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Rotating Savings and Credit Associations, Credit Markets and Efficiency

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This paper examines the allocative performance of rotating savings and credit associations (roscas), a financial institution which is observed world-wide. We develop a model in which individuals save for an indivisible good and study roscas which distribute funds using random allocation and bidding. The allocations achieved by the two types of rosca are compared with that achieved by a credit market and with efficient allocations more generally. We find that neither type of rosca is efficient and that individuals are better off with a credit market than a bidding rosca. Nonetheless, a random rosca may sometimes yield a higher level of ex ante expected utility to prospective participants than would a credit market.

I. INTRODUCTION

Rotating savings and credit associations (roscas) are a widely observed institution for financial intermediation. They are found all over the world, particularly in developing countries, and have heretofore received scant attention from economists.[1] This paper and its companion piece (Besley, Coate and Loury (1993)) constitute a first attempt to analyse their economic role and performance.

Roscas come in two main forms. The first type allocates funds randomly. In a *random rosca*, members commit to putting a fixed sum of money into a "pot" for each period of the rosca's life. Lots are drawn and the pot is randomly allocated to one of the members. In the next period, the process repeats itself, with each previous winner excluded from the draw. The process continues until each rosca member has received the pot once. At this point, the rosca is either disbanded or begins over again. Individuals may also form a *bidding rosca* in which the pot is allocated via a bidding procedure. The individual who

1. Roscas travel under a large number of different names. For example, they are called *Chit Funds* in India, *Susu* in West Africa, *Kye* in Korea. Bouman (1977) reports that 60% of the population in Addis Ababa belongs to a rosca. Radhakrishnan *et al.* (1975), reports that in 1967, there were 12,491 registered chit funds in Kerala state in India alone. The classic anthropological studies are by Ardener (1964) and Geertz (1962). Further references to the literature on roscas can be found in our companion paper.

receives the pot in the present period does so by bidding the most in the form of a pledge of higher future contributions to the rosca or one-time side payments to other rosca members. In a bidding rosca, individuals may still only receive the pot once—the bidding process merely establishes priority.

The extensive informal literature on the subject takes the view that roscas are primarily institutions whose role is to facilitate "saving up" to purchase indivisible goods. In Besley, Coate and Loury (1993) we explained how, in a world with an indivisible good, a group of individuals without access to credit markets could improve their welfare by forming a rosca. Roscas permit the mobilization of savings that would lie idle under autarkic saving and thus take advantage of *gains from intertemporal trade*. That paper also compared the allocations achieved by the two different types of rosca finding that, with homogeneous individuals, a random rosca produces a higher level of expected utility for participants than a bidding rosca (under a plausible restriction on preferences). The ex ante desirability of randomization stems from the non-convexity created by the indivisible good.

Given that a group of individuals can get together to form a rosca[2], they could potentially allocate funds in other ways, such as by organizing an informal credit market. Thus to understand why roscas are sometimes chosen, we propose characterizing the full set of allocations that are feasible for the group. This places roscas in a broader context. As evidenced by their world-wide popularity, roscas are a simple and easily organized method of mobilizing savings. It is important to know how far these simple institutions go towards realizing the maximal possible gains from trade. Do they produce efficient allocations or does their simple structure impose a cost? In what ways do the allocations they produce differ from that achieved from an informal credit market? Are bidding roscas more like a market than random roscas? Does the randomization inherent in a random rosca give it an advantage over a market? These more abstract and theoretically challenging questions about the allocations achieved using roscas are the subject of this paper. Answering them gives insights into both the strengths and weaknesses of roscas as institutions for financial intermediation and, along with appreciating their simplicity, may help to explain their use in practice.

One of our main findings is that roscas do not, in general, produce efficient allocations. Their simple structure allows insufficient flexibility in the rate of accumulation of the indivisible good. We also find that bidding roscas are Pareto dominated by credit markets. Nonetheless, the element of chance offered by random roscas is still of value in comparison with credit markets. We present an example in which an ex post efficient credit market allocation is dominated (under the ex ante expected utility criterion) by a random rosca.

The remainder of the paper is organized as follows. Section II presents the model. Section III describes the allocations achieved by the two types of rosca and a credit market. Section IV develops properties of efficient and optimal allocations and Section V assesses the allocative performance of roscas. Section VI concludes.

II. THE MODEL

The model is essentially as in our companion paper. A group of individuals, without access to an external credit market, would each like to own an indivisible durable consumption

2. The typical scenario for a rosca is a group of individuals who work in the same office block or belong to the same community. Social enforcement is important in explaining why individuals honour their commitment to participate. We are not concerned with enforcement problems here, which are discussed in our companion paper. We shall ask questions about what a group might achieve for a given membership, assuming that there is sufficient social enforcement power for any of the allocations that we describe to be implemented. This seems like a reasonable first step in studying these issues.

good. Each group member lives for a length of time T, receiving an exogenous flow of income over his lifetime of $y>0$. Individuals have identical intertemporally-additive preferences depending on non-durable consumption, c, and on whether or not they own the durable.[3] The durable costs B and does not depreciate, i.e., it yields a constant flow of services for the remainder of an individual's life.

There is no discounting, which precludes any motive for saving or borrowing apart from the desire to acquire the durable. An individual's instantaneous utility with non-durable consumption c is $v(c)+\xi$ if he owns the durable ($\xi>0$), and $v(c)$ otherwise. We make the following assumption about the non-durable utility function.

Assumption 1. The function $v(c)$ is strictly concave, strictly increasing and three times continuously differentiable on $\mathbb{R}_+$, and satisfies $v'(0)=\infty$.

We depart from our companion paper, by adopting the fiction that the group consists of a continuum of individuals. Allowing the fraction of group members holding the indivisible good at any point in time to be treated as a continuous variable greatly simplifies the task of characterizing efficient allocations. We assume, without loss of generality, that the group's members are uniformly distributed over the unit interval and that individuals are indexed with numbers $\alpha\in[0,1]$. A *consumption bundle* for an individual may be described by a pair $\langle s, c(\cdot)\rangle$, where $s\in[0,T]$ denotes the date of receipt of the durable, and $c\colon[0,T]\to\mathbb{R}_+$ gives the consumption of the non-durable at each date.

An *allocation* is a pair of measurable functions, $\langle s(\cdot), c(\cdot,\cdot)\rangle$, such that $s\colon[0,1]\to[0,T]$ and $c\colon[0,1]\times[0,T]\to\mathbb{R}_+$.[4] The function $s(\alpha)$ is referred to hereafter as the *assignment function*. It gives the dates at which different individuals receive the durable. Since we are free to re-label individuals, we assume without loss of generality that those with lower index numbers receive the durable earlier, i.e., $s(\cdot)$ is non-decreasing on $[0,1]$. This implies that at date $s(\alpha)$ a fraction α of the group's members have the durable. The second component of an allocation is the consumption path $\{c(\alpha,\tau)\colon \tau\in[0,T]\}$ of each individual α. Under the allocation $\langle s, c\rangle$, individual α enjoys utility:

$$u(\alpha,\langle s,c\rangle)\equiv\int_0^T v(c(\alpha,\tau))d\tau+\xi(T-s(\alpha)). \tag{2.1}$$

Define $\bar{c}(t)\equiv\int_0^1 c(\alpha,t)d\alpha$, $t\in[0,T]$ to be the aggregate consumption level at time t under allocation $\langle s, c\rangle$. The allocation is *feasible* if it satisfies three conditions:

$$\int_0^{s(\alpha)}[y-\bar{c}(t)]dt\geqq\alpha B\ \forall\alpha\in[0,1], \tag{2.2}$$

$$0\leqq c(\alpha,t),\ \forall(\alpha,t)\in[0,1]\times[0,T], \tag{2.3}$$

and

$$\bar{c}(t)\leqq y,\ \forall t\in[0,T]. \tag{2.4}$$

3. The durable's services are also assumed not to be fungible across individuals; one must own it to benefit from its services.

4. Throughout we restrict attention to allocations in which all the group's members receive the durable at some time during their lives. This will be a property of the allocations achieved by roscas and markets if the value of owning the durable, ξ, is sufficiently large.

The first condition says that aggregate savings are great enough to finance investment at any date. The second says that non-durable consumption be non-negative and the third that aggregate consumption cannot exceed total income.

III. ALLOCATIONS ACHIEVED BY ROSCAS AND A CREDIT MARKET

We aim to understand rosca allocations in the context of the full set of possibilities that are feasible for the group. This section begins by describing the allocations achieved by both types of rosca and, for purposes of comparison, that which can be achieved were the group to form a credit market.

III.1. *Random rosca*

We consider roscas with equally-spaced meeting dates. It follows that the probability of winning the pot in a random rosca of length t is uniformly distributed on $[0, t]$.[5] Contributions will optimally be set so that the pot available to each winning member equals the cost of the durable, B. This implies a flow contribution rate of B/t for each member. Thus if $\tilde{\tau}$ is the random receipt date, each member's lifetime utility is the random variable:

$$\tilde{W}(t, \tilde{\tau}) \equiv t \cdot v(y - B/t) + (t - \tilde{\tau})\xi + (T - t) \cdot (v(y) + \xi), \tag{3.1}$$

and the ex ante expected utility of a representative individual, obtained by taking expectations of (3.1), is:

$$W(t) = t \cdot v(y - B/t) + (t/2)\xi + (T - t) \cdot (v(y) + \xi). \tag{3.2}$$

We suppose the rosca's length t is chosen to maximize (3.2). Denote the solution by t_r and let W_r equal the maximum expected utility. As in our companion paper, we exploit a simple way of writing W_r. Defining $c = y - B/t$ to be the consumption rate during the rosca, we can view the problem as choosing c to maximize $T \cdot (v(y) + \xi) - B \cdot [(v(y) + \xi/2 - v(c))/y - c]$. Then, defining

$$\mu(\lambda) \equiv \operatorname*{Min}_{0 \leqq c \leqq y} \left[\frac{v(y) + \xi(1 - \lambda) - v(c)}{y - c}\right], \quad \text{for } 0 \leqq \lambda \leqq 1, \tag{3.3}$$

we write:

$$W_r = T \cdot (v(y) + \xi) - B \cdot \mu(1/2). \tag{3.4}$$

The first term in (3.4) represents lifetime utility were the durable free, while the second is the minimized utility cost of saving up for the durable. This minimization trades off the benefit of a shorter accumulation period (or rosca length) against the benefit of higher consumption during this period (or smaller contributions). Letting $c^*(\lambda)$ denote the

5. In our companion paper we analyse a finite rosca with n members. It has meeting dates $\{t/n, 2t/n, \ldots, t\}$. The case that we are considering here is the limit as n approaches infinity. The particular limit obtained depends upon the assumption that the spacing of meeting dates is uniform and that the contribution at each meeting is constant. By having the meetings occur with different frequencies at different times during the life of the rosca, and by varying the rate of contribution across meeting dates, it may be shown that one can, in the limit, generate *every* feasible allocation $\langle s, c \rangle$ in which the consumption paths $c(\alpha, \tau)$ are constant in α, as the ex post outcome of a random rosca. (Moreover, one can in similar fashion generate every feasible allocation $\langle s, c \rangle$ in which the utility $u(\alpha; \langle s, c \rangle)$ is constant in α, as the limiting outcome of a bidding rosca.) However, we do not attempt to exploit these facts in this paper. To do so would run contrary to the spirit of our analysis. The point of this exercise is to analyse the allocative performance of roscas as they operate in practice.

consumption level which solves (3.3), the optimal consumption rate in the random rosca is $c_r \equiv c^*(1/2)$. As established in our companion paper, the minimized cost $\mu(\cdot)$ is a decreasing, strictly concave function of λ, and the cost-minimizing consumption rate $c^*(\cdot)$ is an increasing function of λ. Both are twice-continuously differentiable on [0, 1], where they satisfy the identity $\mu(\lambda) \equiv v'(c^*(\lambda))$.

Let $\langle s_r, c_r \rangle$ be the allocation in the optimal random rosca. By relabelling as required, individual α receives the durable at meeting date αt_r and the assignment function is linear, i.e. $s_r(\alpha) = \alpha t_r$. Thus the fraction of members who have received the durable is increasing and linear over the accumulation interval. All individuals have identical consumption paths that fall into two phases. During the rosca's life they consume at rate c_r. After it ends they consume at rate y. Thus, $c_r(\alpha, \tau) = c_r$, for $\tau \in [0, t]$; and, $c_r(\alpha, \tau) = y$, for $\tau \in (t_r, T]$. While group members have identical expected utilities, they enjoy different ex post utility levels.

III.2. *Bidding rosca*

As in our analysis of a random rosca, we consider a bidding rosca which meets at equally spaced meeting dates. The bidding rosca determines the order of receipt by bidding that takes place when the rosca is formed at time zero. Individuals bid by committing to various contribution rates over the life of the rosca. Since individuals are identical, any bidding equilibrium must make everyone indifferent between bid/receipt pairs. Any efficient auction procedure must also ensure that total contributions to the rosca are just adequate to finance acquisition of the durable. In fact, these two requirements completely determine the outcome of the bidding procedure, making it unnecessary to commit to a particular auction protocol.[6]

In a bidding rosca of length t, bidding determines the order of receipt over the interval $[0, t]$. Let $b(\alpha)$ denote the contribution of member α who, without loss of generality, is assumed to receive the durable at date αt. A set of bids $\{b(\alpha): \alpha \in [0, 1]\}$ constitutes an equilibrium if: (i) no individual could do better by out bidding another for his place in the queue; and (ii) contributions are sufficient to allow each member to acquire the durable upon receiving the pot.

Member α receiving the pot at date αt, will have non-durable consumption $c(\alpha) = y - b(\alpha)/t$ at each moment during the rosca's life. Thus the bidding rosca can be characterized in terms of consumption rates $\{c(\alpha): \alpha \in [0, 1]\}$. Condition (ii) implies that individual α's equilibrium utility level is $t \cdot v(c(\alpha)) + t(1-\alpha)\xi + (T-t) \cdot (v(y) + \xi)$ in a bidding rosca of length t. Condition (i) implies, for all individuals α and some number x, that:

$$v(c(\alpha)) + (1-\alpha)\xi = x, \tag{3.5}$$

where x represents the members' common average utility during the life of a bidding rosca of length t, in a bidding equilibrium.

Now define $\bar{c}$ to be the average non-durable consumption rate of members during the life of the rosca, i.e., $\bar{c} \equiv \int_0^1 c(\alpha)d\alpha$. Then condition (ii) is equivalent to:

$$t \cdot (y - \bar{c}) = B. \tag{3.6}$$

Given the rosca's length t, the relations (3.5) and (3.6) uniquely determine members' non-durable consumption rates and their average utility over the life of the rosca. Equivalently,

6. Our companion paper discusses how an ascending bid auction could implement this outcome.

one could take the equilibrium average utility level for the duration of the rosca, x, as given. Then (3.5) gives individuals' equilibrium consumption levels, $\{c(\alpha): \alpha \in [0, 1]\}$; and these, via (3.6) can be used to find the rosca's length, t.

We again assume that the bidding rosca's length is chosen to maximize the common utility level of its members. The foregoing discussion and (3.5) imply that the latter is $T \cdot (v(y)+\xi)-B \cdot \{[v(y)+\xi-x]/[y-\bar{c}]\}$. Now let $\hat{c}(\alpha, x)$ be the function satisfying $v(\hat{c})+(1-\alpha)\xi \equiv x$, and define $\bar{c}(x) \equiv \int_0^1 \hat{c}(\alpha, x)d\alpha$. If the equilibrium average utility during a bidding rosca is x, then $\hat{c}(\alpha, x)$ is individual α's non-durable consumption rate during the rosca, and $B/[y-\bar{c}(x)]$ is the rosca's length. Let t_b and W_b be the duration and common utility level of the optimal bidding rosca. Then, we write:

$$W_b = T \cdot (v(y)+\xi)-B \cdot \mu_b, \tag{3.7}$$

where

$$\mu_b \equiv \operatorname*{Min}_x \left[\frac{v(y)+\xi-x}{y-\bar{c}(x)}\right]. \tag{3.8}$$

If x^* is the minimum in (3.8), then $t_b = B/[y-\bar{c}(x^*)]$ is the length of the optimal bidding rosca. Individual α's consumption rate during the life of the rosca is $c_b(\alpha) \equiv \hat{c}(\alpha, x^*)$.

Lifetime utility expressed in (3.7) can be interpreted in the same way as for the random rosca: the difference between lifetime utility were the durable free, and the minimal cost of saving up. The latter, determined in (3.8), again trades off higher welfare during the rosca vs. faster acquisition of the durable.

Let $\langle s_b, c_b \rangle$ denote the allocation generated by the optimal bidding rosca. The assignment function is linear, i.e. $s_b(\alpha) = \alpha t_b$. Unlike the random rosca, each individual receives a different consumption path in a bidding rosca. However the general pattern is similar, an accumulation phase followed by a phase in which members consume all of their income. Thus the allocation of non-durable consumption is described by $c_b(\alpha, \tau) = c_b(\alpha)$, for $\tau \in [0, t_b]$ and $c_b(\alpha, \tau) = y$, for $\tau \in (t_b, T]$.

III.3. *A credit market*

We now study the credit market allocation. Organizing a market for funds is an option open to a group and we do indeed observe many informal credit markets in less developed countries. We will analyse the operation of an idealized competitive credit market. While this may not perfectly characterize the reality of informal credit, to do otherwise would be to risk stacking the deck against credit markets in our comparisons below. Moreover, there is no generally agreed upon model of how such informal markets do function.

Let $\delta(\tau)$ be the present value of a dollar at time τ. It is best to think of the market as determining a sequence of present value prices $\{\delta(\tau): \tau \in [0, T]\}$ at which the supply of and demand for loanable funds are equated.[7] Hence, an individual who buys the durable good at time s pays $\delta(s)B$ for it. Given a price path, an individual α chooses a purchase date $s(\alpha)$ and a consumption path $\{c(\alpha, \tau): \tau \in [0, T]\}$ to maximize utility; i.e., he solves

$$\operatorname*{Max}_{\{c(\cdot),s\}} \int_0^T v(c(\tau))d\tau + \xi(T-s) \text{ subject to}$$

$$\int_0^T \delta(\tau)c(\tau)d\tau + \delta(s)B \leqq y \int_0^T \delta(\tau)d\tau, \; s \in [0, T]. \tag{3.9}$$

7. It would be simple to map this back into a sequence of market clearing interest rates.

A *market equilibrium* is an allocation $\langle s_m, c_m \rangle$ and a price path $\delta(\cdot)$ satisfying two conditions. First, $\langle s_m(\alpha), c_m(\alpha, \tau) \rangle$ must solve (3.9) for all $\alpha \in [0, 1]$ and second,

$$\int_0^{s_m(\alpha)} [y - \bar{c}_m(t)]dt = \alpha B, \forall \alpha \in [0, 1]; \text{and}, \bar{c}_m(t) = y, \forall t \in (s_m(1), T], \tag{3.10}$$

where $\bar{c}_m(t)$ is aggregate consumption at time t. Since individuals are identical, the first condition implies that, in equilibrium, all individuals are indifferent between durable purchase times. The second says that savings equals investment at each date. We use W_m to denote the equilibrium level of utility enjoyed by group members with a credit market. Below, we show that this can be written in a form analogous to (3.4) and (3.7).

Direct computation of equilibrium prices and the associated allocation is difficult, even in the simple case of logarithmic utility studied in Section V. However, we are able to infer the existence and some of the properties of the credit market equilibrium in our model by using the fact that, with identical individuals, it must coincide with a Pareto efficient allocation which gives equal utility to every individual. Hence, describing properties of the credit market allocation must await consideration of efficient allocations more generally.

IV. EFFICIENT AND EX ANTE OPTIMAL ALLOCATIONS

The previous section described the allocations achieved by roscas and a credit market. We now turn to characterizing the group's "best" feasible allocations. Two criteria are natural here. The first is ex post Pareto efficiency, or more simply, *efficiency*. An allocation is *efficient* if it is feasible and if there exists no feasible allocation which makes almost every individual $\alpha \in [0, 1]$ at least as well off, and which makes a set of individuals of positive measure strictly better off.

The second criterion is defined in terms of ex ante expected utility. The allocation $\langle s, c \rangle$ is better than $\langle \hat{s}, \hat{c} \rangle$, in this sense if $\int_0^1 u(\alpha; \langle s, c \rangle) d\alpha > \int_0^1 u(\alpha; \langle \hat{s}, \hat{c} \rangle) d\alpha$. The thought experiment required is as follows: an individual will be assigned to any position in the queue for the durable with equal probability. We then ask which allocation would be best for *any* individual viewed from behind this "veil of ignorance". This allocation will thus be that which yields the highest level of expected utility. We call this the ex ante *optimal allocation*. It is obvious that this must correspond to a particular ex post efficient allocation.

Since all group members have the same utility level in a bidding rosca and a credit market, the criteria of ex post utility and ex ante expected utility coincide when applied to either of these institutions. This is not true for the random rosca; an allocation generated by a random rosca might be Pareto dominated though still preferred to some Pareto efficient allocation ex ante. In fact we present an example of this below.

IV.1. *Efficient allocations*

The standard approach to characterizing efficient allocations would introduce weights for each individual and maximize a weighted sum of utilities. This will also be our approach here. Thus we introduce the set of weights $\Theta \equiv \{\theta: [0, 1] \to \mathbb{R}_{++} | \theta$ is continuous and $\int_0^1 \theta(\alpha) d\alpha = 1\}$. Note, however, that the environment we are studying does not permit the use of a standard separation argument to justify this method of characterizing efficient allocations. This is because our economy has both a continuum of individuals and an indivisible good. The proof of Lemma 1 therefore constructs the weights associated with each efficient allocation.

Lemma 1. *Let $\langle s, c\rangle$ be an efficient allocation. Then there exist weights $\theta \in \Theta$ such that $\langle s, c\rangle$ maximizes the weighted sum of utilities over all feasible allocations.*

Proof: See the Appendix. ||

We now investigate properties of efficient allocations by studying, for fixed $\theta \in \Theta$, the problem:

$$\max_{\langle s,c\rangle} W(\theta; \langle s, c\rangle) \equiv \int_0^1 \theta(\alpha)u(\alpha; \langle s, c\rangle)d\alpha \text{ subject to (2.2), (2.3) \& (2.4),} \tag{4.1}$$

for $u(\alpha; \langle s, c\rangle)$ as defined in (2.1). Let $\langle s_\theta, c_\theta\rangle$ denote the allocation that solves this problem and $W_\theta \equiv W(\theta; \langle s_\theta, c_\theta\rangle)$ denote the maximized value of the objective function.

Our first observation is that the efficiency problem can be solved in two stages, loosely corresponding to static and dynamic efficiency. The first requires that aggregate consumption be optimally allocated across group members, i.e. maximizes the weighted sum of instantaneous utility at each date, while the second determines the optimal acquisition path for the durable good.

Let $\bar{c}_\theta(\tau)$ denote aggregate consumption in period τ, i.e. $\bar{c}_\theta(\tau) \equiv \int_0^1 c_\theta(\alpha, \tau)d\alpha$. Assumption 1 implies that the non-negativity constraint (2.3) will not be binding for almost all individuals at any time, so we may assume without loss of generality that $\bar{c}_\theta(\tau) > 0$. Inspection of problem (4.1) reveals that this aggregate consumption should be distributed among group members so as to maximize the weighted sum of utilities from consumption in period τ. To make this more precise, for all $w > 0$, consider the problem:

$$\operatorname*{Max}_{\chi(\cdot)} \int_0^1 \theta(\alpha)v(\chi(\alpha))d\alpha \quad \text{subject to} \int_0^1 \chi(\alpha)d\alpha = w. \tag{4.2}$$

Let $\chi_\theta(\cdot, w)$ denote the solution and let $V_\theta(w)$ denote the value of the objective function. Then individual α's consumption at time $\tau \in [0, T]$ is given by $c_\theta(\alpha, \tau) = \chi_\theta(\alpha, \bar{c}_\theta(\tau))$ and total weighted utility from non-durable consumption is given by $V_\theta(\bar{c}_\theta(\tau))$.

It remains, therefore, to determine $s_\theta(\alpha)$ and $\bar{c}_\theta(\tau)$. Note first, that, given our assumptions on preferences, constraint (2.2) may without loss of generality be written as:

$$\int_0^{s(\alpha)} [y - \bar{c}(\tau)]d\tau = \alpha B, \text{ for all } \alpha \in [0, 1], \tag{4.3}$$

which says that savings must be put to immediate use. Note also that in the absence of discounting, if the flow of aggregate savings $y - \bar{c}(\tau)$ equals zero at some date τ, then efficiency demands that it is also zero at any later date $\tau' > \tau$. Otherwise, moving later savings forward in time could give some individuals an earlier receipt date for the durable without reducing anyone's utility from non-durable consumption. This implies that any assignment function solving (4.1) must be continuous, increasing, and satisfy $s(0) = 0$. Such an assignment function is differentiable almost everywhere. It follows that constraint (4.3) can be rewritten as:

$$s'(\alpha) = B/[y - \bar{c}(s(\alpha))], \text{ for all } \alpha \in [0, 1]. \tag{4.4}$$

Next observe that after the date $s(1)$ at which the last individual in the group receives the durable, no further savings are necessary. Thus, we may assume with no loss of generality that:

$$\bar{c}(\tau)=y, \text{ for all } \tau\in(s(1), T] \tag{4.5}$$

Equations (4.4) and (4.5) are the analogue of "production efficiency"; they imply no waste of resources.

It follows from the above discussion that $s_\theta(\alpha)$ and $\bar{c}_\theta(\tau)$ solve the problem

$$\operatorname{Max} \int_0^{s(1)} V_\theta(\bar{c}(\tau))d\tau+(T-s(1))[V_\theta(y)+\xi]+\xi\left(s(1)-\int_0^1 \theta(\alpha)s(\alpha)d\alpha\right)$$

$$\text{subject to (4.4).} \tag{4.6}$$

Now define the function $\mu_\theta(\cdot)$ as follows:

$$\mu_\theta(\lambda)\equiv \min_{0\leqq\sigma\leqq y}\left[\frac{V_\theta(y)+(1-\lambda)\xi-V_\theta(\sigma)}{y-\sigma}\right], 0\leqq\lambda\leqq 1, \tag{4.7}$$

and for each λ denote the solution of the minimization in (4.7) by $\sigma_\theta(\lambda)$. Then we have:

Lemma 2. *Let $\langle s, c\rangle$ be an efficient allocation, and let θ be the weights for which $\langle s, c\rangle$ provides a solution in* (4.1). *Then the maximized value can be written in the form*

$$W_\theta=T\cdot(V_\theta(y)+\xi)-B\cdot\int_0^1 \mu_\theta\left(1-\int_x^1 \theta(z)dz\right)dx$$

and the assignment function satisfies:

$$s(\alpha)=B\cdot\int_0^\alpha\left[y-\sigma_\theta\left(1-\int_x^1 \theta(z)dz\right)\right]^{-1}dx, \qquad \forall\alpha\in[0, 1].$$

Moreover, for all $\alpha\in[0, 1]$, non-durable consumption obeys

$$c(\alpha, s(x))=\chi_\theta\left(\alpha, \sigma_\theta\left(1-\int_x^1 \theta(z)dz\right)\right), \text{ for } x\in[0, 1];$$

and

$$c(\alpha, \tau)=\chi_\theta(\alpha, y), \text{ for } \tau\in(s(1), T].$$

Proof: See the Appendix. ||

Pareto efficiency requires two conditions beyond no waste of resources: Any aggregate level of non-durable consumption should be allocated efficiently among individuals and the intertemporal trade-off between aggregate non-durable consumption and faster diffusion of durable ownership is optimally managed. Above we discussed how $V_\theta(\cdot)$ summarized the first of these stages. We now discuss the dynamic efficiency part in greater detail, in particular the relevance of the minimization in (4.7).

The expression for W_θ in Lemma 2 is the difference between two terms. The first, $T\cdot(V_\theta(y)+\xi)$, is the maximal weighted utility sum if the durable were a free good, while

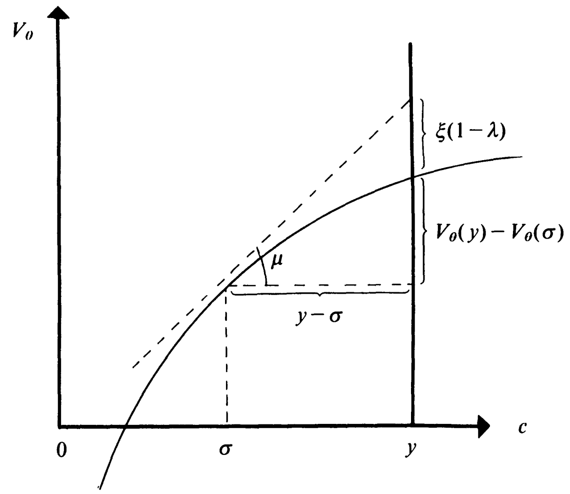

FIGURE 1

the second is the (utility equivalent) cost of acquiring the durable. It is this cost that is minimized in (4.7). It has two competing components: non-durable consumption foregone in the process of acquiring the durable (since $\bar{c}(s(\alpha))<y$) and durable services foregone in allowing some non-durable consumption (since $s'(\alpha)<B/y$). During the small interval of time that the durable is being acquired by individuals $\tilde{\alpha}\in(\alpha,\alpha+d\alpha)$, then the sum of these two components is approximately $[V_\theta(y)+\xi\int_\alpha^1\theta(z)dz-V_\theta(\bar{c}(s(\alpha)))]$, while the duration of this time interval is $s'(\alpha)d\alpha=Bd\alpha/[y-\bar{c}(s(\alpha))]$. Efficient accumulation therefore means minimizing the product of these terms at each $\alpha\in[0,1]$. This is precisely the problem described by (4.7).

A geometric treatment of the minimization problem (4.7) may also be helpful (see Figure 1). The function $V_\theta(\cdot)$ is smooth, increasing and strictly concave because we have assumed that $v(\cdot)$ has these properties. Therefore, choosing σ to minimize the ratio $[V_\theta(y)+(1-\lambda)\xi-V_\theta(\sigma)]/[y-\sigma]$ means finding that point $(\sigma, V_\theta(\sigma))$ on the graph of $V_\theta(\cdot)$ such that the straight line containing it, and containing the point $(y, V_\theta(y)+(1-\lambda)\xi)$, is tangent to the graph of $V_\theta(\cdot)$. Notice from the diagram that $\sigma_\theta(\lambda)$ must be increasing, rising to y as λ increases to 1.

This observation, together with Lemma 2, permits us to deduce some properties of efficient allocations that embody the key economic insights behind the results:

Theorem 1. *Let $\langle s, c\rangle$ be an efficient allocation. Then*

(i) *the assignment function $s(\cdot)$ is increasing, strictly convex and satisfies $\lim_{\alpha\to 1}s'(\alpha)=+\infty$, and*
(ii) *for all $\alpha\in[0,1]$, $c(\alpha,\cdot)$ is increasing on the interval $[0, s(1)]$, and constant thereafter.*

Proof. (i) In view of (4.4) and Lemma 2, we know that any efficient allocation $\langle s, c\rangle$ satisfies $s'(\alpha)=B/[y-\sigma_\theta(1-\int_\alpha^1\theta(z)dz)]$, for some θ. As noted above $\sigma_\theta(\lambda)$ is increasing and approaches y as λ increases to 1. Hence, the result.

(ii) This follows immediately from Lemma 2 after noting that $\chi_\theta(\alpha,\cdot)$ is increasing and that $\sigma_\theta(\lambda)$ is increasing. ||

The properties of the assignment function imply that, in an efficient allocation, the fraction of the group who have received the durable by time τ is increasing and strictly concave. In addition, the rate of accumulation (the time derivative of the inverse of the assignment function) approaches zero as τ goes to $s(1)$.

The analogy between the characterization in Lemma 2 and the expressions for the random and bidding roscas in (3.4) and (3.7) is worth noting. These all take the same general form: welfare is the hypothetical utility achieved if the durable were free, less the utility cost of acquiring the durable. This observation underpins the results in Section V.

IV.2. *The optimal allocation*

The *optimal allocation* is that efficient allocation in which individuals are equally weighted, $\theta(\alpha)\equiv 1, \forall\alpha\in[0, 1]$. Since individuals are identical and are assigned types randomly, this maximizes the *ex ante* expected utility of a representative group member (see also (4.1)). Hence, we can write *ex ante* expected utility as $W(1; \langle s, c\rangle)$, and the *optimal allocation* $\langle s_o, c_o\rangle$ must satisfy: $W(1; \langle s_o, c_o\rangle)\geqq W(1; \langle s, c\rangle)$, for all feasible allocations $\langle s, c\rangle$.

Since $V_1(w)\equiv v(w)$, and $\chi_1(\alpha, w)\equiv w$, for all (α, w), aggregate non-durable consumption is allocated equally among group members. In addition, $\mu_1(\lambda)\equiv\mu(\lambda)$, where $\mu(\cdot)$ is defined in (3.3), and $\mu'(\lambda)=-\xi/[y-\sigma_1(\lambda)]$, using the Envelope Theorem. These facts, together with Lemma 2, yield:

Theorem 2. *Let $\langle s_o, c_o\rangle$ be the optimal allocation. Then, ex ante expected utility can be written in the form*

$$W(1; \langle s_o, c_o\rangle)\equiv W_o=T\cdot(v(y)+\xi)-B\cdot\int_0^1 \mu(\alpha)d\alpha,$$

and the optimal assignment function satisfies

$$s_o(\alpha)=-\frac{B}{\xi}\cdot\int_0^\alpha \mu'(x)dx.$$

Moreover, for all $\alpha\in[0, 1]$, non-durable consumption obeys

$$c_o(\alpha, s_o(x))=y+\xi/\mu'(x) \text{ for } x\in[0, 1], \text{ and } c_o(\alpha, \tau)=y \text{ for } \tau\in(s_o(1), T].$$

In addition to the properties in Theorem 1, the optimal allocation gives each individual a consumption path that rises smoothly to y at the end of the accumulation phase. The fraction of the group who owns a durable is increasing and concave over the interval of accumulation.

There is a close relationship between the problem solved by the optimal allocation and that of a single individual accumulating a perfectly divisible good. If the durable good were perfectly divisible, then there would be no gains from trade in the latter case and autarkic saving would be optimal.[8] It is the indivisibility of the durable that creates the

8. Even with indivisibility the allocation problem would reduce in this way if the durable's services were fungible across agents—if there were, e.g. a perfect rental market for its services. There are, of course, good (adverse selection/moral hazard) reasons why such trade in durable services might not obtain, especially in a LDC setting. Moreover, some reports on the use of roscas stress their role in financing personal expenditures (daughter's wedding, feast for fellow villagers, tin roof for house) which, though not producing a fungible asset, generate private consumption benefits lasting for some time that are not transferable to others.

problem. However, the group may approximately replicate perfect divisibility in an ex ante sense by randomly assigning individuals to positions in the queue at the initial date. In effect, each individual is given a "share" of the aggregate amount of the durable good available at any subsequent date. The optimal non-durable consumption path, $\bar{c}_o(\cdot)$, is precisely that attained by an individual accumulating a perfectly divisible durable good.[9]

The credit market and rosca allocations can be related to the efficient and optimal ones. The random rosca maximizes ex ante expected utility subject to the assignment function being linear. The bidding rosca also imposes the constraint that utilities be equal. A credit market corresponds to the efficient allocation where life-time utilities are equal. These relationships are important in understanding the results of the next section.

V. THE ALLOCATIVE PERFORMANCE OF ROSCAS

Our characterization of efficient and optimal allocations, gives us a useful starting point for evaluating the performance of roscas. We begin by discussing efficiency.

Proposition 1. *The allocations achieved by bidding and random roscas are inefficient.*

Proof. By Theorem 1(i) efficient allocations have *strictly convex* assignment functions, while the analysis of Sections III.1 and III.2 showed that roscas, with their uniformly spaced meeting dates and constant contribution rates, lead to *linear* assignment functions. ||

This says that the simple structure of roscas has a cost and identifies the nature of it. The convexity of efficient assignment functions follows from the fact that, as the remaining horizon becomes shorter, the value of the durable good to a group member who acquires it diminishes, so the amount of current consumption foregone to finance diffusion of durable goods should also decline. Roscas, with their uniformly spaced meeting dates and constant contribution rates, cannot achieve this subtle intertemporal shift in resource allocation. Their simple form therefore prevents the realization of maximal gains from trade.[10]

Notwithstanding, the best random rosca does yield maximal ex ante expected utility to its members subject to the constraint of the assignment function being linear. Moreover, the best bidding rosca generates the highest *common* level of utility for its members, among all feasible allocations with linear assignment functions.

Our companion paper established that random roscas resulted in higher expected utility than bidding roscas; that is $W_r > W_b$. The analysis of this paper extends this ranking

9. To be more precise, it can be shown that the optimal aggregate consumption path $\{\bar{c}_o(\tau): \tau \in [0, T]\}$ solves the problem:

$$\max_{c(\cdot)} \int_0^T [v(c(\tau)) + \xi(T-\tau)K'(\tau)]d\tau$$

$$\text{subject to } B \cdot K'(\tau) = y - c(\tau);\ K(0) = 0;\ K(T) = 1;\ 0 \leq c(\tau) \leq y.$$

Here, the function $K(\tau)$ is to be interpreted as the stock of the divisible asset the individual holds at time τ.

10. As a referee points out, linear assignment functions may be efficient in an environment with overlapping generations of agents. Consider, for example, an overlapping-generations model with stationary demographic structure in which agents consume a durable good like that in our model. If the durable good cannot be passed on to future generations or if it depreciates, it seems quite plausible that a constant rate of accumulation of durables could be efficient. There is thus the intriguing possibility that by combining individuals from different generations roscas could achieve efficient allocations in such environments.

by noting that neither achieves maximal possible expected utility. This can be proved directly by using the fact in Theorem 2, that the cost of saving up equals $\int_0^1 \mu(\alpha)d\alpha$. Using Jensen's inequality and the strict concavity of $\mu(\cdot)$, we obtain $\int_0^1 \mu(\alpha)d\alpha < \mu(1/2)$ to prove that the cost of saving up will be greater under a random rosca. The Optimum is better than a random rosca precisely because it offers a non-linear assignment function.

The credit market allocation which is constrained by definition to provide group members with equal utilities, generates lower ex ante expected utility than the optimal allocation $\langle s_o, c_o \rangle$.[11] In general, however a credit market Pareto dominates a bidding rosca. To see this, recall that, in addition to being constrained to provide individuals with equal utilities, the bidding rosca is also constrained to have a linear assignment function. We summarize these observations in

Proposition 2. *While not achieving the optimal allocation, a credit market is preferred to a bidding rosca from an ex ante viewpoint, i.e.* $W_o > W_m > W_b$.

Proof. Since each individual's utility is constant in both $\langle s_m, c_m \rangle$ and $\langle s_b, c_b \rangle$, and since $\langle s_b, c_b \rangle$ is Pareto inefficient while by the First Fundamental Theorem of Welfare Economics $\langle s_m, c_m \rangle$ is efficient, we must have $W_m > W_b$. Moreover, the constancy of individuals' utility in a competitive equilibrium implies:

$$\forall \theta \in \Theta: W_\theta \equiv W(\theta; \langle s_\theta, c_\theta \rangle) \geqq W(\theta; \langle s_m, c_m \rangle) = W(\theta_m; \langle s_m, c_m \rangle) \equiv W_m. \quad (5.1)$$

where θ_m are the weights associated with the competitive allocation. The inequality in (5.1) reflects the fact that $\langle s_\theta, c_\theta \rangle$ maximises the weighted sum of utilities with weights θ; the equality is due to the fact that the weighted average of a constant function does not depend on the weights. So $W_m = \text{Min}_{\theta \in \Theta} \{\text{Max}_{\langle s, c \rangle} \{W(\theta; \langle s, c \rangle) \mid \langle s, c \rangle \text{ is feasible}\}\}$! The competitive equilibrium solves an elegant mini—max problem. Thus, not only is $W_m < W_o$ (equality is impossible since then, by the strict concavity of $v(\cdot)$ and the fact that $c_m \neq c_o$, a strict convex combination of $\langle s_m, c_m \rangle$ and $\langle s_o, c_o \rangle$ would be feasible and would dominate $\langle s_o, c_o \rangle$), but W_m is less than *any* maximized weighted sum of utilities. ||

This proof demonstrates that the credit market equilibrium uses weights which minimize W_θ. This is key to our constructive demonstration, in Proposition 3 below, that there exist circumstances under which the credit market allocation is strictly dominated, in terms of ex ante expected utility, by the optimal random rosca. Hence, our final result on welfare comparisons shows that the "equal utility" constraint can be more of an impediment to generating ex ante welfare than the "linear assignment function" constraint. As already mentioned, W_r is the maximal ex ante welfare subject to having a linear assignment function, while W_m maximizes the same criterion subject to the constraint that utilities are equal. The question naturally arises whether one can prove a general result on the relation of these values. One might have suspected that under some plausible conditions the competitive allocation would dominate the inefficient random rosca. However, this is not the case. What follows is an illustration of the fact that a simple institution of financial intermediation, allocating its funds by lot, can actually outperform an idealized competitive credit market.

11. The failure of the market to achieve the ex ante optimum parallels results in other literatures where indivisibilities are important. See, for example, the model of conscription in Bergstrom (1986), the location models of Mirrlees (1972) and Arnott and Riley (1977), the club membership model of Hillman and Swan (1983), and the labour market model of Rogerson (1988).

Proposition 3. *In the case of logarithmic utility, there exists a $\bar{\xi}$ such that for all $\xi > \bar{\xi}$, a random rosca dominates the credit market; i.e. $W_r > W_m$.*

Proof. See the Appendix. ||

The technique of proof is indirect, since explicit representation of credit market allocations, even in the case of logarithmic utility, seems intractable. We use the fact, from the proof of Proposition 3, that the market gives the least maximized weighted utility sum, over all possible weights. We then construct a set of weights whose maximized utility sum is less than W_r, to infer the result. Intuitively the result may be understood by recognizing that, when ξ is very large, respecting the equal utility constraint means those receiving the durable early must get much lower non-durable consumption than those acquiring it late. This causes individuals' *marginal* utilities of income to diverge. However, since preferences are additive and there is no discounting, an ideal intertemporal path of consumption would equate marginal utilities of income through time, something which is achieved under a random rosca. The effect of increasing ξ is thus to increase this divergence in marginal utilities thereby lowering ex ante expected utility in a market. The magnitude of ξ does not, however, affect the utility cost of having a linear assignment function. Thus when ξ is sufficiently large the random rosca dominates.

VI. CONCLUSION

Given the world-wide prevalence of roscas, it is important to understand their economic role and performance. Following the large informal literature, we have sought their rationale in the fact that some goods are indivisible. This makes autarkic saving inefficient. Our companion paper spelled out how, in a world with an indivisible good, a group of individuals without access to credit markets could improve their welfare by forming a rosca and compared the allocations achieved by the two different types of rosca. It found that with homogeneous individuals, randomization is preferred to bidding as a method of allocating funds within roscas. With heterogeneous individuals, however, this result may not hold.

This paper completes the picture by considering roscas in the larger context of the set of feasible allocations that can be attained by a group of individuals. One important finding is that roscas do not, in general, produce efficient allocations. Their simple structure allows less flexibility in the rate of accumulation of the indivisible good than is necessary to achieve maximal gains from trade. A further finding is that bidding roscas are Pareto dominated by credit markets. This is not so surprising since both institutions use prices to allocate access to the indivisible good, but the credit market has greater flexibility. Nonetheless, the element of chance offered by random roscas is still of value. Credit market allocations may be dominated (under the ex ante expected utility criterion) by those produced by a random rosca.[12] In light of the significantly greater complexity of a credit market, this is a noteworthy finding.

APPENDIX

Proof of Lemma 1. Let $\langle s, c\rangle$ be an efficient allocation. We begin with a few preliminary observations about this allocation. It will simplify notation to let t^* denote the date at which the last individual receives the

12. Given the results of our companion paper it seems reasonable to conjecture that this may cease to hold if individuals are sufficiently heterogeneous.

durable; i.e. $t^* = s(1)$. First note that, since $v'(0) = \infty$ and $v'(y) > 0$, it is obvious that (2.2) holds as an identity for $\alpha \in [0, 1]$, that (2.3) holds strictly for all (α, t) and that (2.4) holds as an equality for all $t \in [t^*, T]$. From this we deduce that $s(\cdot)$ is differentiable at every point α in [0, 1] such that $\bar{c}^*(\alpha) \equiv \bar{c}(s(\alpha))$ is continuous and

$$s'(\alpha)[y - \bar{c}^*(\alpha)] = B. \tag{A.1}$$

Since $v(c)$ is strictly concave, we can also conclude that the consumption of almost every individual is constant on $[t^*, T]$. Hence

$$c(\alpha, t) \equiv c(\alpha, t^*) > 0 \ \forall t \in [t^*, T]. \tag{A.2}$$

For all $\alpha \in [0, 1]$, define $\lambda(\alpha) \equiv 1/v'(c(\alpha, t^*))$.

Now notice that the non-durable consumption functions $c(\alpha, \cdot)$, $\alpha \in [0, 1]$ must be continuous at any date t, for almost all individuals. For if a non-negligible set of consumers experienced a discontinuity at the same date, then, because utility is strictly concave, by smoothing their consumption in the neighbourhood of the point of discontinuity while keeping cumulative consumption constant, one can increase their utility, while negligibly affecting the rate at which aggregate savings allow new acquisitions of the durable.

It follows that, at each date $t \in [0, T]$, the ratio $v'(c(\alpha, t))/v'(c(\alpha, t^*))$ is equal to the same number, $\psi(t)$, for almost every individual α. Otherwise some shift of consumption between individuals at dates near t and t^* would yield a Pareto improvement for a non-negligible set of consumers. Specifically, in view of the continuity of $c(\alpha, \cdot)$, were $v'(c(\alpha, t))/v'(c(\alpha, t^*))$ not the same across almost every individual, then there would exist an open interval $I \subset [0, T]$ with $t \in I$, positive numbers ψ_1 and ψ_2, and non-negligible, disjoint sets of individuals A_1 and A_2, such that

$$\frac{v'(c(\alpha, \tau))}{v'(c(\alpha, \rho))} \geqq \psi_1 > \psi_2 \geqq \frac{v'(c(\beta, \tau))}{v'(c(\beta, \rho))}$$

for all $\alpha \in A_1$, $\beta \in A_2$, $\tau \in I$ and $\rho \geqq t^*$. Thus moving consumption from individuals in A_1 to those in A_2 at some dates $\rho \geqq t^*$, while moving an equal amount of consumption in the opposite direction at some dates $\tau \in I$, could make all of the individuals in both sets strictly better off.

Thus there exist functions $\lambda(\alpha)$ and $\psi(t)$ such that $\lambda(\alpha) > 0$, $\psi(t) > 0$, $\psi(t) = \psi(t^*) = 1 \ \forall t > t^*$, and for which, for all t and almost all α:

$$v'(c(\alpha, t)) = \psi(t)/\lambda(\alpha). \tag{A.3}$$

For all $\alpha \in [0, 1]$, define $\psi^*(\alpha) \equiv \psi(s(\alpha))$.

We now prove the lemma stated in the text for weights $\theta(\alpha) = \lambda(\alpha)/\int_0^1 \lambda(\beta)d\beta$, where $\lambda(\alpha)$ is as defined above. The proof has three steps. First, we show that in an efficient allocation at each date t, aggregate consumption $\bar{c}(t)$ must be spread among individuals to maximize the weighted sum of contemporaneous utility from non-durable consumption. Second, we show that the aggregate consumption in an efficient allocation at each point in time must be such that a certain equation that has a unique solution is satisfied. Third, we show that the solution to the problem of maximizing the weighted sum of utilities, with the weights indicated above, implies a level of aggregate consumption that satisfies the same equation.

Claim 1. *For all* $t \in [0, T]$

$$\int_0^1 \lambda(\alpha)v(c(\alpha, t))d\alpha = \text{Max}\left\{\int_0^1 \lambda(\alpha)v(\chi(\alpha))d\alpha \,\middle|\, \int_0^1 \chi(\alpha)d\alpha \leqq \bar{c}(t)\right\} \equiv V(\bar{c}(t)). \tag{A.4}$$

Proof. This follows, given the concavity of $v(c)$ from the fact that, for any $\chi(\alpha)$:

$$v(c(\alpha, t)) - v(\chi(\alpha)) \geqq v'(c(\alpha, t)) \cdot [c(\alpha, t) - \chi(\alpha)].$$

Hence multiplying both sides by $\lambda(\alpha)$, integrating from 0 to 1, and using the supposition that $\chi(\alpha)$ does not integrate to more than $\bar{c}(t)$, gives the result. Note well that:

$$\psi(t) \equiv V'(\bar{c}(t)), \forall t \in [0, T],$$

and that $V'(\cdot)$ is twice-continuously differentiable, given our assumptions on $v(\cdot)$. ||

Claim 2. *For all* $\alpha \in [0, 1]$

$$V(y) - V(\bar{c}^*(\alpha)) + \xi \int_\alpha^1 \lambda(\beta) d\beta = V'(\bar{c}^*(\alpha)) \cdot [y - \bar{c}^*(\alpha)]. \tag{A.5}$$

Proof. We first show that the following identity holds:

$$\xi \int_0^1 \lambda(x) s'(x) dx \equiv B \cdot [\psi^*(\alpha) - 1] \ \forall \alpha \in [0, 1]. \tag{A.6}$$

We prove this by showing that were it not true, then changing the pattern of the durable accumulation during $[0, t^*]$, by reallocating the consumption of those receiving the durable on or after one of the dates $s(\alpha)$, can be done so as to create a Pareto improvement. Thus suppose the individuals in the interval $(\alpha - d\alpha, \alpha)$ were all to receive the durable at the date $s(\alpha - d\alpha)$, instead of over the interval $(s(\alpha - d\alpha), s(\alpha))$. Financing the durable's acquisition by these individuals would cost $B \cdot d\alpha$ units of non-durable consumption. Removing these individuals from the "queue" would allow each individual $\beta \in [\alpha, 1]$ to receive the durable at the date $s(\beta - d\alpha)$ instead of $s(\beta)$, increasing utility by $\xi s'(\beta) d\alpha$. We now see how much these individuals $\beta \in [\alpha, 1]$ would be willing to pay for this accelerated accumulation.

If individuals $\beta \in [\alpha, 1]$ were to reduce their consumption for a short time-interval of length Δ just prior to the date $s(\alpha - d\alpha)$, and then increase their consumption by the same amount and for the same length of time prior to the date t^*, they could generate resources that might be sufficient to finance the durable's accelerated acquisition. Let the consumption change for agent β be $dc(\beta)$. Then the utility cost of the change is:

$$[v'(c(\beta, s(\alpha))) - v'(c(\beta, t^*))] \cdot dc(\beta) \cdot \Delta.$$

By equating this cost to the utility gain from accelerating the durable's acquisition, we infer the amount of consumption that each agent $\beta \in [\alpha, 1]$ would be willing to shift:

$$\Delta dc(\beta) = \frac{\xi s'(\beta) d\alpha}{v'(c(\beta, s(\alpha))) - v'(c(\beta, t^*))}.$$

Multiply the numerator and denominator above by $v'(c(\beta, t^*))$, use the result above that $v'(c(\alpha, t))/v'(c(\alpha, t^*))$ equals $\psi(t)$ and the definition of $\lambda(\alpha)$, and integrate over $\beta \in [\alpha, 1]$ to conclude that the total "willingness to pay" for the accelerated accumulation by the individuals $[\alpha, 1]$ is as follows:

$$\Delta \int_\alpha^1 dc(\beta) d\beta = \frac{\xi}{\psi(\alpha) - 1} \cdot \left[\int_\alpha^1 \lambda(\beta) s'(\beta) d\beta \right] d\alpha.$$

When this quantity is greater than $Bd\alpha$ the cost of financing the acceleration, then increasing consumption by individuals $\beta \in [\alpha, 1]$ near the end of the accumulation period, while reducing consumption for some time just before $s(\alpha)$ so as to permit faster acquisition of the durable, can produce a Pareto improvement. When this quantity is less than $Bd\alpha$, increasing consumption just before $s(\alpha)$, thereby delaying the acquisition of the durable for the individuals $[\alpha, 1]$, while reducing consumption by corresponding amount near the end of the accumulation period, can generate a Pareto improvement for these individuals. We conclude that efficient accumulation requires (A.6) to hold.

Claim 2 now follows easily. Differentiate (A.6), using the fact that $\psi^*(\alpha) = V'(\bar{c}^*(\alpha))$, and use (A.1) to get:

$$\xi \theta(\alpha) = -[y - \bar{c}^*(\alpha)] \cdot \frac{d\psi^*}{d\alpha}(\alpha), \ \alpha \in [0, 1].$$

Add $\psi^*(\alpha) \cdot d\bar{c}^*(\alpha)/d\alpha = d[V(\bar{c}^*(\alpha))]/d\alpha$ to both sides of this equation, and integrate to get:

$$\int_\alpha^1 \left\{ \xi \lambda(\beta) + \frac{d}{d\beta} [V'(\bar{c}^*(\beta))] \right\} d\beta = - \int_\alpha^1 \frac{d}{d\beta} \{ \psi^*(\beta) [y - \bar{c}^*(\beta)] \} d\beta,$$

or

$$V(y) - V(\bar{c}^*(\alpha)) + \xi \int_\alpha^1 \lambda(\beta) d\beta = V'(\bar{c}^*(\alpha)) \cdot [y - \bar{c}^*(\alpha)].$$

This proves Claim 2. ||

Notice that, given $\lambda(\cdot)$, equation (A.5) uniquely determines $\bar{c}^*(\alpha)$ at each $\alpha \in [0, 1]$, and that $\bar{c}^*(\alpha)$ is an increasing, differentiable function on $[0, 1]$, satisfying $\bar{c}^*(1) = y$. Equation (A.5) also determines $c(t) = \bar{c}^*(s^{-1}(t))$, since $s(\cdot)$ solves the differential equation: $s'(\alpha) = B/[y - \bar{c}^*(\alpha)]$, $s(0) = 0$. Thus, an efficient allocation is completely determined by the function $\lambda(\alpha) = 1/v'(c(\alpha, t^*))$.

Now consider the problem of maximizing the weighted sum of lifetime utilities of all individuals, given arbitrary weights $\theta \in \Theta$. Let $\langle s_\theta, c_\theta \rangle$ be the solution. This solution has the following two properties, as is demonstrated in the text following Lemma 1 and in the proof of Lemma 2. First, for all $t \in [0, T]$,

$$\int_0^1 \theta(\alpha) v(c_\theta(\alpha, t)) d\alpha = \underset{\chi(\cdot)}{\text{Max}} \left\{ \int_0^1 \theta(\alpha) v(\chi(\alpha)) d\alpha \,\middle|\, \int_0^1 \chi(\alpha) d\alpha \leqq \bar{c}_\theta(t) \right\} \equiv V_\theta(\bar{c}_\theta(t)).$$

Second, the optimal aggregate consumption function $\bar{c}_\theta^*(\alpha) \equiv \bar{c}_\theta(s(\alpha))$, $\alpha \in [0, 1]$ satisfies the first order condition

$$V_\theta(y) - V_\theta(\bar{c}_\theta^*(\alpha)) + \xi \int_\alpha^1 \theta(\beta) d\beta = V'_\theta(\bar{c}_\theta^*(\alpha)) \cdot [y - \bar{c}_\theta^*(\alpha)].$$

Note that if $\theta(\alpha) = \lambda(\alpha) / \int_0^1 \lambda(\beta) d\beta$, then $V_\theta(w) = V(w) / \int_0^1 \lambda(\beta) d\beta$. In light of Claims 1 and 2 and the discussion following Claim 2, we may therefore conclude that the efficient allocation $\langle s, c \rangle$ solves the weighted utility maximization problem for these weights. ||

Proof of Lemma. 2. In view of the discussion preceding the statement of the Lemma:

$$W_\theta \equiv \text{Max} \left\{ \int_0^{s(1)} V_\theta(\bar{c}(\tau)) d\tau + (T - s(1)) \cdot V_\theta(y) + \xi T - \xi \int_0^1 \theta(\alpha) s(\alpha) d\alpha \right\}$$

subject to

$$s'(\alpha) = B/[y - \bar{c}(s(\alpha))], \; \alpha \in [0, 1].$$

Now employ the change of variables: $\tau = s(\alpha)$, $d\tau = s'(\alpha) d\alpha$, $\tau \in [0, s(1)]$; note that $s(1) = \int_0^1 s'(\alpha) d\alpha$; and use (4.4) and then definition in (4.7) to get the following:

$$\begin{aligned} W_\theta &= \text{Max} \left\{ \int_0^{s(1)} V_\theta(\bar{c}(\tau)) d\tau + (T - s(1)) \cdot V_\theta(y) + \xi \cdot \left(T - \int_0^1 \theta(\alpha) s(\alpha) d\alpha \right) \right\} \\ &= \text{Max} \left\{ T \cdot (V_\theta(y) + \xi) - \int_0^1 s'(\alpha) \cdot \left[V_\theta(y) + \xi \int_\alpha^1 \theta(z) dz - V_\theta(\bar{c}(s(\alpha))) \right] d\alpha \right\} \\ &= T \cdot (V_\theta(y) + \xi) - \text{Min} \left\{ B \int_0^1 \left\{ \left[V_\theta(y) + \xi \int_\alpha^1 \theta(z) dz - V_\theta(\bar{c}(s(\alpha))) \right] \Big/ [y - \bar{c}(s(\alpha))] \right\} d\alpha \right\} \\ &= T \cdot (V_\theta(y) + \xi) - B \int_0^1 \mu_\theta \left(1 - \int_\alpha^1 \theta(z) dz \right) d\alpha. \end{aligned}$$

This proves the first claim. Note that the minimization above is pointwise, with respect to $\bar{c}(s(\alpha))$, at each $\alpha \in [0, 1]$. So it implies that for $\alpha \in [0, 1]$, $\bar{c}(s(\alpha)) = \sigma_\theta(1 - \int_\alpha^1 \theta(z) dz)$. In view of this and (4.4) we conclude that $s(\alpha) = \int_0^\alpha s'(x) dx$ satisfies the second claim. Now also from (4.5) we know that $\bar{c}(\tau) = y$, for $\tau > s(1)$, and we noted earlier in the text that $c(\alpha, t) = \chi_\theta(\alpha, \bar{c}(t))$, $\forall \alpha \in [0, 1]$, $\forall t \in [0, T]$. Taken together, these prove the final claim. ||

Proof of Proposition 3. When $v(c) = ln(c)$, simple but tedious calculation reveals that welfare under the random rosca is:

$$W_r = T[\ln(y) + \xi] - \frac{B}{y}[1 + \chi(\xi/2)]$$

where the function $\chi(\cdot)$ is implicitly defined by $\chi(\xi)-\ln(1+\chi(\xi))\equiv\xi$, $\xi\geqq 0$. Similarly, for weights $\theta\in\Theta$ we have

$$W_\theta=T\left[\ln(y)+\xi+\int_0^1\theta(a)\ln(\theta(a))da\right]-\frac{B}{y}\int_0^1\left[1+\chi\left(\xi\int_x^1\theta(z)dz\right)\right]dx.$$

We know from the proof of Proposition 2 that for the market that $W_m=\min_{\theta\in\Theta}W_\theta$. Hence, $W_r>W_m$ if and only if $\exists\theta$ such that $W_r>W_\theta$. The proof constructs some weights for which this is so. First we need:

Lemma 3. *Let $f(\cdot)$ be an increasing, strictly concave function satisfying $f(0)=0$, and let $g(\cdot)$ be a function on* [0, 1], *strictly decreasing satisfying $g(1)=0$ and $g(0)=1$. Then*

$$\int_0^1 f(g(x))dx>f(1)\int_0^1 g(x)dx.$$

Proof. Let $\tilde{x}$ be a random variable which is uniformly distributed on [0, 1]. Define $\tilde{y}=g(\tilde{x})$, and $\tilde{z}=1$, if $\tilde{x}\leqq\int_0^1 g(x)dx$, and $\tilde{z}=0$, *if* $\tilde{x}>\int_0^1 g(x)dx$. Then

$$E(\tilde{y})=\int_0^1 g(x)dx=E(\tilde{z}),$$

where $E(\cdot)$ denotes the expectations operator. Moreover, $\tilde{z}$ is riskier than $\tilde{y}$ in the sense of second-order stochastic dominance. Therefore, since $f(\cdot)$ is strictly concave:

$$E(f(\tilde{y}))=\int_0^1 f(g(x))dx>E(f(\tilde{z}))=f(1)\int_0^1 g(x)dx.$$

This proves the lemma. ||

This lemma implies that

$$\int_0^1\chi\left(\xi\int_x^1\theta(z)dz\right)dx>\chi(\xi)\int_0^1\left(\int_x^1\theta(z)dz\right)dx.$$

This in turn implies that

$$W_\theta<T\left[\ln(y)+\xi+\int_0^1\theta(x)\ln(\theta(x))dx\right]-\frac{B}{y}\left[1+\left(\int_0^1 x\theta(x)dx\right)\cdot\chi(\xi)\right],$$

where we have also used the fact that $\int_0^1\int_x^1\theta(z)dzdx=\int_0^1 x\theta(x)dx$. Hence, a sufficient condition for $W_r>W_m$, is that:

$$\exists\theta\in\Theta\text{ such that }-\frac{B}{Ty}\chi(\xi/2)>\int_0^1\theta(x)\ln(\theta(x))dx-\frac{B}{Ty}\chi(\xi)\int_0^1 x\theta(x)dx.$$

Now we define $E(\bar{\theta})\equiv[\min_{\theta\in\Theta}\int_0^1\theta(x)\ln(\theta(x))dx$ s.t. $\int_0^1 x\theta(x)dx=\bar{\theta}]$. Then $W_r>W_m$ if:

$$\exists\bar{\theta}\in(0,1)\text{ such that }\left(\frac{B}{Ty}\right)\cdot[\bar{\theta}\chi(\xi)-\chi(\xi/2)]>E(\bar{\theta}).$$

Let $B/Ty\equiv\gamma\in(0,1)$, and consider the problem: $\max_{\bar{\theta}\in[0,1]}\{\gamma\chi(\xi)\bar{\theta}-E(\bar{\theta})\}\equiv\Omega^*$. We conclude that it is sufficient for $W_r>W_m$ that $\Omega^*>\gamma\chi(\xi/2)$.

Lemma 4. *(i) $E(\bar{\theta})$ is strictly convex, and $E(\bar{\theta})\geqq E(\frac{1}{2})$, $\forall\bar{\theta}\in[0,1]$; and, (ii) if $E'(\bar{\theta})=\lambda$, then $E(\bar{\theta})=\lambda\bar{\theta}+\ln(\lambda(e^\lambda-1)^{-1})$.*

Proof. Define the Lagrangean

$$L=\int_0^1\theta(x)\ln(\theta(x))dx+\lambda\left[\bar{\theta}-\int_0^1 x\theta(x)dx\right]+\mu\left[1-\int_0^1\theta(x)dx\right].$$

The first-order condition with respect to $\theta(x)$ is: $\ln(\theta(x))+1-\lambda x-\mu=0$, $x\in[0, 1]$. Inverting and integrating this condition, using the constraint, yields: $e^{\mu-1}[e^{\lambda}-1/\lambda]=1$. Solving this for μ, substituting into the first-order condition, multiplying by $\theta(x)$ and integrating yields (ii). To prove (i) observe that integrating the first-order condition, after inverting and multiplying through by x, and using the above derived expression for μ yields:

$$\int_0^1 x\theta(x)dx \equiv \bar{\theta} = e^{\mu-1}\int_0^1 xe^{\lambda x}dx = \left[\frac{\lambda e^{\lambda}-(e^{\lambda}-1)}{\lambda^2}\right] \Big/ \left[\frac{e^{\lambda}-1}{\lambda}\right] = e^{\lambda}/(e^{\lambda}-1)-\lambda^{-1} \equiv \phi(\lambda).$$

It is straightforward now to see that $\phi(\lambda)\to 0$ as $\lambda\to-\infty$; $\phi(\lambda)\to 1$ as $\lambda\to+\infty$ and $\phi(\lambda)\to 1/2$ as $\lambda\to 0$. Part (i) is now proved by noting that $E'(\bar{\theta})=\phi^{-1}(\bar{\theta})$, from the envelope condition. ||

This result and simple calculation reveals: $\Omega^*>\gamma\chi(\xi/2)$ if and only $(e^{\gamma\chi(\xi)}-1)/\gamma\chi(\xi)>e^{\gamma\chi(\xi/2)}$. Note also that, from the definition of $\chi(\cdot)$, that $\chi'(\xi)=(1+\chi(\xi))/\chi(\xi))>1$. Thus $\chi(\xi)>\chi(\xi/2)+\xi/2$. Moreover, $(e^z-1)/z=\int_0^1 e^{zx}dx$ is a strictly increasing function.

So:

$$\frac{e^{\gamma\chi(\xi)}-1}{\gamma\chi(\xi)} > \frac{e^{\gamma\chi(\xi/2)}\,e^{\gamma\xi/2}-1}{\gamma\chi(\xi/2)+\gamma\xi/2} = e^{\gamma\chi(\xi/2)}\left[\frac{e^{\gamma\xi/2}}{\gamma\chi(\xi/2)+\gamma\xi/2}\right]-[\gamma(\chi(\xi/2)+\xi/2)]^{-1}.$$

The first of the two terms on the right-hand side grows unboundedly as $\xi\to\infty$. Moreover, for a sufficiently large ξ, the second term vanishes. Hence, for large enough ξ, $[e^{\gamma\chi(\xi)}-1]/\gamma\chi(\xi)>e^{\gamma\chi(\xi/2)}$ and $W_r>W_m$. (Note that $\bar{\xi}$, the critical value of ξ, depends only on $\gamma\equiv B/Ty$.) ||

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