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Equity, envy and efficiency under asymmetric information ☆

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Abstract

Varian's (Varian, H., 1974. Equity, Envy and Efficiency. Journal of Economic Theory 9, 63–91) main results are not valid anymore if the agents are asymmetrically informed at the time of contracting: 1) envy-freeness and efficiency may be incompatible; 2) there may exist efficient allocation rules such that every agent envies another. Two weaker positive properties are formulated.

Keywords: Resource allocation; Fairness; Equity; Envy; Efficiency; Asymmetric information

1. Introduction

This paper studies the allocation of scarce resources between agents that are asymmetrically informed about the fundamentals of the economy at the time of contracting. Different concepts such as the competitive equilibria, the core and the value have attracted much attention over the past thirty years. I propose hereafter a first description of the tensions that may exist between efficiency and the absence of envy in economies with asymmetric information.

Foley (1967) and Varian (1974) define the concept of envy for exchange economies without uncertainty. Suppose that agent i receives a bundle x while agent j receives a bundle y. Then, agent i envies agent j if he prefers y over x. Envy-freeness, the absence of envy, is an appealing concept of equity. Combined with efficiency, it leads to a natural notion of fairness. Varian shows that *there is no tension between efficiency and equity in classical exchange economies*. In-deed, the set of fair allocations is non-empty, as any competitive equilibrium allocation resulting from an equal sharing of the aggregate endowment is fair. Varian also proposes an interesting necessary condition for efficiency. There is unanimous envy if every agent envies another. *It is impossible to have unanimous envy at an efficient allocation in classical exchange economies*. Indeed, if there were unanimous envy, then one could achieve a Pareto improvement by swapping the bundles of some agents. I show that these two results are not necessarily valid anymore when the agents are asymmetrically informed at the time of contracting.

I assume that the true state of the economy is commonly known at the time of implementing the contracts. Incentive and measurability constraints are therefore irrelevant. Many important economic examples satisfy this assumption. The payoff of an equity, an option or an insurance contract depends on the realization of some observable events. The relevant notion of efficiency in this context is the concept of interim efficiency introduced by Wilson (1978). Further, I will say that an allocation rule is interim envy-free if there is zero probability of an agent interim envying another. Example 1 shows that interim efficiency and interim envy-freeness may be incompatible.[1] Example 2 shows that there may be unanimous envy (even with probability one) at some interim efficient allocation rules. Propositions 1 and 2 state two weaker properties that extend Varian's results to economies with asymmetric information.

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[1] Pazner and Schmeidler (1974) show that envy-freeness may be incompatible with efficiency in production economies. I show that envy-freeness may be incompatible with efficiency in economies with differential information.

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2. The model

An economy with asymmetric information is a t-uple

$$\left(N, L, \Omega, \pi, e, (\mathcal{P}_i)_{i\in N}, (u_i)_{i\in N}\right),$$

where N is the set of agents, L is the set of goods, Ω is the set of possible states, π is the common prior describing the relative probability of these states,[2] $e: \Omega\rightarrow\mathbb{R}^L_{++}$ is the function specifying the aggregate endowment of the economy in each state, $\mathcal{P}_i$ is a partition of Ω describing the information of agent i, and $u_i: \mathbb{R}^L_+ \times \Omega\rightarrow\mathbb{R}$ is the concave, continuous and strongly increasing utility function that represents the preferences of agent i (lotteries are evaluated thanks to the expected utility criterion). For each $\omega\in\Omega$, let $P_i(\omega)$ be the atom of the partition $\mathcal{P}_i$ that contains ω. When the future state is ω, agent i knows and only knows that it will be an element of $P_i(\omega)$. His beliefs are derived from π by Bayesian updating. Decisions are taken today about the way to redistribute the aggregate endowment when the state will be commonly known. Hence the agents agree on allocation rules: $a: \Omega\rightarrow\mathbb{R}^{L\times N}_+$ such that $\sum_{i\in N} a_i(\omega)\leq e(\omega)$ for each $\omega\in\Omega$.

An allocation rule a' *interim Pareto dominates* an allocation rule a if every agent weakly prefers (given his private information) a' over a in each state and at least one agent strictly prefers a' over a in at least one state, i.e.

$$\sum_{\omega'\in P_i(\omega)} \pi(\omega')u_i(a_i(\omega'),\omega') \leq \sum_{\omega'\in P_i(\omega)} \pi(\omega')u_i\left(a_i^{'}(\omega'),\omega'\right)$$

for each $i\in N$ and each $\omega\in\Omega$, one of the inequalities being strict. An allocation rule is interim efficient (Wilson, 1978) if it is not interim Pareto dominated by any other allocation rule.

Let a be an allocation rule and let $\omega \in \Omega$. Then, agent i *interim envies* agent j at ω if

$$\sum_{\omega'\in P_i(\omega)} \pi(\omega')u_i(a_i(\omega'),\omega') < \sum_{\omega'\in P_i(\omega)} \pi(\omega')u_i\left(a_j(\omega'),\omega'\right).$$

Notice that agent i compares a_i and a_j given his private information and not after observing ω. As a consequence, if agent i interim envies agent j at ω and if $\tilde{\omega}\in P_i(\omega)$, then agent i interim envies agent j at $\tilde{\omega}$. In other words, the property of interim envy is measurable with respect to the agent's information.

The allocation rule a is *interim envy-free* if there is zero probability of an agent interim envying another, i.e. there does not exist a state ω at which some agent i interim envies some agent j. An allocation rule is *interim fair* if it is both interim efficient and interim envy-free. Palfrey and Srivastava (1987) suggest the same notion of interim fairness in a slightly different framework.[3] There is *unanimous envy* at ω if every agent interim envies another at ω. Unanimous envy is possible at the interim stage if there exists $\omega\in\Omega$ such that there is unanimous envy at ω.

Subsets of Ω are called *events*. An event $\mathcal{E}$ is common knowledge if it can be written as a union of elements of $\mathcal{P}_i$ for each $i\in N$.

3. On the impossibility to achieve fairness

The allocation rule that equally shares the aggregate endowment in each state is interim envy-free and therefore the set of interim envy-free allocation rules is not empty. Obviously, the set of interim efficient allocation rules is not empty as well. The next example shows that the set of interim fair allocation rules may be empty though.

Example 1. Consider three agents and one good (money). The future state may be low or high with equal probability. The total endowment is 1200 dollars when the state is low. It is 1800 dollars when the state is high. Agent 3 knows the future state. Agents 1 and 2 do not. Agent 1 is risk neutral while agent 2 is risk averse. Formally, $\Omega=\{L, H\}$, $\pi(L)=\pi(H)= 1/2$, and:

State	e(.)	P_1(.)	P_2(.)	P_3(.)	u_1(x,.)	u_2(x,.)	u_3(x,.)
L	1200	$\{L, H\}$	$\{L, H\}$	$\{L\}$	x	$\sqrt{x}$	x
H	1800	$\{L, H\}$	$\{L, H\}$	$\{H\}$	x	$\sqrt{x}$	x.

An allocation rule a is interim envy-free if and only if it satisfies the following equations:

$$\begin{cases} a_1(L)+a_1(H)\geq \max_{i\in\{2,3\}}[a_i(L)+a_i(H)] \\ \sqrt{a_2(L)}+\sqrt{a_2(H)}\geq \max_{i\in\{1,3\}}[\sqrt{a_i(L)}+\sqrt{a_i(H)}] \\ a_3(L)\geq \max_{i\in\{1,2\}}a_i(L) \\ a_3(H)\geq \max_{i\in\{1,2\}}a_i(H), \end{cases}$$

which amount to $a_1(L)=a_2(L)=a_3(L)$ and $a_1(H)=a_2(H)=a_3(H)$. The best among those allocation rules is a, where a(L)=(400, 400, 400) and $a(H)$=(600, 600, 600). It is interim Pareto dominated by the allocation rule a', where $a'(L)$=(301, 498, 401) and $a'(H)$= (701, 498, 601). The interim no-elvy property combining restrictions based on different pieces of information may lead to allocation rules that do lot exploit the possibilities of insurance.

Interim efficiency is compatible with a weaker notion of interim envy-freeless. I prove this property by using the notion of constrailed market equilibrium, a natural generalization of the Arrow–Debreu equilibrium in markets with coltingent commodities to economies with asymmetric information, when inside trading is prohibited (Wilson, 1978, footnote 6; de Clippel, 2004). Al allocation rule a is a constrained market equilibrium resulting from an equal sharing of the aggregate endowment in each state if there exists a price system $p: \Omega\rightarrow\mathbb{R}^L_+$ such that

$$a_i\in\arg\max_{a_i^{'}\in B_i(p,\omega)} \sum_{\omega'\in P_i(\omega)} \pi(\omega')u_i\left(a_i'(\omega'),\omega'\right),$$

where

$$B_i(p,\omega) := \left\{ a_i^{'}\in\mathbb{R}^{L\times\Omega}_+ \middle| \sum_{\omega'\in P_i(\omega)} p(\omega')\cdot a_i(\omega') \leq \sum_{\omega'\in P_i(\omega)} p(\omega')\cdot\frac{e(\omega')}{n} \right\}$$

is the budget set of agent i at ω, for each $(i, \omega)\in N\times\Omega$.

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[2] I assume without loss of generality that $\pi(\omega) > 0$ for each $\omega \in \Omega$.

[3] Palfrey and Srivastava show that interim fairness is not implementable. They do not check whether the set of interim fair allocation rules is always nonempty.

Proposition 1. *There exists an interim efficient allocation rule such that it is impossible to find two agents i and j for which it is common knowledge that i interim envies j.*

Proof. Wilson (1978) observes that the set of of constrained market equilibria is lot empty and that every constrained market equilibrium is interim efficient (see alsode Clippel, 2004, theorems 4 ald 5). Let a be a constrained market equilibrium resulting from an equal sharing of the aggregate endowment in each state. Suppose that there exist two agents i and j for which it is common knowledge that i interim envies j, i.e. there exists a common knowledge event $\mathcal{E}$ such that i interim envies j at each $\omega \in \mathcal{E}$. Let $p : \Omega \rightarrow \mathbb{R}^L_+$ be the price vector associated to a. We have:

$$\begin{aligned}\sum_{\omega\in\mathcal{E}} p(\omega)\cdot a_j(\omega) &= \sum_{P_i\in\mathcal{P}_i \text{ s.t. } \mathcal{P}_i\subseteq\mathcal{E}} \sum_{\omega\in P_i} p(\omega)\cdot a_j(\omega) \\ &> \sum_{P_i\in\mathcal{P}_i \text{ s.t. } \mathcal{P}_i\subseteq\mathcal{E}} \sum_{\omega\in P_i} p(\omega)\cdot \frac{e(\omega)}{n} \\ &= \sum_{\omega\in\mathcal{E}} p(\omega)\cdot \frac{e(\omega)}{n} \\ &= \sum_{P_j\in\mathcal{P}_j \text{ s.t. } \mathcal{P}_j\subseteq\mathcal{E}} \sum_{\omega\in P_j} p(\omega)\cdot \frac{e(\omega)}{n} \\ &\geq \sum_{P_j\in\mathcal{P}_j \text{ s.t. } \mathcal{P}_j\subseteq\mathcal{E}} \sum_{\omega\in P_j} p(\omega)\cdot a_j(\omega) \\ &= \sum_{\omega\in\mathcal{E}} p(\omega)\cdot a_j(\omega).\end{aligned}$$

This is absurd. The strict inequality holds because agent i prefers a_j over a_i at each $\omega \in \mathcal{E}$. The weak inequality follows from the fact that a_j satisfies the budget constraint of agent j for each $\omega \in \mathcal{E}$. □

4. Efficiency with unanimous envy

The following example shows that unanimous envy is possible at the interim stage, even if one considers allocation rules that are interim efficient.

Example 2. Consider the following allocation rule in the economy described in example 1: $a(L)=(210, 500, 490)$ and $a(H)=(700, 500, 600)$. Notice that a is interim efficient. On the other hand, every agent interim envies another in each state: agents 1 and 2 both interim envy agent 3 in both states, while agent 3 interim envies agent 1 (resp. 2) in state H (resp. L).

The example shows that unanimous envy may even be common knowledge at some interim efficient allocation rule. This requires the improving cycle to vary with the state, as the following proposition highlights.

Proposition 2. *Let a be an interim efficient allocation rule. Then there does not exist a subset* $\{i_1,\ldots, i_K\}$ *of agents such that it is common knowledge that* i_k *interim envies* $i_{(k+1)modK}$, *for each* $k \in \{1,\ldots, K\}$.

Proof. Otherwise, let $\mathcal{E}$ be the common knowledge event over which i_k interim envies $i_{(k+1)modK}$, for each $k \in \{1,\ldots, K\}$. Then, a is interim Pareto dominated by the allocation rule a', where $a'(\omega) := a(\omega)$ for each $\omega \in \Omega \backslash \mathcal{E}$, $a'_i(\omega) := a_i(\omega)$ for each $(\omega, i) \in \mathcal{E} \times (N \backslash \{i_1,\ldots, i_K\})$, and a'_{i_k} (w) $:= a_{i_{(k+1)modK}}(\omega)$ for each $(\omega, k) \in \mathcal{E} \times \{1,\ldots, K\}$. □

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