Online Appendix

This appendix presents further results and technical details for *Bounded Rationality and Limited Datasets* (by Geoffroy de Clippel and Kareen Rozen).

Contents

A Our Methodology Applied to Additional Theories

We introduce additional theories in Section A.1, and show in Section A.2 how their empirical content can be captured by generalized SARP conditions, that is, the existence of an acyclic relation that satisfies a set of restrictions inferred from choices. Such results thus mirror those presented in Section 3 of the paper for Limited Attention and Categorization/Rationalization. In Section A.3, we show that, for all these additional theories, restrictions are elementary, and that making a valid guess in each step of the enumeration is easy. As a consequence of our results in the paper, all these theories can thus be tested through enumeration in a way that is roughly as easy as rationality.

A.1 Additional Theories

Order Rationalization (CFS13) is simply the variant of Rationalization where the preference is required, in addition, to be an ordering.

Under *Consistent Reference Points*, the DM views one alternative in each choice problem as his reference, and picks the best alternative according to his reference-dependent preference ordering. Reference points are assumed to

be consistent in the following sense: if x is the reference point in a choice problem S, then x remains the reference point in subsets of S containing x. This theory is essentially equivalent to Rubinstein and Salant's theory of Triggered Rationality, where the most salient alternative triggers the rationale used to make a choice.[1] It can also be seen as capturing a form of Ariely, Lowenstein and Prelec's 'coherent arbitrariness'.[2]

The class of *Minimal Consideration* theories extends rational choice by bounding from below the number of options considered. Theories in this class are indexed by a function k that associates to each problem S an integer between 1 and $|S|$. The DM uses an ordering P to pick the best element in his consideration set $\Gamma(S)$, which must contain at least $k(S)$ elements. The function k, which fixes a theory, limits the extent of a DM's 'mistakes.' If $k(S) = |S| - 1$ for all S, then the DM always picks from the top two options in a choice problem; if $k(S) = \lceil (1 - \alpha)|S| \rceil$ for all S, then the DM always picks from the top α-percentile. Theories in this class can also capture a DM who becomes overwhelmed in large choice problems, with $k(S)$ decreasing in $|S|$.

A.2 Testable Implications via Generalized SARP Conditions

We start by studying the testable implications of Order Rationalization. As in Rationalization, the choice from a set is also considered in subsets. Thus y is revealed preferred to x, denoted $y \succ^*_{OR} x$, if y is chosen in the presence of x, which itself is the choice from a superset.[3] Let $\mathcal{R}_{OR}(c_{obs})$ be the collection of restrictions that y is ranked above x for any x, y with $y \succ^*_{OR} x$. The next result shows that CFS13's full-data characterization of Order Rationalization in terms of the acyclicity of $\succ^*_{OR}$ extends to limited datasets.

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[1]Rubinstein, Ariel and Yuval Salant (2006), Two Comments on the Principle of Revealed Preference, *mimeo.* In addition, studying choice from lists, Rubinstein and Salant [*Theoretical Economics*, **1**, 3 (2006)] propose a model (their Example 4) where the DM's preference depends on the first element presented. Consistent Reference Points can be seen as the case where the list is unknown, or subjectively determined.

[2]Ariely, D., G. Loewenstein, and D. Prelec. (2003), 'Coherent Arbitrariness': Stable Demand Curves Without Stable Preferences, *Quarterly Journal of Economics*, 118, 73–106.

[3]This revealed preference is identified by CFS13 when $\mathcal{D} = \mathcal{P}(X)$.

Proposition 6. *The observed choice function c_{obs} is consistent with Order Rationalization if and only if there is an acyclic relation satisfying $\mathcal{R}_{OR}(c_{obs})$.*

We now turn our attention to Consistent Reference Points. Since the DM's reference point satisfies IIA, it can be interpreted as maximal for a 'salience ordering' $\succ_{REF}$. If x is the reference point in a choice problem, then it remains the reference point in smaller problems containing it; and the DM's choices in those problems all arise from maximizing the same preference ordering $\succ_x$. For any choice problem S and any $x \in S$, we say that $a \succ^*_{S,x} b$ if $c_{obs}(R) = a$ for some $R \subseteq S$ that contains b and x. This is the revealed preference under the *supposition* that x is the DM's reference point in S. Observe that when $\succ^*_{S,x}$ is cyclic, then x cannot be the reference point in S, and therefore cannot be the most salient alternative in S. Let $\mathcal{R}_{REF}$ be the following collection of restrictions on $\succ_{REF}$: for each $S \in \mathcal{P}(X)$ and $x \in S$ such that $\succ^*_{S,x}$ is cyclic, there is $y \in S \setminus \{x\}$ with $y \succ_{REF} x$.

Proposition 7. *The observed choice function c_{obs} is consistent with Consistent Reference Points if and only if there is an acyclic relation satisfying $\mathcal{R}_{REF}$.*

Interestingly, the proof reveals that it is without loss of generality to require that if x is preferred to y when the reference point is y, then x is also preferred to y when the reference point is x.[4]

To understand the testable implications of Minimal Consideration theories, we start by fixing a theory in this class, which is described by a given function $k : \mathcal{P}(X) \to \mathbb{N}$. If the DM picks x from S, then there must exist at least $k(S)-1$ alternatives in S that are inferior to x. These restrictions are summarized by $\mathcal{R}_k = \{(c_{obs}(S), \mathcal{T}_S) \mid S \in \mathcal{D}\}$, where $\mathcal{T}_S$ denotes the set of subsets of $S \setminus \{c_{obs}(S)\}$ with exactly $k(S) - 1$ elements.

Proposition 8. *For each $k : \mathcal{P}(X) \to \mathbb{N}$, the observed choice function c_{obs} is consistent with k-Minimal Consideration if and only if there is an acyclic relation satisfying $\mathcal{R}_k$.*

The proofs of Propositions 6-8 appear in Section A.4 below.

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[4]Such a reference effect is related to status quo bias; see Tversky and Kahneman [*The Quarterly Journal of Economics*, **106**, 1039 (1991)] and Masatlioglu and Ok [*Journal of Economic Theory*, **121**, 1 (2005)].

A.3 Enumeration

In this section, we apply Proposition 4 from the paper with Propositions 6-8 to show that testing each of the theories discussed in this appendix is roughly as easy as testing rationality.

For Order Rationalization and Minimal Consideration theories, one sees that Proposition 4 applies since, for any c_{obs}, all the restrictions in $\mathcal{R}_{OR}(c_{obs})$ and $\mathcal{R}_k(c_{obs})$ are elementary. Thus, in each case, the existence of an acyclic relation satisfying the restrictions can be tested using a path-independent enumeration procedure. The associated guess correspondences derived from (6) can be written as $G_{OR}(S) = \{x \in S \mid \text{If } x \succ_{OR}^* y \text{ then } y \notin S\}$ and $G_k(S) = \{x \in S \mid \text{For all } R \in \mathcal{D}, \text{ if } x = c_{obs}(R) \text{ then } |R \setminus S| \geq k(R) - 1\},^5$ and hence it is easy to determine whether a valid guess exists in each step.

Strictly speaking, restrictions associated to Consistent Reference Points are not elementary, but become elementary when reversed. Formally, let $\mathcal{R}_{REF}^*(c_{obs})$ be the following set of restrictions on a relation O: for each $S \in \mathcal{P}(X)$ and $x \in S$ such that $\succ_{S,x}^*$ is cyclic, there is $y \in S \setminus \{x\}$ with xOy. Existence of an acyclic relation satisfying $\mathcal{R}_{REF}^*(c_{obs})$ is equivalent to the existence of an acyclic relation satisfying $\mathcal{R}_{REF}(c_{obs})$ (simply by reversing the relation). Restrictions in $\mathcal{R}_{REF}^*(c_{obs})$ are elementary, and hence Proposition 4 applies once again. Thus consistency can be checked using a path-independent enumeration procedure. The associated guess correspondence derived from (6) can be written as $G_{REF}(S) = \{x \in S \mid \succ_{S,x}^* \text{ is acyclic}\},^6$ and hence it is easy to determine whether a valid guess exists in each step.

A.4 Proofs

Proof of Proposition 6 Necessity was given earlier. For sufficiency, suppose there is an acyclic relation satisfying $\mathcal{R}_{OR}$, and let P be a transitive com-

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[5] Indeed, there exists $T \in \mathcal{T}_R$ such that $S \cap T = \emptyset$ if and only if one can find $k(R) - 1$ elements that are in R but not S.

[6] To see whether $x \in G_{REF}(S)$, first note that it suffices to check only those restrictions $(x, \{y\}_{y \in R \setminus \{x\}})$ corresponding to $R \subseteq S$ (as there trivially exists $y \in R \setminus \{x\}$ such that $S \cap \{y\} = \emptyset$ when $R \not\subseteq S$). Next, if $\succ_{R,x}^*$ is cyclic for some $R \subset S$, then so is $\succ_{S,x}^*$, and of course there is no $y \in S \setminus \{x\}$ such that $S \cap \{y\} = \emptyset$.

pletion (hence P still satisfies $\mathcal{R}_{OR}$). Define the filter Ψ_P as in Lemma 1 (using P for O). CFS13 (Section 4.1) show that a filter is the set of rationalizable elements for some rationales $\{R_k\}_k$. Let c be the choice function arising from $(P, \{R_k\}_k)$ under the theory. For any $S \in \mathcal{D}$, we show $c(S) = c_{obs}(S)$. Suppose otherwise; then $\Psi_P(S)$ contains at least two elements, and $c(S)$ must be the observed choice from some $T \in \mathcal{D}$ with $S \subset T$. This implies $c_{obs}(S) \succ^*_{OR} c(S)$. But then $c_{obs}(S)Pc(S)$, contradicting P-maximality of $c(S)$ in $\Psi_P(S)$. $\square$

Proof of Proposition 7 Necessity was given earlier. For sufficiency, suppose an acyclic relation satisfying $\mathcal{R}_{REF}$ exists, and let $\succ_{REF}$ be a transitive completion (hence $\succ_{REF}$ still satisfies $\mathcal{R}_{REF}$). Let x_i denote the i-th maximal element according to $\succ_{REF}$. For each i, let $\succ_{x_i}$ be a transitive completion of $\succ^*_{X_i, x_i}$. Such a completion exists, because x_i being $\succ_{REF}$-maximal in $X_i = \{x_i, x_{i+1}, \ldots, x_n\}$ implies $\succ^*_{X_i, x_i}$ is acyclic. The choice function $c : \mathcal{P}(X) \to X$ generated by these primitives will now be shown to coincide with c_{obs} on $\mathcal{D}$. Take any $S \in \mathcal{D}$. Let k be the smallest index such that $x_k \in S$. Then $S \subseteq X_k$. By definition of $\succ_{x_k}$, $c_{obs}(S) \succ_{x_k} y$ for all $y \in S \setminus \{c_{obs}(S)\}$. $\square$

Proof of Proposition 8 Necessity was given earlier. For sufficiency, suppose an acyclic relation satisfying $\mathcal{R}_k$ exists, and let P be a transitive completion (P still satisfies $\mathcal{R}_k$). Let $\Gamma(S)$ be the (weak) P-lower contour set of $c_{obs}(S)$ for $S \in \mathcal{D}$, and $\Gamma(S) = S$ otherwise. The choice function obtained by maximizing P over Γ clearly extends c_{obs}. Since P satisfies $\mathcal{R}_k$, for any $S \in \mathcal{D}$ there exists $T \in \mathcal{T}_S$ with $k(S) - 1$ elements such that $c_{obs}(S)Px$ for all $x \in T$. The condition $|\Gamma(S)| \geq k(S)$ thus holds for $S \in \mathcal{D}$ (it is trivial for $S \notin \mathcal{D}$). $\square$

B Complexity Results

Proposition 9. *The classic SAT problem is reducible in polynomial time into the problem of determining whether observed choices are consistent with Limited Attention.*

Proof. Fix an instance of SAT with a set $\mathcal{L}$ of literals and a set $\mathcal{C}$ of clauses. Consider the abstract set of options X that contains all literals and their negations, all clauses, plus three options denoted x, y, and z. Let $\mathcal{L}_c$ denote

the set of literals in clause c and let the literal $\bar{\ell}$ denote the negation of literal ℓ. Construct the following observed choice function:

S	cy	xz	yz	cxz	xyz	$\ell\bar{\ell}x$	$\ell\bar{\ell}xz$	$\ell\bar{\ell}y$	$cy\mathcal{L}_c$
$c_{obs}(S)$	y	x	z	z	y	ℓ	x	$\bar{\ell}$	c

for all $c \in \mathcal{C}$ and all $\ell \in \mathcal{L}$. Applying Proposition 2, c_{obs} is consistent with Limited Attention if and only if there is an acyclic relation O on X such that:

(i) yOx, from $c_{obs}(\{y, z\}) = z$ and $c_{obs}(\{x, y, z\}) = y$.

(ii) xOz, from $c_{obs}(\{\ell, \bar{\ell}, x\}) = \ell$ and $c_{obs}(\{\ell, \bar{\ell}, x, z\}) = x$.

(iii) For all $c \in \mathcal{C}$: zOc, from $c_{obs}(\{x, z\}) = x$ and $c_{obs}(\{c, x, z\}) = z$.

(iv) For all $\ell \in \mathcal{L}$: ℓOx or $\bar{\ell}Oy$, from $c_{obs}(\{\ell, \bar{\ell}, x\}) = \ell$ and $c_{obs}(\{\ell, \bar{\ell}, y\}) = \bar{\ell}$.

(v) For all $c \in \mathcal{C}$, there exists $\ell \in \mathcal{L}_c$ such that $cO\ell$, from $c_{obs}(\{c, y\}) = y$ and $c_{obs}(\{c, y\} \cup \mathcal{L}_c) = c$.

(vi) For all $\ell \in \mathcal{L}$ and $c \in \mathcal{C}$, $xO\ell$ or $xO\ell'$ or zOc, from $c_{obs}(\{c, x, z\}) = x$ and $c_{obs}(\{\ell, \bar{\ell}, x, z\}) = x$.

Note that (vi) is redundant in view of (ii). These conditions are exhaustive, since we have used all the pairs $R, R' \in \mathcal{D}$ which cause a WARP violation.

We show SAT has a truthful assignment if and only if there exists an acyclic relation O satisfying (i)-(v). Suppose SAT has a truthful assignment. We construct an ordering O by putting the false literals (in any order) at the top of the ordering; then y; then x; then z; then the clauses; and then the true literals (in any order). It is easy to check that O satisfies (i) to (v). Conversely, suppose an acyclic relation satisfying (i)-(v) exists, and let O be a transitive completion. We construct an assignment for SAT: if $xO\ell$ then ℓ is true; if $xO\bar{\ell}$ then ℓ is false; and if both ℓOx and $\bar{\ell}Ox$, then assign ℓ an arbitrary value. This is well-defined since, by (i) and (iv), it cannot be that both $xO\ell$ and $xO\bar{\ell}$. By (v), for all $c \in \mathcal{C}$, there exists $\ell \in \mathcal{L}_c$ such that $cO\ell$. Combined with (ii)-(iii), we conclude ℓ is true. Hence the assignment is truthful for SAT. $\square$

Proposition 10. *The classic SAT problem is reducible in polynomial time into the problem of determining whether observed choices are consistent with psychological filter theory.*

Proof. Fix an instance of SAT with a set $\mathcal{L}$ of literals and a set $\mathcal{C}$ of clauses. Consider the abstract set of options X that contains all literals and their negations, all clauses, plus options denoted w, w', w'', x, y, and one option z_c for each clause c. Let V be the sets of variables defining the literals, let $\mathcal{L}_c$ be the set of literals appearing in clause c, and let $\bar{\ell}$ be the negation of the literal ℓ. Construct the following observed choice function:

S	cx	cy	cz_c	ℓc	vx	vy	$\bar{v}x$	$\bar{v}y$	wx	xz_c	cxy	wxz_c
$c_{obs}(S)$	x	y	c	c	x	v	$\bar{v}$	y	w	z_c	c	x

S	$vwxy$	$vw''xy$	$\bar{v}w''xy$	$\bar{v}w'xy$	$cz_c\mathcal{L}_c$
$c_{obs}(S)$	x	y	x	y	z_c

for all $c \in \mathcal{C}$, all $v \in V$, and all $\ell \in \mathcal{L}_c$.

We show that SAT has a truthful assignment if and only if there exist a filter Ψ and a relation P that generates a choice function c which coincides with c_{obs} on $\mathcal{D}$. Suppose first that SAT has a truthful assignment. First we pick a relation P such that yPx, xPw'', yPw'', and $z_cP\ell$, for each clause c and true literal ℓ in c, and aPb, for all $a, b \in X$ such that $c_{obs}(\{a, b\}) = a$. Next we consider the enumeration of the elements in X that starts with w''; followed by all true literals (in any order); followed by all clauses (in any order); followed by x, y, w, and w' (in that order); followed by z_c for each clause c (in any order); followed by all literals and their negations that did not already appear (in any order). For each choice problem R, let $\Psi(R)$ be the set containing the first element in the enumeration that belongs to R plus any element $a \in R$ such that $a = c_{obs}(S)$ for some $S \in \mathcal{D}$ containing R. It is easy to check that Ψ is a filter. It remains to show that the choice function generated by (Ψ, P) coincides with c_{obs} on $\mathcal{D}$. This follows by definition of P on pairs. By definition of Ψ, we have $\Psi(\{c, x, y\}) = \{c\}$ and $\Psi(\{w, x, z_c\}) = \{x\}$, and hence $c = c_{obs}$ on these two choice problems as well. For each variable v, $\Psi(\{v, w, x, y\}) = \{x\}$ or $\{v, x\}$ depending on whether v comes after or before x. In either case, the choice is x since xPv. For each variable v, $\Psi(\{\bar{v}, w'', x, y\}) = \{w'', x\}$ and $\Psi(\{v, w'', x, y\}) = \{w'', y\}$. The choices are x and y, respectively, since xPw''

and yPw''. For each variable v, $\Psi(\{\bar{v}, w', x, y\}) = \{x, y\}$ or $\{\bar{v}, y\}$ depending on whether $\bar{v}$ comes after or before x. In either case, the choice is y since yPx and $yP\bar{v}$. Finally, for each clause c, $\Psi(\{c, z_c\} \cup \mathcal{L}_c)$ contains z_c and a true literals appearing in c. By definition of P, $c(\{c, z_c\} \cup \mathcal{L}_c) = z_c$, as desired.

Conversely, suppose the filter Ψ and relation P generate a choice function c which coincides with c_{obs} on $\mathcal{D}$. We can assume without loss of generality that the DM pays attention to both options in each pair under Ψ, so that aPb if and only if $c(\{a, b\}) = a$ for all $a, b \in X$. We construct an assignment for SAT as follows. Consider the enumeration of X defined by $x_1 = c(X)$ and $x_k = c(X \setminus \{x_1, \ldots, x_{k-1}\})$, for all $k \geq 2$. Say a literal is true if it appears before both x and y. (It need not be that every literal or its negation is true, but that will not matter.) We show it is impossible to have both a literal and its negation true. Suppose, on the contrary, that there is a variable v such that both v and $\bar{v}$ come before both x and y in the enumeration. Assume that $c(\{x, y\}) = x$ so that xPy (a similar reasoning applies in the other case where yPx). From the corresponding pairwise choices, we infer $\bar{v}Px$ and $yP\bar{v}$. Notice that $x, y \in \Psi(\{\bar{v}, x, y\})$ since x is picked out of $\{\bar{v}, w'', x, y\}$ and y is picked out of $\{\bar{v}, w', x, y\}$. Also, $\bar{v} \in \Psi(\{\bar{v}, x, y\})$ since $\bar{v}$ is picked from the set consisting of elements of X that succeed $\bar{v}$ in the enumeration. We reach a contradiction since P is cyclic over $\{\bar{v}, x, y\}$ and all three receive attention.

Given this well-defined truth assignment, we check that all clauses in SAT are satisfied. Let c be a clause. Since x is picked out of $\{w, x, z_c\}$, but also z_cPx and wPx, x must precede both w and z_c in the enumeration. Otherwise, the first element in $\{w, x, z_c\}$ appearing in the enumeration is the first element in $\{w, z_c\}$ in the enumeration. That element must be paid attention when choosing from $\{w, x, z_c\}$, contradicting that x is picked. Similarly, c precedes both x and y in the enumeration since c is picked out of $\{c, x, y\}$, xPc and yPc. From $c(\{c, z_c\} \cup \mathcal{L}_c) = z_c$ and cPz_c, we conclude that c is not the first element of that choice problem to appear in the enumeration. If z_c comes first, then z_c precedes c. This would contradict the fact that c precedes x and x precedes z_c. Hence, one of the literals in $\mathcal{L}_c$ appears first, and precedes c. That literal is true since c precedes x and y. The assignment is thus truthful for SAT. □