Reason-Based Choice: a Bargaining Rationale for the Attraction and Compromise Effects

Online Appendix

Geoffroy de Clippel and Kfir Eliaz*

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1 Independence of the Axioms in Theorem 1

EFF: Consider a set X with four elements, let's say $X = \{a, b, c, d\}$, and let C be the preference-based bargaining solution that coincides with C^f, except as follows: $C_\succ(X) = C^f_\succ(X) \cup \{x\}$, for all $x \in X$ and all $\succ \in L(X) \times L(X)$ such that $\succ_1$ and $\succ_2$ are completely opposite on $X \setminus \{x\}$, and x is Pareto dominated by either the $\succ_1$-optimal element or the $\succ_2$-optimal element, but by no other element of X. For instance, $C_{\succ^*}(X) = \{a, b, c\}$, while $C^f_{\succ^*}(X) = \{b, c\}$, when $b \succ^*_1 a \succ^*_1 c \succ^*_1 d$ and $d \succ^*_2 c \succ^*_2 b \succ^*_2 a$. The modification thus amounts to add some options to the fallback solution in some cases, and will satisfy RA *a fortiori*. By construction, C is regular, but violates EFF. ATT does not apply in those cases where C is different from C^f (because the Pareto dominated option falls below an option that is not chosen in the triplet obtained by deleting that Pareto dominated option), and hence C satisfies it (since C^f does). It is straightfoward to check NBC. Finally, SYM is satisfied because C^f satisfies it, and a Pareto dominated option is never selected out of any triplet.

ATT: Consider the fallback solution applied only to the set of Pareto efficient alternatives, $C_\succ(S) = C^f_\succ[EFF_\succ(S)]$, where

$$EFF_\succ(S) = \{x \in S \mid \textit{for all } y \in S,\ x \succ_i y \textit{ for some } i \in \{1, 2\}\} \tag{1}$$

Note that the fallback solution is applied here to a subset of options, whose score is unaffected by dominated elements. Hence, $C_\succ$ violates ATT. It is straightforward to verify that $C_\succ$ is regular and satisfies NBC, RA, EFF, EX and IPUA. To see that it also satisfies SYM, suppose $x, y \in C_\succ(S)$ but $x \notin C_\succ(S \setminus \{z\})$ for some $z \in S \setminus \{x, y\}$. Then $z \in EFF_\succ(S)$. Let $T \equiv EFF_\succ(S)$, then $x, y \in C^f_\succ(T)$ but $x \notin C^f_\succ(T \setminus \{z\})$ for some

*Brown University, Department of Economics, Providence, Rhode Island - declippel@brown.edu, kfir_eliaz@brown.edu.

$z \in T\backslash\{x, y\}$. Then by SYM, $y \notin C^f_{\succ}(T\backslash\{z'\})$ for some $z' \in T\backslash\{x, y\}$, which implies that $y \notin C_{\succ}(S\backslash\{z'\})$.

NBC: Consider the analogue of the Borda rule in our setting:

$$C_{\succ}(S) = \arg\max_{x\in S}[s_1(x, S, \succ) + s_2(x, S, \succ)],$$

for each subset S of X. It is straightforward to check that this defines a regular preference-based bargaining solution that satisfies EFF and ATT. It violates NBC. For instance, it does not refine the set of Pareto efficient options when the two preferences are strict opposite to each others. It remains to show that the solution satisfies both RA and SYM. Since it satisfies EFF, the sum of the scores must decrease by at least one point for each option that is chosen, when removing x from the original problem S. Any element of $C_{\succ}(S)$ such that the sum of the scores decreases by exactly one point when removing x clearly belongs to $C_{\succ}(S \setminus \{x\})$. Hence we must consider the case where the sum of the scores decreases by two points, for each element of $C_{\succ}(S)$. This implies that x is Pareto dominated by some elements of S, and the set of Pareto efficient options remains unchanged when removing x. The sum of the scores of any element of the Pareto frontier decreases by at least one point when removing x, and hence $C_{\succ}(S) \subseteq C_{\succ}(S\backslash\{x\})$, and we are done proving RA. For SYM, suppose on the contrary that one can find $x, y \in C_{\succ}(S)$ and $z \in S \setminus \{x, y\}$ such that $x \notin C_{\succ}(S \setminus \{z\})$ and $y \in C_{\succ}(S \setminus \{z\})$. Both x and y being Pareto efficient in S, it must be that the sum of the scores decreases by at least one point for both of them when removing z. Since y remains chosen, but not x, it must be that the sum of the scores of x decreases by two points while the sum of the scores of y decreases by exactly one point. In other words, x Pareto dominates z, but y does not Pareto dominates z. It is easy to check that one would get a contradiction with $x, y \in C_{\succ}(S)$ if there does not exist $z' \in S$ that is Pareto dominated by y, but not by x. For any such z', we'll have $y \notin C_{\succ}(S \setminus \{z'\})$, and we are done proving SYM.

RA: Let CL^f be the lexicographic refinement of the fallback solution,

$$CL^f_{\succ}(S) = \{x \in C^f_{\succ}(S) \mid s_i(x, S, \succ) \geq s_i(y, S, \succ)\ \forall y \in C^f_{\succ}(S)\},$$

for each $S \subseteq X$, amd each $\succ \in L(X) \times L(X)$. It is easy to check that CL^f inherits the properties of regularity, EFF, ATT, and NBC from C^f. To see that it violates RA, consider $S = \{a, b, c, d\}$ and the preference pair $\succ^*$ that give rise to the following ranking on S: $b \succ^*_1 a \succ^*_1 c \succ^*_1 d$ and $d \succ^*_2 c \succ^*_2 b \succ^*_2 a$. Then $CL^f_{\succ^*}(S) = \{b\}$ while $CL^f_{\succ^*}(X\backslash\{a\}) = \{c\}$. All what remains is to check SYM. Suppose that $x, y \in CL^f_{\succ}(S)$ and that there exists $z \in S \setminus \{x, y\}$ such that $x \notin CL^f_{\succ}(S \setminus \{z\})$. This implies that $x, y \in C^f_{\succ}(S)$. If there exists $z \in S \setminus \{x, y\}$ such that $x \notin C^f_{\succ}(S \setminus \{z\})$, then there exists $z' \in S\setminus\{x, y\}$ such that $y \notin C^f_{\succ}(S\setminus\{z'\})$, by SYM. If there exists $z' \in S\setminus\{x, y\}$ such that $y \notin C^f_{\succ}(S\setminus\{z'\})$, then $y \notin CL^f_{\succ}(S\setminus\{z'\})$, as desired, since CL^f refines C^f. Hence the last case that could lead to a possible violation of SYM for CL^f is when $x, y \in C^f(S \setminus \{z\})$, for all $z \in S\setminus\{x, y\}$. But we know from Lemma 1 that this configuration of choice for C^f is possible only if $x \succ z$ and $y \succ z$, for all $z \in S \setminus \{x, y\}$. In such cases, it is impossible to have $x \notin CL^f_{\succ}(S \setminus \{z\})$, and we are done proving SYM.

SYM: Consider a set X with five elements, let's say $X = \{a, b, c, d, e\}$, and let C be the preference-based bargaining solution that coincides with C^f, except as follows: $C_{\succ}(X) = C^f_{\succ}(X \setminus \{x\})$, for all $x \in X$ and all $\succ \in L(X) \times L(X)$ such that $\succ_1$ and $\succ_2$ are completely opposite on $X \setminus \{x\}$, and x is Pareto dominated by either the $\succ_1$-optimal element or the $\succ_2$-optimal element, but by no other element of X. For instance, $C_{\succ^*}(X) = \{c, d\}$, while $C^f_{\succ^*}(X) = \{c\}$, when $b \succ^*_1 a \succ^*_1 c \succ^*_1 d \succ^*_1 e$ and $e \succ^*_2 d \succ^*_2 c \succ^*_2 b \succ^*_2 a$. The modification thus amounts to add some options to the fallback solution in some cases, and will satisfy RA *a fortiori*. By construction, C is regular and satisfies EFF. ATT does not apply in those cases where C is different from C^f (because the Pareto dominated option falls below an option that is not chosen in the quadruplet obtained by deleting that Pareto dominated option), and hence C satisfies it (since C^f does). Finally, SYM is violated. For instance, $C_{\succ^*}(X) = \{c, d\}$, c is selected from any quadruple that includes it, but $d \notin C_{\succ^*}(X \backslash \{e\})$.

2 Independence of the Axioms in Theorem 2

EFF: Consider the choice correspondence $C_{\succ^*}$ introduced when showing that EFF does not follow from the other axioms in Theorem 1. A similar argument implies that $C_{\succ^*}$ satisfies the current versions of ATT, NBC, RA, and SYM, but violates EFF. EC and PC are satisfied since $C_{\succ^*}$ coincides with the fallback solution on pairs and triplets. OC does not apply, and is thus satisfied trivially.

ATT: Let $\succ$ be a pair of linear orderings on X satisfying that there exists at least one pair of elements, $x, y \in X$ such that $x \succ y$. Define $C(S)$ be a choice correspondence defined as the fallback solution applied only to the set of $\succ$-Pareto efficient alternatives in S, $C(S) = C^f_{\succ}[EFF_{\succ}(S)]$, where $EFF_{\succ}(S)$ is defined in (1). We've already shown that this choice correspondence violates ATT, while satisfying NBC, RA, EFF and SYM. $C(\{x, y\}) = \{x\}$ and $C(\{y, z\}) = \{y\}$ imply that $x \succ y$ and $y \succ z$, which in turn implies that $x \succ z$, and hence, $C(\{x, z\}) = \{x\}$. This verifies PC. If the choice out of any pair in $\{x, y, z\}$ is the pair itself, then $EFF_{\succ}(\{x, y, z\}) = \{x, y, z\}$. Since $C^f_{\succ}(\{x, y, z\})$ is a singleton, so is $C(\{x, y, z\})$, which verifies EC. Suppose the choice out of any pair in $\{x, y, z\}$ is the pair itself and $C(\{x, y, z\}) = \{y\}$. Then both individuals must rank y in between x and z. If $C(\{w, x, y, z\}) = \{w\}$, then w cannot be Pareto dominated by any of the elements. It is easy to check that, if w does not Pareto dominate y, then y also belongs to $C(\{x, y, z, w\})$, a contradiction. Hence, it must be that $w \succ y$, confirming OC.

NBC: Consider two orderings $\succ_1$ and $\succ_2$ that are opposite on X: $x \succ_1 y$ if and only if $y \succ_2 x$. Let then C be the choice correspondence defined as follows: $C(S) = C^f_{\succ}(S)$ if S has exactly three elements, and $C(S) = S$ otherwise. C satisfies EFF and ATT trivially since no two elements are Pareto comparable under $\succ$. NBC is clearly violated in sets with at least four elements. C is larger than the fallback solution applied to $\succ$, and hence C satisfies RA. SYM are trivially satisfied when applied to any set whose cardinality is not equal to four. For any element in a quadruplet, there exists a triplet where that element is available, yet not selected, and hence SYM is verified. PC and EC are satisfied since C coincides with the fallback on pairs and triplets. OC does not apply since C never selects a singleton in quadruplets.

RA: As in the previous example, two orderings $\succ_1$ and $\succ_2$ that are opposite on X. Inspired by Masatlioglu et al. (2009), suppose that the decision maker can pay attention to at most five options. Formally, he has an attention filter $\alpha : P(X) \to P(X)$: $\alpha(S) \subset S$, for all $S \subseteq X$ such that $|\alpha(S)| = \min\{5, |S|\}$ and $x \in \alpha(T)$ if $x \in T \subseteq S$. We will assume in addition that, if x and y belong to $\alpha(S)$, then there does not exist $z \in S \backslash \alpha(S)$ that falls in between x and y according to $\succ$ (it is easy to construct various attention filters with this property). Let then $C(S) = C^f_\succ(\alpha(S))$. EFF and ATT are both satisfied because the choice out of any pair is the pair itself. EC is satisfied because the choice out of any triplet is a singleton. NBC is satisfied because of the second property we imposed on the attention filter. OC is satisfied because there is no singleton choice out of quadruplets. PC is satisfied because the choice out of any pair is the pair itself. If $C(S)$ contains two elements, then it must be that S contains at most four elements, in which case C coincides with the fallback, and hence C satisfies SYM. Finally, let's check that C violates RA. Indeed, let S be a set that contains six elements. Let y be the element of S that does not belong to $\alpha(S)$. Let i be such that y is better than the element selected in S for $\succ_i$. Let then $z \in \alpha(S)$ be an option that is worse than the element selected in S for $\succ_i$. It is easy to check that the element selected in $S \setminus \{z\}$ is different from the element selected in S, thereby showing that RA is violated.

SYM: Consider the choice correspondence $C_{\succ^*}$ introduced when showing that SYM does not follow from the other axioms in Theorem 1. A similar argument implies that $C_{\succ^*}$ satisfies the current versions of EFF, ATT, NBC, and RA, but violates SYM. PC, EC and OC are satisfied since $C_{\succ^*}$ coincides with the fallback solution on pairs and triplets.

PC: Let P be a strict complete and transitive ordering on X, and let C be the choice obtained by maximizing this ordering, except that $C(\{x, y\}) = \{x, y\}$, where x is the best element in X and y is the worst element in X. It is easy to check that C satisfies all the axioms of Theorem 2, except PC.

EC: Consider the analogue of the Borda rule introduced in the previous section to show that NBC is not implied by the other axioms in Theorem 1. We already proved that it satifies EFF, ATT, NBC, RA, and SYM, for any given pair of preferences. PC is straightforward to check, and OC never applies because the choice out of any triplet is the triplet itself if no two elements are Pareto comparable. For the same reason, the choice correspondence will violate EC, as soon as there are at least three elements that are not Pareto comparable.

OC: See the two examples given before introducing OC.

3 Proof of Proposition 1

EFF and PC follow from the definition of the fallback bargaining solution. As already observed in the main text, the case where the choice out of any pair in $\{x, y, z\}$ is the pair itself, and the choice out of $\{x, y, z\}$ is $\{z\}$, is consistent with fallback bargaining only if some orderings are of the form $x \succ_i z \succ_i y$, some other orderings are of the form $y \succ_j z \succ_j x$, and any other ordering (if any) places z above both x and y. So indeed, if both x and y belong to the fallback solution of some problem S, there cannot exist a z as claimed in NBC, as it would obtain a minimal score that is strictly larger than the minimal score of both x and y.

We now establish property 1. If $y \in C(S)$, then

$$\min_{i=1,\dots n} s_i(y, \succ, S) \geq \min_{i=1,\dots n} s_i(z, \succ, S),$$

for all $z \in S$. On the other hand, $C(\{x, y\}) = \{y\}$ implies that $y \succ_i x$, for all i. Hence

$$\min_{i=1,\dots n} s_i(y, \succ, S \cup \{x\}) = \min_{i=1,\dots n} s_i(z, \succ, S) + 1.$$

At the same time,

$$\min_{i=1,\dots n} s_i(z, \succ, S \cup \{x\}) \leq \min_{i=1,\dots n} s_i(z, \succ, S) + 1,$$

and hence $y \in C(S \cup \{x\})$, as desired.

We now move to the first part of property 2, which will also imply RA. For any $z \in C(S) \setminus \{x\}$ and any $j \in \arg\min_{i=1,\dots n} s_i(z, \succ, S)$, we must have $x \succ_j z$, because $x \in C(S)$. Hence

$$\min_{i=1,\dots n} s_i(z, \succ, S \setminus \{x\}) = \min_{i=1,\dots n} s_i(z, \succ, S),$$

while the minimal score of the other elements of S does not increase, and hence $z \in C(S \setminus \{x\})$. As for the second part of property 2, suppose that $x \notin C(S)$ and that there exists $y \in C(S \setminus \{x\}) \setminus C(S)$. Let $z \in C(S)$. The fact that $y \notin C(S)$ implies that the minimal score of y in S is at least one point smaller than the minimal score of z in S. Since, in addition, $y \in C(S \setminus \{x\})$, it must be that the minimal score of y in S is exactly one point smaller than the minimal score of z in S, as must be the minimal score of z in $S \setminus \{x\}$, and hence $z \in C(S \setminus \{x\})$, as desired.

We conclude the proof of Proposition 1 by establishing property 3. As already oberved earlier, the choice pattern on $\{a, b, c\}$ implies that some orderings are of the form $a \succ_i b \succ_i c$, some other orderings are of the form $c \succ_j b \succ_j a$, and any other ordering (if any) places b above both a and c. Since d is chosen uniquely out of the quadruplet, it must be that d is ranked first or second best out of $\{a, b, c, d\}$ in all ranking (otherwise b would also be chosen), and it is easy to conclude that d is selected uniquely out of any triplet that contains it. ■