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Games and Economic Behavior 53 (2005) 73–82

GAMES and Economic Behavior

www.elsevier.com/locate/geb

Values for cooperative games with incomplete information: An eloquent example ☆

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Received 10 June 2002

Available online 10 December 2004

Abstract

Myerson's [Cooperative games with incomplete information. Int. J. Game Theory 13 (1984) 69–96] extension of the λ-transfer value to cooperative games with incomplete information focuses among other things on the strength of the incentive constraints at the solution for determining the power of coalitions. We construct an intuitive three-player game where the player whose *only* contribution is to partly release the two other players from the incentive constraints they face when they cooperate, receives a zero payoff according to Myerson's solution. On the contrary, the random order arrival procedure attributes a strictly positive payoff to him. Our example is thus an analog of the banker game of Owen [Values of games without side payments. Int. J. Game Theory 1 (1972) 95–109] that was designed for evaluating Shapley's λ-transfer value under complete information. Asymmetric information now takes up the role that was formerly attributed to the lack of transferability of utilities.

JEL classification: C71; C78; D82

Keywords: Cooperative games; Incomplete information; Virtual utility

☆ The present paper is a revised version of the fifth chapter of my PhD dissertation written at CORE (Université Catholique de Louvain, Belgium).

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0899-8256/$ – see front matter
doi:10.1016/j.geb.2004.09.004

1. Introduction

Cooperative game theory proposes models explaining how the benefits of cooperation are shared between the participants.

The Shapley (1953) value plays a prominent role in the class of fair solutions defined for games with transferable utility (TU). Numerous axiomatic and non-cooperative justifications were given.

Later on, techniques were developed in order to extend the Shapley value to games with non-transferable utility (NTU). For instance, the fictitious-transfer procedure of Shapley (1969) (see also Myerson, 1992) allows to extend any solution concept defined for TU games to some larger class of NTU games. When applied to the Shapley (1953) value, it gives the so-called λ-transfer value or the Shapley NTU value. The pertinence of the λ-transfer value was tested afterwards in many different ways. In particular, the banker game of Owen (1972) appeared to be a very constructive example. Two players generate some surplus by cooperating (e.g. via the provision of a public good, or via mutually beneficial exchanges), but are limited in their ability to share this surplus. The third player (the "banker") can only release them from this restriction. In this context, should he receive a strictly positive payoff? According to the λ-transfer value, the answer is no. Other fairness criteria, such as the random order arrival procedure (see Maschler and Owen, 1989), imply a positive answer to the question.

Adding the possibility of asymmetric information raises two conceptual issues. First, the players may have an interest to communicate and hence to agree on mechanisms. As a consequence of the revelation principle, we have to restrict ourselves to direct mechanisms that are Bayesian incentive compatible. Second, the bargaining stage itself may convey information, a player insisting heavily on a particular incentive compatible mechanism could be a signal about his private information. Myerson (1984b) proposed a general notion of value that takes both aspects into account. He generalized the fictitious-transfer procedure in order to extend the Shapley (1953) value to a large class of cooperative games with incomplete information. Computing the solution can be difficult and, as far as we know, no example with more than two players has ever been studied before.

The solution focuses among other things on the strength of the incentive constraints at the solution for determining the power of coalitions. Starting from the bargaining problem studied in Section 10 of Myerson (1984a), we add a third player whose *only* contribution is to partly release the two original players from the incentive constraints they face when they cooperate. Some may consider this contribution important enough for giving a strictly positive payoff to the third player in any fair solution. The random order arrival procedure appears to abide to this principle, while Myerson's (1984b) solution does not. Our example is thus an analog of the banker game of Owen (1972) where asymmetric information now takes up the role that was formerly attributed to the lack of transferability of utilities.

Other values for exchange economies under asymmetric information have been proposed by Allen (1991) and by Krasa and Yannelis (1994). Their main objective is to study fair solutions when bargaining takes place at the ex-ante stage, i.e. before the agents learn their private information. This constitutes an important difference with respect to Myerson's model. We show in section 5 that, in this context, the third player does not generate any additional surplus by cooperating with the two other players. Hence he receives a zero payoff according to both the λ-transfer value and the random order arrival procedure. There is no puzzle.

2. The example

We consider a three-player cooperative game whose intuitive interpretation goes as follows. Players 1 and 2 face a bilateral trade problem under asymmetric information. Player 2 owns one unit of a good that has no value for himself, but that has some value for player 1. This value can be relatively low ($30), with probability 0.2, or relatively high ($90), with probability 0.8. The problem is then to determine the quantity sold by player 2 and the price paid by player 1. Thus far, the example is quite similar to the one treated by Myerson (1979, 1984a). Although utilities are transferable ex-post, the two players are limited in their abilities to share the cooperative gains at the interim stage. For instance, the mechanism that gives the entire surplus to player 2 in both states (i.e. "give the good to player 1 in exchange of 30 or 90 dollars depending on the value he attributes to the good") is not incentive compatible. Player 3 allows to weaken these incentive constraints. We model this fact by adding collective decisions that give the whole surplus in both states to either players 2 or 3 when the three players cooperate. The question is then: how much should players 1 and 2 reward the third player for his services?

Following the formalism introduced by Myerson (1984b), we define the game as follows: $N = \{1, 2, 3\}$, $T_1 = \{L, H\}$, $T_2 = T_3 = \{*\}$, $\text{Prob}(L) = 0.2$, $D_{\{1\}} = \{d_1\}$, $D_{\{2\}} = \{d_2\}$, $D_{\{3\}} = \{d_3\}$, $D_{\{1,3\}} = \{[d_1, d_3]\}$, $D_{\{2,3\}} = \{[d_2, d_3]\}$, $D_{\{1,2\}} = \{[d_1, d_2], d_{12}, d_{21}\}$, $D_{\{1,2,3\}} = \{[d_1, d_2, d_3], [d_{12}, d_3], [d_{21}, d_3], d_{213}, d_{312}\}$, and

	$[d_1, d_2, d_3]$	$[d_{12}, d_3]$	$[d_{21}, d_3]$	d_{213}	d_{312}
$u_1(\cdot, L)$	0	30	−60	0	0
$u_1(\cdot, H)$	0	90	0	0	0
$u_2(\cdot, L)$	0	0	90	30	0
$u_2(\cdot, H)$	0	0	90	90	0
$u_3(\cdot, L)$	0	0	0	0	30
$u_3(\cdot, H)$	0	0	0	0	90

For each coalition S, D_S represents the set of collective decisions available to its members. As far as coalition $\{1, 2\}$ is concerned, decision $[d_1, d_2]$ represents the no-exchange alternative. Decision d_{12} (respectively d_{21}) represents the situation where player 2 receives the good from player 1 for free (respectively in exchange of $90). Any other transfer of money from player 1 to player 2 (between $0 and $90) can be represented by a lottery defined on $\{d_{12}, d_{21}\}$. As far as the grand coalition is concerned, the collective decisions d_{213} and d_{312} are added to $D_{\{1,2\}} \times D_{\{3\}}$. Decision d_{213} (respectively d_{312}) amounts to give the whole surplus to player 2 (respectively 3) in both states of the world. Putting some positive weight on d_{312} allows to reward player 3 for his services.

Let S be a coalition. A *mechanism for* S is a function $m_S : T_S \to \Delta(D_S)$. When S is different from $\{1, 2\}$ or $\{1, 2, 3\}$, there exists only one mechanism, as D_S then corresponds to a singleton. Let S be the coalition $\{1,2\}$ or the grand coalition and let m_S be a mechanism. Then, m_S is *incentive compatible* if[1] $\sum_{d\in D_S} m_S(d|t)u_1(d,t) \geqslant \sum_{d\in D_S} m_S(d|t')u_1(d,t)$, for each $(t,t') \in \{L,H\} \times \{L,H\}$ such that $t \neq t'$. In other words, m_S is incentive compatible if and only if player 1 does not have a strict interest to misreport his type. Let m and m' be two mechanisms for the grand coalition. Then, m' *interim Pareto dominates* m if $u_1(m'(L),L) \geqslant u_1(m(L),L)$, $u_1(m'(H),H) \geqslant u_1(m(H),H)$, $0.2u_2(m'(L),L)+0.8u_2(m'(H),H) \geqslant 0.2u_2(m(L),L)+0.8u_2(m(H),H)$ and $0.2u_3(m'(L),L) + 0.8u_3(m'(H),H) \geqslant 0.2u_3(m(L),L) + 0.8u_3(m(H),H)$, with at least one of the four inequalities being strict. The mechanism m is *interim incentive efficient* if it is incentive compatible and there does not exist any other incentive compatible mechanism that interim Pareto dominates it.

3. Virtual utility solution

A *game with transferable utility* is a function $v : P(N) \to \mathbb{R}$. Shapley's (1953) value is denoted *Sh*, i.e.

$$Sh_i(v) := \sum_{S\in P(N) \text{ s.t. } i\in S} \frac{(s-1)!(n-s)!}{n!}\big(v(S) - v(S\setminus\{i\})\big),$$

for each $i \in N$. We assume that it appropriately represents fairness for TU-games under complete information.

Myerson's (1984b) solution is an extension of the λ-transfer value (see Shapley, 1969) defined for cooperative games with incomplete information. The *virtual utilities* used in order to express interpersonal comparisons of utilities do not only involve a possible rescaling of the individual utilities, but also some adjustment associated to the presence of incentive constraints. For our example, a mechanism m for the grand coalition is an *M-solution* if it is incentive compatible and there exists a vector $(\lambda_{1.L}, \lambda_{1.H}; \lambda_2; \lambda_3) \in \mathbb{R}^4_{++}$ and a vector $(\alpha(L|H), \alpha(H|L)) \in \mathbb{R}^2_+$ such that

$$\begin{cases}
(1) \quad \alpha(H|L)(u_1(m(L),L) - u_1(m(H),L)) = 0, \\
(2) \quad \alpha(L|H)(u_1(m(H),H) - u_1(m(L),H)) = 0, \\
(3) \quad v_1^{(\lambda,\alpha)}(m(L),L) = Sh_1(v^{(\lambda,\alpha)}(L)), \\
(4) \quad v_1^{(\lambda,\alpha)}(m(H),H) = Sh_1(v^{(\lambda,\alpha)}(H)), \\
(5) \quad 0.2v_2^{(\lambda,\alpha)}(m(L),L) + 0.8v_2^{(\lambda,\alpha)}(m(H),H) = 0.2Sh_2(v^{(\lambda,\alpha)}(L)) \\
\qquad\quad + 0.8Sh_2(v^{(\lambda,\alpha)}(H)), \\
(6) \quad 0.2v_3^{(\lambda,\alpha)}(m(L),L) + 0.8v_3^{(\lambda,\alpha)}(m(H),H) = 0.2Sh_3(v^{(\lambda,\alpha)}(L)) \\
\qquad\quad + 0.8Sh_3(v^{(\lambda,\alpha)}(H)),
\end{cases}$$

[1] Let $S \in P(N)$, let $d_S \in D_S$, let $t \in T$ and let $i \in S$. Then, $u_i(d_S,t)$ is well-defined, as $u_i((d_S, d_{N\setminus S}), t) = u_i((d_S, d'_{N\setminus S}), t)$ for each $(d_{N\setminus S}, d'_{N\setminus S}) \in D_{N\setminus S} \times D_{N\setminus S}$. The coalitions are *orthogonal* in the terminology of Myerson (1984b).

where, for each $d \in D_{\{1,2,3\}}$,

$$\begin{cases} v_1^{(\lambda,\alpha)}(d,L) := \frac{1}{0.2}[(\lambda_{1.L} + \alpha(H|L))u_1(d,L) - \alpha(L|H)u_1(d,H)], \\ v_1^{(\lambda,\alpha)}(d,H) := \frac{1}{0.8}[(\lambda_{1.H} + \alpha(L|H))u_1(d,H) - \alpha(H|L)u_1(d,L)], \\ v_2^{(\lambda,\alpha)}(d,L) := \lambda_2 u_2(d,L), \\ v_2^{(\lambda,\alpha)}(d,H) := \lambda_2 u_2(d,H), \\ v_3^{(\lambda,\alpha)}(d,L) := \lambda_3 u_3(d,L), \\ v_3^{(\lambda,\alpha)}(d,H) := \lambda_3 u_3(d,H), \end{cases}$$

and, for each $t \in \{L,H\}$, $v^{(\lambda,\alpha)}(t)$ is the TU-game whose value is zero for each coalition different from $\{1,2\}$ and $\{1,2,3\}$. When S equals $\{1,2\}$ or $\{1,2,3\}$, then

$$v^{(\lambda,\alpha)}(S,t) := \max_{d_S \in D_S} \sum_{i \in S} v_i^{(\lambda,\alpha)}(d_S,t_S).$$

Conditions (1) and (2) are traditional complementary slackness conditions. Given λ and α, the number $v_i^{(\lambda,\alpha)}(d,t)$ is the *virtual utility* of decision d in state t for player i. It amounts to a rescaling of $u_i(d,t)$ that is then adjusted by the utility received for d by the other types of player i. The vector α influences the virtual utility for player 1 only if some incentive constraint is binding at the solution. It can be shown that the M-solution is interim incentive efficient, that the vector λ supporting it is orthogonal to the set of interim allocations achievable by incentive compatible mechanisms at the interim allocation $(u_1(m(L),L); u_1(m(H),H); 0.2u_2(m(L),L) + 0.8u_2(m(H),H); 0.2u_3(m(L),L) + 0.8u_3(m(H),H))$, and that the vector α corresponds to a list of dual variables associated to the incentive constraints involved in this linear programming problem. The *warrant equations* (3)–(6) can be interpreted as follows. Considering that, in (λ,α)-virtual utilities, types are verifiable and utilities are transferable, the players should agree on a mechanism that generates the (λ,α)-virtual utility profile $(Sh_1(v^{(\lambda,\alpha)}(L)), Sh_1(v^{(\lambda,\alpha)}(H)); 0.2Sh_2(v^{(\lambda,\alpha)}(L)) + 0.8Sh_2(v^{(\lambda,\alpha)}(H)); 0.2Sh_3(v^{(\lambda,\alpha)}(L)) + 0.8Sh_3(v^{(\lambda,\alpha)}(H)))$. Following Shapley's (1969) philosophy the vector (λ,α) specifying the relevant virtual utility scales, is then determined endogenously in order to obtain the feasibility of the corresponding (λ,α)-virtually fair utility profile.

Proposition 1. $(15,45;39;0)$ *is the only interim allocation that can be supported by some M-solution.*

Proof. The pooling mechanism[2] $(\frac{1}{2}[d_{12},d_3] + \frac{1}{2}d_{213}, \frac{1}{2}[d_{12},d_3] + \frac{1}{2}d_{213})$ is an M-solution. Indeed, the conditions are satisfied for $\lambda = (0.2, 0.8; 1; 1)$ and $\alpha = (0,0)$. It is natural to take $\alpha(L|H) = \alpha(H|L) = 0$ as the mechanism is ex-post optimal, which means that the

[2] By convention, a mechanism is denoted by a couple, the first (respectively second) component corresponding to the lottery chosen after player 1 reported a low (respectively high) type.

incentive constraints are not essential. In fact, the mechanism gives the Shapley value in both ex-post games. It generates the interim allocation $(15, 45; 39; 0)$.

We now show that no other interim allocation can be supported by some M-solution. Let m be an M-solution. Let $(\lambda_{1.L}, \lambda_{1.H}; \lambda_2; \lambda_3) \in \mathbb{R}^4_{++}$ and $(\alpha(L|H), \alpha(H|L)) \in \mathbb{R}^2_+$ be the associated vectors.

(1) $u_1(m(L), L) \geqslant 10$ and $u_1(m(H), H) \geqslant 30$.
We have

$$\begin{cases} Sh_1(v^{(\lambda,\alpha)}(L)) \geqslant \frac{1}{0.2}[(\lambda_{1.L} + \alpha(H|L))10 - \alpha(L|H)30], \\ Sh_1(v^{(\lambda,\alpha)}(H)) \geqslant \frac{1}{0.8}[(\lambda_{1.H} + \alpha(L|H))30 - \alpha(H|L)10]. \end{cases}$$

Indeed, player 1 of type t has non-negative marginal contributions in the game $v^{(\lambda,\alpha)}(t)$, for both $t \in \{L, H\}$, $v^{(\lambda,\alpha)}(N, L) \geqslant \frac{1}{0.2}[(\lambda_{1.L} + \alpha(H|L))30 - \alpha(L|H)90]$, and $v^{(\lambda,\alpha)}(N, H) \geqslant \frac{1}{0.8}[(\lambda_{1.H} + \alpha(L|H))90 - \alpha(H|L)30]$.

Developing conditions (3) and (4) appearing in the definition of M-solutions and using the complementary slackness conditions, the two previous inequalities may be rewritten as follows:

$$A \cdot \begin{pmatrix} u_1(m(L), L) \\ u_1(m(H), H) \end{pmatrix} \geqslant A \cdot \begin{pmatrix} 10 \\ 30 \end{pmatrix},$$

where

$$A := \begin{bmatrix} \lambda_{1.L} + \alpha(H|L) & -\alpha(L|H) \\ -\alpha(H|L) & \lambda_{1.H} + \alpha(L|H) \end{bmatrix}.$$

The matrix A is invertible and its inverse has non-negative entries.[3] It is then easy to conclude.

(2) $\sum_{i=1}^{3} v_i^{(\lambda,\alpha)}(m(t), t) = v^{(\lambda,\alpha)}(N, t)$ for each $t \in \{L, H\}$.
Indeed, taking a weighted sum of the warrant equations, we obtain that $0.2 \sum_{i=1}^{3} v_i^{(\lambda,\alpha)}(m(L), L) + 0.8 \sum_{i=1}^{3} v_i^{(\lambda,\alpha)}(m(H), H) = 0.2 v^{(\lambda,\alpha)}(N, L) + 0.8 v^{(\lambda,\alpha)}(N, H)$.

(3) $v^{(\lambda,\alpha)}(N, L) = \frac{1}{0.2}[(\lambda_{1.L} + \alpha(H|L))30 - \alpha(L|H)90]$.
Let us suppose on the contrary that $v^{(\lambda,\alpha)}(N, L) > \frac{1}{0.2}[(\lambda_{1.L} + \alpha(H|L))30 - \alpha(L|H)90]$. By item (2), $m([d_{12}, d_3]|L) = 0$. So, $u_1(m(L), L) \leqslant 0$ which contradicts item (1).

(4) $v^{(\lambda,\alpha)}(N, H) = \frac{1}{0.8}[(\lambda_{1.H} + \alpha(L|H))90 - \alpha(H|L)30]$: similar to item (3).

(5) $Sh(v^{(\lambda,\alpha)}(t)) = (\frac{1}{2} v^{(\lambda,\alpha)}(N, t), \frac{1}{2} v^{(\lambda,\alpha)}(N, t), 0)$ for both $t \in \{L, H\}$.
Indeed, the third player is a null player (given items (3) and (4)) and the two first players are symmetric in $v^{(\lambda,\alpha)}(t)$, for both $t \in \{L, H\}$.

(6) $m(d_{312}|L) = m(d_{312}|H) = 0$.

[3] This observation constitutes in fact a particular case of Lemma 1 in Myerson (1983).

Indeed, given item (5), Eq. (6) appearing in the definition of M-solutions becomes $\lambda_3(0.2u_3(m(L), L) + 0.8u_3(m(H), H)) = 0$. In particular, the third player expects a null payoff from m.

(7) $u_1(m(L), L) = 15$ and $u_2(m(H), H) = 45$.
Using items (3)–(5), as well as the complementary slackness conditions, the warrant equations (3) and (4) may be rewritten as follows:

$$A \cdot \begin{pmatrix} u_1(m(L), L) \\ u_1(m(H), H) \end{pmatrix} = A \cdot \begin{pmatrix} 15 \\ 45 \end{pmatrix},$$

where the matrix A was defined in item (1). A being invertible, it is easy to conclude.

(8) Player 2 expects a payoff of 39 from m.
By item (7), $m([d_{12}, d_3]|H) = 1/2$. As m is incentive compatible, $m([d_{12}, d_3]|L) \leqslant 1/2$ and we must have equality together with $m([d_{21}, d_3]|H) = 0$ in order to have $u_2(m(L), L) = 15$. At the same time, $m([d_1, d_2, d_3]|L) = m([d_1, d_2, d_3]|H) = 0$ by item (2). Then, $m(d_{213}|L) = 1/2$ and $m(d_{213}|H) + m([d_{21}, d_3]|H) = 1/2$. Anyway, d_{213} and $[d_{21}, d_3]$ are utility equivalent in the high state. $\square$

4. Random order arrival

We observe that player 3 is considered de facto as a null player according to the M-solution. This is due to the fact that the virtual value of coalition $\{1, 2\}$ is computed while using the vector (λ, α) as specified for the grand coalition. By doing so, we act as if incentive constraints do not matter in coalition $\{1, 2\}$, although they do. Even though it is true that player 3 does not create any surplus per se, it could look fair to give him some positive payoff, as players 1 and 2 have to rely on him in order to weaken the incentive constraints they face when they cooperate. As it was the case for the banker game under complete information, the *random order arrival procedure* generates an interesting alternative to the virtual utility solution in our example. This procedure could also be considered as being the natural generalization of the *random dictatorship* approach proposed by Myerson (1984a) to games involving more than two players. We formalize explicitly the procedure via a Bayesian game in extensive form, in order to be sure that we correctly take into account the influence of asymmetric information.

After player 1 has learned his type, a specific order o of the three players is chosen at random according to a uniform probability distribution. Player $o(3)$ proposes a mechanism m. Afterwards, player $o(2)$ chooses whether to accept player $o(3)$'s proposal. If he accepts, then player $o(1)$ chooses whether to accept m. If both players accept m, then it is implemented. If player $o(2)$ rejects m, then he proposes a mechanism $m' : T_{o(1)} \times T_{o(2)} \to D_{\{o(1),o(2)\}}$ and player $o(3)$ is left alone, having to choose $d_{o(3)}$. Player $o(1)$ chooses whether to accept player $o(2)$'s proposal. If he accepts, then m' is implemented. Whenever player $o(1)$ rejects some proposition, there is no cooperation and the outcome is $[d_1, d_2, d_3]$. For simplicity, we assume that the players accept a proposal when they are indifferent between accepting and rejecting, and that player 1 tells the truth when he is indifferent between lying and telling the truth.

A *strategy* for a player specifies an action at each of his information sets. For player 1, it is contingent on his type. A *belief system* specifies a probability distribution over the set of types of player 1 at each information set of players 2 and 3. A *weak sequential equilibrium* (see Myerson (1991)) specifies a strategy for each player and a belief system such that

(1) the belief system is consistent in the sense that the beliefs are obtained from the initial beliefs (i.e. $\text{Prob}(\text{L}) = 0.2$) by Bayesian updating on the equilibrium path;
(2) the action of each player at each of his information sets is optimal given his beliefs and the equilibrium strategies played in the continuation games.

Proposition 2. $(15, 45; 38; 1)$ *is the only interim allocation that can be supported by some weak sequential equilibrium.*

Proof. We characterize the set of allocations that can be supported by some weak sequential equilibrium in each sub-game starting after some specific order of the players has been chosen. If $o(3) = 2$, player 2 obtains an expected payoff of 78 in equilibrium. Indeed, for any proposal of player 2, his expected payoff in the continuation game is lower or equal to $78 (= 0, 2.30 + 0, 8.90)$. On the other hand, he can obtain 78 by proposing, for instance, the mechanism (d_{213}, d_{213}). If $o = (1, 2, 3)$, then player 2 proposes the mechanism $([d_1, d_2], d_{21})$ to player 1. This amounts to solve a simple screening game. Given the associated reservation utilities $(0, 0; 72)$ for players 1 and 2, each proposal of player 3 will give him at most an expected payoff of 6. He can obtain this expected payoff of 6 by proposing, for instance, the mechanism (d_{312}, d_{213}). When player 1 is the first mover, matters could be more complicated as we have to solve a signaling game. Nevertheless, in our case, the mechanism $([d_{12}, d_3], [d_{12}, d_3])$ is the most preferred by both types of player 1 and it satisfies the ex-post individual rationality constraints of both players 2 and 3 (it is a kind of strong solution using the terminology introduced by Myerson, 1983). In equilibrium, player 1 of type L (respectively H) obtains a payoff of 30 (respectively 90). Similarly, for the enumeration $(2, 1, 3)$, players 1 and 2 "agree" on the mechanism (d_{12}, d_{12}), so that there is no more surplus left for player 3 who has to distribute the reservation utilities $(30, 90; 0)$ to players 1 and 2. We record the various orderings and the associated "marginal contribution vectors" in Table 1.

The result is obtained by computing the mean of the vectors appearing in the right column. □

Table 1

o	$MCV(o)$
$(1, 2, 3)$	$(0, 0; 72; 6)$
$(1, 3, 2)$	$(0, 0; 78; 0)$
$(2, 1, 3)$	$(30, 90; 0, 0)$
$(2, 3, 1)$	$(30, 90; 0; 0)$
$(3, 1, 2)$	$(0, 0; 78; 0)$
$(3, 2, 1)$	$(30, 90; 0; 0)$

5. Ex-ante bargaining

Let us briefly study our example when bargaining takes place at the ex-ante stage as in Allen (1991) and in Krasa and Yannelis (1994). The players are symmetrically informed at the time of contracting but are asymmetrically informed at the time of implementing the contracts. The main conceptual issue amounts to determine the set of feasible agreements for each coalition. The two previous papers focused on measurability conditions. Others impose incentive compatibility constraints, see Section 4.1.1 of Forges et al. (2002) for a thorough discussion on that point. Once the set of feasible agreements is specified, we have a characteristic function and we may apply any solution concept defined for NTU games. Allen, Krasa and Yannelis study, for instance, the λ-transfer value. Let us follow this approach in our example.

The analysis is valid for both the private measurability and the incentive constraints because of the simple information structure. The ex-ante characteristic function is given by: $V(\{1\}) = V(\{2\}) = V(\{3\}) = \{0\}$, $V(\{1,3\}) = V(\{2,3\}) = \{(0,0)\}$, $V(\{1,2\}) = ch\{(0,0),(78,0),(-12,90)\}$ and $V(\{1,2,3\}) = ch\{(0,0,0),(78,0,0),(-12,90,0),(0,0,78)\}$, where ch denotes the convex hull operator. The unique λ-transfer value is $(36;36;0)$. Although player 3 receives a null payoff, there is no puzzle in the present case. Indeed, players 1 and 2 are not constrained at the ex-ante stage by the information that player 1 will acquire. Interim individual rationality constraints are not relevant anymore. For instance, the incentive compatible mechanism $(\frac{2}{15}d_{12} + \frac{13}{15}d_{21}, \frac{2}{15}d_{12} + \frac{13}{15}d_{21})$ for coalition $\{1,2\}$ gives the whole ex-ante surplus to player 2 as it generates the ex-ante allocation $(0;78)$.[4] It is not surprising then that the random order arrival procedure supports the same ex-ante allocation $(36;36;0)$.

6. Conclusion

The banker game highlighted the fact that the λ-transfer value defined for NTU games does not reward the players for some kinds of contribution. This example was at the origin of various papers that increased our understanding of the concepts of fairness for NTU games. Aumann (1985) axiomatized the λ-transfer value, Maschler and Owen (1989, 1992) proposed the consistent Shapley value, an attractive alternative value based on the random order arrival procedure, and Hart and Mas-Colell (1996) developed explicit bargaining procedures in order to support it non-cooperatively.

Our example shows that Myerson's (1984b) value for games with incomplete information is itself insensitive to some informational contributions. Far from being a criticism against Myerson's approach, we hope that it will stimulate further research on the topic of cooperative games with incomplete information.

[4] The associated interim allocation $(-48,12;78)$ is not interim individually rational.

Acknowledgments

I thank Professors Jean-François Mertens, Enrico Minelli and Roger B. Myerson for their useful comments, as well as the three anonymous referees and the associate editor whose constructive advices helped me a lot for improving the paper. The usual disclaimer applies.

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