screening game.

Appendix: The core and the Arrow–Debreu equilibria of the type-agent representation

I refer to the last paragraph of Section 3 for the definition of the type-agent representation of the economy. I denote by $\mathfrak{e}_{(i,\mathcal{E})} : \Omega \to \mathbb{R}^L_+$ the endowment of type-agent $(i, \mathcal{E})$: $\mathfrak{e}_{(i,\mathcal{E})}(\omega) := e_i(\omega)$ if $\omega \in \mathcal{E}$, $\mathfrak{e}_{(i,\mathcal{E})}(\omega) := 0$ if $\omega \in \Omega \setminus \mathcal{E}$, for each $(i, \mathcal{E}) \in \mathcal{N}$. Subsets of $\mathcal{N}$ are called *coalitions*. An allocation rule $\mathfrak{a}$ is *feasible for coalition* $\mathbb{S}$ if $\sum_{(i,\mathcal{E})\in\mathbb{S}} \mathfrak{a}_{(i,\mathcal{E})}(\omega) \leqslant \sum_{(i,\mathcal{E})\in\mathbb{S}} \mathfrak{e}_{(i,\mathcal{E})}(\omega)$ for each $\omega \in \Omega$. An allocation rule $\mathfrak{a}'$ *Pareto dominates* an allocation rule $\mathfrak{a}$ *for coalition* $\mathbb{S}$ if $E(u_{(i,\mathcal{E})}(\mathfrak{a}'_{(i,\mathcal{E})})) > E(u_{(i,\mathcal{E})}(\mathfrak{a}_{(i,\mathcal{E})}))$ for each $(i, \mathcal{E}) \in \mathbb{S}$. The *core* is the set of allocation rules that are feasible for $\mathcal{N}$ and such that there do not exist a coalition $\mathbb{S}$ and an allocation rule $\mathfrak{a}'$ feasible for $\mathbb{S}$ that Pareto dominates $\mathfrak{a}$ for $\mathbb{S}$.

[6] Serrano and Vohra [18] is another paper going in that direction. They characterize the set of allocation rules that are robust to coalitional deviations that require the unanimous support of its members in some explicit voting games.

Proposition A1. *Let a be an allocation rule that belongs to the type-agent core. Then the associated allocation rule $\mathfrak{a}$ belongs to the core of the type-agent representation of the economy.*

Proof. Notice that $\mathfrak{a}$ is feasible for $\mathcal{N}$. Indeed, for each $\omega \in \Omega$, $\sum_{i\in N} a_i(\omega) \leqslant \sum_{i\in N} e_i(\omega)$ implies that $\sum_{i\in N} \mathfrak{a}_{(i,P_i(\omega))}(\omega) \leqslant \sum_{i\in N} \mathfrak{e}_{(i,P_i(\omega))}(\omega)$ which, in turn, implies that $\sum_{(i,\mathcal{E})\in\mathcal{N}} \mathfrak{a}_{(i,\mathcal{E})}(\omega) \leqslant \sum_{(i,\mathcal{E})\in\mathcal{N}} \mathfrak{e}_{(i,\mathcal{E})}(\omega)$. Suppose now that there exist a coalition $\mathbb{S}$ and an allocation rule $\mathfrak{a}'$ feasible for $\mathbb{S}$ that Pareto dominates $\mathfrak{a}$ for $\mathbb{S}$ in the type-agent representation of the economy. Given the preferences of the type-agents, I may assume without loss of generality that $\mathfrak{a}'_{(i,\mathcal{E})}(\omega) = 0$ for each $(i,\mathcal{E}) \in \mathcal{N}$ and each $\omega \in \Omega$ such that $\omega \notin \mathcal{E}$. Let then a' be the allocation rule defined as follows: $a'_i(\omega) := \mathfrak{a}'_{(i,P_i(\omega))}(\omega)$ for each couple $(i,\omega) \in N \times \Omega$ such that $(i, P_i(\omega)) \in \mathbb{S}$, and $a'_i(\omega) := 0$ for each other couple $(i,\omega) \in N \times \Omega$. Notice that, for each $\omega \in \Omega$, $i \in D(a,a',\omega)$ if and only if $(i, P_i(\omega)) \in \mathbb{S}$. We have

$$\begin{aligned}
\sum\nolimits_{i\in D(a,a',\omega)} a'_i(\omega) &= \sum\nolimits_{i\in N \text{ s.t. } (i,P_i(\omega))\in\mathbb{S}} a'_i(\omega) \\
&= \sum\nolimits_{i\in N \text{ s.t. } (i,P_i(\omega))\in\mathbb{S}} \mathfrak{a}'_{(i,P_i(\omega))}(\omega) \\
&= \sum\nolimits_{(i,\mathcal{E})\in\mathbb{S}} \mathfrak{a}'_{(i,\mathcal{E})}(\omega) \\
&\leqslant \sum\nolimits_{(i,\mathcal{E})\in\mathbb{S}} \mathfrak{e}_{(i,\mathcal{E})}(\omega) \\
&= \sum\nolimits_{i\in N \text{ s.t. } (i,P_i(\omega))\in\mathbb{S}} \mathfrak{e}_{(i,P_i(\omega))}(\omega) \\
&= \sum\nolimits_{i\in N \text{ s.t. } (i,P_i(\omega))\in\mathbb{S}} e_i(\omega) \\
&= \sum\nolimits_{i\in D(a,a',\omega)} e_i(\omega),
\end{aligned}$$

for each $\omega \in \Omega$. If a' is not *strictly* feasible when proposed against a, then one may slightly modify a' as follows. There exist $\omega \in \Omega$ and $i \in D(a,a',\omega)$ such that $a'_i(\omega) > 0$. One of the positive components of $a'_i(\omega)$ is decreased while keeping $D(a,a',\omega)$ unchanged. The resulting allocation rule is strictly feasible when proposed against a. □

Proposition A2. *Let $\mathfrak{a}$ be an allocation rule that belongs to the core of the type-agent representation of the economy. Then the associated allocation rule a belongs to the type-agent core.*

Proof. (a) Notice that a is feasible. Indeed, for each $\omega \in \Omega$, $\sum_{(i,\mathcal{E})\in\mathcal{N}} \mathfrak{a}_{(i,\mathcal{E})}(\omega) \leqslant \sum_{(i,\mathcal{E})\in\mathcal{N}} \mathfrak{e}_{(i,\mathcal{E})}(\omega)$ implies that $\sum_{i\in N} \mathfrak{a}_{(i,P_i(\omega))}(\omega) \leqslant \sum_{i\in N} \mathfrak{e}_{(i,P_i(\omega))}(\omega)$ which, in turn, implies that $\sum_{i\in N} a_i(\omega) \leqslant \sum_{i\in N} e_i(\omega)$. Suppose now that a is blocked by an allocation rule a' in the original economy. Let then $\mathbb{S}$ be the coalition defined as follows: $\mathbb{S} = \{(i,\mathcal{E}) \in \mathcal{N} | E(u_i(a'_i)|\mathcal{E}) > E(u_i(a_i)|\mathcal{E})\}$. Let also $\mathfrak{a}'$ be an allocation rule such that $\mathfrak{a}'_{(i,\mathcal{E})}(\omega) = a'_i(\omega)$ if $\omega \in \mathcal{E}$ and $\mathfrak{a}'_{(i,\mathcal{E})}(\omega) = 0$ if $\omega \in \Omega \setminus \mathcal{E}$, for each $(i,\mathcal{E}) \in \mathbb{S}$. Observe that $\mathfrak{a}'$ is feasible for $\mathbb{S}$. Indeed, we have

$$\begin{aligned}
\sum\nolimits_{(i,\mathcal{E})\in\mathbb{S}} \mathfrak{a}'_{(i,\mathcal{E})}(\omega) &= \sum\nolimits_{(i,\mathcal{E})\in\mathbb{S} \text{ s.t. } \omega\in\mathcal{E}} a'_i(\omega) \\
&= \sum\nolimits_{i\in D(a,a',\omega)} a'_i(\omega) \\
&\leqslant \sum\nolimits_{i\in D(a,a',\omega)} e_i(\omega) \\
&= \sum\nolimits_{(i,\mathcal{E})\in\mathbb{S} \text{ s.t. } \omega\in\mathcal{E}} e_i(\omega) \\
&= \sum\nolimits_{(i,\mathcal{E})\in\mathbb{S}} \mathfrak{e}_{(i,\mathcal{E})}(\omega),
\end{aligned}$$

for each $\omega \in \Omega$. Observe also that $\mathfrak{a}'$ Pareto dominates $\mathfrak{a}$ for $\mathbb{S}$. Indeed, we have

$$\begin{aligned} E(u_{(i,\mathcal{E})}(\mathfrak{a}'_{(i,\mathcal{E})})) &= \sum\nolimits_{\omega \in \mathcal{E}} \pi(\omega) u_i(a'_i(\omega), \omega) \\ &= \pi(\mathcal{E}) E(u_i(a'_i)|\mathcal{E}) \\ &> \pi(\mathcal{E}) E(u_i(a_i)|\mathcal{E}) \\ &= \sum\nolimits_{\omega \in \mathcal{E}} \pi(\omega) u_i(a_i(\omega), \omega) \\ &= E(u_{(i,\mathcal{E})}(\mathfrak{a}_{(i,\mathcal{E})})), \end{aligned}$$

for each $(i, \mathcal{E}) \in \mathbb{S}$. $\square$

An allocation rule $\mathfrak{a}$ is an *Arrow–Debreu equilibrium* in the type-agent representation of the economy if it is feasible for $\mathcal{N}$ and there exists a price system $p: \Omega \to \mathbb{R}^L_+$ such that

$$\mathfrak{a}_{(i,\mathcal{E})} \in \arg \max_{\mathfrak{a}'_{(i,\mathcal{E})} \in \mathfrak{B}_{(i,\mathcal{E})}(p)} E(u_{(i,\mathcal{E})}(\mathfrak{a}'_{(i,\mathcal{E})})),$$

where

$$\mathfrak{B}_{(i,\mathcal{E})}(p) := \left\{ \mathfrak{a}'_{(i,\mathcal{E})} \in \mathbb{R}^{L \times \Omega}_+ \,\middle|\, \sum_{\omega' \in \Omega} p(\omega') \cdot \mathfrak{a}'_{(i,\mathcal{E})}(\omega') \leqslant \sum_{\omega' \in \Omega} p(\omega') \cdot \mathfrak{e}_{(i,\mathcal{E})}(\omega') \right\}$$

for each $(i, \mathcal{E}) \in \mathcal{N}$.

Proposition B1. *Let a be a constrained market equilibrium. Then the associated allocation rule* $\mathfrak{a}$ *is an Arrow–Debreu equilibrium in the type-agent representation of the economy.*

Proof. Notice that $\mathfrak{a}$ is feasible for $\mathcal{N}$, as in Proposition A1. Let p be the price system supporting a as a constrained market equilibrium. Let $(i, \mathcal{E}) \in \mathcal{N}$. Notice that $\mathfrak{a}_{(i,\mathcal{E})} \in \mathfrak{B}_{(i,\mathcal{E})}(p)$, as $a_i \in B_i(p, \omega)$ for each $\omega \in \mathcal{E}$. Let $\mathfrak{a}'_{(i,\mathcal{E})} \in \mathfrak{B}_{(i,\mathcal{E})}(p)$. I have to prove that $E(u_{(i,\mathcal{E})}(\mathfrak{a}'_{(i,\mathcal{E})})) \leqslant E(u_{(i,\mathcal{E})}(\mathfrak{a}_{(i,\mathcal{E})}))$. Given the preferences of type-agent $(i, \mathcal{E})$, I may assume without loss of generality that $\mathfrak{a}'_{(i,\mathcal{E})}(\omega) = 0$ for each $\omega \in \Omega \setminus \mathcal{E}$. Let $\omega \in \mathcal{E}$ and let $a'_i \in \mathbb{R}^{L \times \Omega}_+$ be defined as follows: $a'_i(\omega') := \mathfrak{a}'_{(i,P_i(\omega))}(\omega')$ for each $\omega' \in \Omega$. Then, $a'_i \in B(p, \omega)$ and hence $E(u_i(a'_i)|P_i(\omega)) \leqslant E(u_i(a_i)|P_i(\omega))$. It is then easy to conclude, as $E(u_{(i,\mathcal{E})}(\mathfrak{a}'_{(i,\mathcal{E})})) = \pi(P_i(\omega))$ $E(u_i(a'_i)|P_i(\omega))$ and $E(u_{(i,\mathcal{E})}(\mathfrak{a}_{(i,\mathcal{E})})) = \pi(P_i(\omega)) E(u_i(a_i)|P_i(\omega))$. $\square$

Proposition B2. *Let* $\mathfrak{a}$ *be an Arrow–Debreu equilibrium in the type-agent representation of the economy. Then the associated allocation rule a is a constrained market equilibrium.*

Proof. Notice that a is feasible, as in Proposition A2. Let p be the price system supporting $\mathfrak{a}$ as an Arrow–Debreu equilibrium in the type-agent representation of the economy. Let $i \in N$ and let $\omega \in \Omega$. Notice that $a_i \in B_i(p, \omega)$, as $\mathfrak{a}_{(i,P_i(\omega))} \in \mathfrak{B}_{(i,P_i(\omega))}(p)$. Let $a'_i \in B_i(p, \omega)$ and let $\mathfrak{a}'_{(i,P_i(\omega))} \in \mathbb{R}^{L \times \Omega}_+$ be defined as follows: $\mathfrak{a}'_{(i,P_i(\omega))}(\omega') := a'_i(\omega')$ for each $\omega' \in P_i(\omega)$ and $\mathfrak{a}'_{(i,P_i(\omega))}(\omega') := 0$ for each $\omega' \in \Omega \setminus P_i(\omega)$. Then, $\mathfrak{a}'_{(i,P_i(\omega))} \in \mathfrak{B}_{(i,P_i(\omega))}(p)$. Hence, $E(u_i(a'_i)|P_i(\omega)) = \frac{E(u_{(i,P_i(\omega))}(\mathfrak{a}'_{(i,P_i(\omega))}))}{\pi(P_i(\omega))} \leqslant \frac{E(u_{(i,P_i(\omega))}(\mathfrak{a}_{(i,P_i(\omega))}))}{\pi(P_i(\omega))} = E(u_i(a_i)|P_i(\omega))$. $\square$

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