Online Appendix to Adjustment Costs, Firm Responses, and Labor Supply Elasticities: Evidence from Tax Records

Raj Chetty, Harvard University and NBER
John N. Friedman, Harvard University and NBER
Tore Olsen, Harvard University and CAM
Luigi Pistaferri, Stanford University and NBER

September 22, 2010

Appendix A: Theoretical Derivations

Predictions 1-3 for Tax Reforms. We introduce a second period in the model to analyze the effects of tax reforms. At the beginning of the second period, the government announces an unexpected tax reform that raises the linear tax rate for workers of type $s_i = L$ from $\tau$ to $\bar{\tau}$. Let $\Delta \tau = \bar{\tau} - \tau$ and $\Delta \log(1-\tau) = \log(1-\bar{\tau}) - \log(1-\tau)$. In the interest of space, we consider only the union bargaining equilibrium here; see Chetty et al. (2009) for an analogous analysis for the market equilibrium case.

We model the search process in period 1 exactly as above. Because the tax reform is unanticipated, worker and union behavior in period 1 is the same as in the static model. In period 2, the union can change the hours they bargain from firms at no cost. The equilibrium wage rate is $w(h) = w = p$ in period 2 as in period 1. Workers associated with a firm that changes its hours requirement are forced to work that new level of hours unless they switch jobs. After seeing the full distribution of new hours in period 2, workers can pay a search cost $\phi_i$ to switch to their optimal job.

In the second period, as in the first, the union sets the aggregate distribution such all workers are employed. A full characterization of dynamics requires assumptions about the specific firms that move in order to shift the old equilibrium distribution of jobs to the new equilibrium distribution. The results we derive below rely only on aggregate dynamics and thus do not require such assumptions.

Let $h_{it}^*$ denote worker $i$’s optimal labor supply choice in period $t$ and $h_{it}$ her actual choice in equilibrium. We characterize the observed elasticity from the tax reform $\tilde{\varepsilon}_{TR} = \frac{\mathbb{E} \log h_{i2} - \mathbb{E} \log h_{i1}}{\Delta \log(1-\tau)}$ in each of the special cases analyzed in Section 2 in turn.

Special Case 1. In the frictionless benchmark model, $\phi_i = 0$ for all workers, in which case workers set $h_{it}^*(\tau) = \alpha_i (w(1-\tau))^\varepsilon$ in both periods. It follows immediately that the observed elasticity from a tax reform $\tilde{\varepsilon}_{TR} = \frac{\mathbb{E} \log h_{i2} - \mathbb{E} \log h_{i1}}{\Delta \log(1-\tau)} = \varepsilon$.

Special Case 2. In the second special case, $\phi_i = \phi$ is constant and a measure zero set of workers faces the linear tax schedule ($\zeta = 1$), so the equilibrium distribution of hours $G(h)$ is unchanged across the two periods. In the second period, a worker’s first-period job $h_{i1}$ functions as an initial offer, just as the initial draw $h_i^0$ did in the first period. A worker pays to switch to his optimal job $h_{i2}^*$ iff $h_{i1} \notin [h_{i2}, \bar{h}_{i2}]$, where the thresholds are defined as the text. When $\Delta \tau = 0$, the new bounds coincide with the old: $\bar{h}_{i2} = \bar{h}_{i1}$ and $h_{i2} = h_{i1}$. As the size of the tax reform grows, more workers have $h_{i1} \notin [h_{i2}, \bar{h}_{i2}]$ because $\frac{\partial h_{i2}}{\Delta \tau} < 0$ and $\frac{\partial \bar{h}_{i2}}{\Delta \tau} < 0$. Therefore the fraction of workers paying to
search increases. Average labor supply for those with $s_i = L$ in the second period can be written as

$$h_2 = \int \left[ q_{i2} h_{i2}^* + \int_{h_{i1}}^{\bar{h}_{i2}} hG(h) \right] dF(a_i)$$

(1)

where $q_{i2} = 1 - G(\bar{h}_{i2}) + G(h_{i1})$ is the fraction of workers that switch jobs after the reform.

As the size of the tax reform grows large, the observed elasticity converges to $\varepsilon$: $\lim_{\Delta \tau \to \infty} \hat{\varepsilon}_{TR} = \varepsilon$. Intuitively, for a sufficiently large tax reform, $\bar{h}_{i2} \leq h_{i1}$, in which case all workers pay to search ($q_{i2} = 1$) and set $h_{i2} = h_{i1}^*$. Although workers do not all have $h_{i1} = h_{i1}^*$, the change in average hours grows large relative to $h_{i1} - h_{i1}^*$ as $\Delta \tau \to \infty$, and thus $\hat{\varepsilon}_{TR} \to \varepsilon$. While $\hat{\varepsilon}_{TR}$ always converges to $\varepsilon$, the derivative $\frac{\partial \hat{\varepsilon}_{TR}}{\partial \Delta \tau}$ can only be signed by making assumptions about the job offer distribution $G(h)$. Suppose that the distribution of preferences are such that the equilibrium distribution of jobs $G(h)$ is uniform for those with $s_i = NL$, who do not face the tax reform. Under this assumption, the fraction of workers who reoptimize following the tax change $q_{i2}$ increases monotonically from 0 to 1 as the size of the reform increases and hence $\frac{\partial \hat{\varepsilon}_{TR}}{\partial \Delta \tau} > 0$.

Combining these results yields a prediction for tax reforms analogous to Prediction 1.

**Prediction A1:** When workers face search costs,

(a) the observed elasticity from tax reforms converges to $\varepsilon$ as the size of the tax change grows:

$$\lim_{\Delta \log(1 - \tau) \to \infty} \hat{\varepsilon}_{TR} = \varepsilon$$

(b) If the offer distribution $G(h)$ is uniform, $\hat{\varepsilon}$ rises with $\Delta \tau$:

$$\partial \hat{\varepsilon}_{TR}/\partial \ln (1 - \tau) > 0$$

**Special Case 3.** In the third special case, $\phi_i = 0$ for a fraction $\delta$ of workers and $\phi_i = \infty$ for the rest, and $\zeta \in (0, 1)$. In both periods, the equilibrium distribution of hours $G(h(\tau)) = G^*(h(\tau))$, the optimal distribution of hours, following the same logic as in the text. Let $\Delta \log h_L^* = \varepsilon \cdot (\log(1 - \tau) - \log(1 - \tau))$ denote the optimal change in hours for those facing the linear tax. The actual change in hours for this group is $\Delta \log h_L = (\delta + (1 - \delta) (1 - \zeta)) \Delta \log h_L^*$. The first term in this expression is the individual response (the analog of individual bunching), $\Delta \log h_L = \delta \Delta \log h_L^*$. The second term is the firm response (the analog of aggregate bunching), $\Delta \log h_F = (1 - \delta) (1 - \zeta) \Delta \log h_L^*$. The change in hours for those with $s_i = NL$ is $\Delta \log h_{NL} = (1 - \delta) (1 - \zeta) \Delta \log h_L^* = \Delta \log h_F$, providing an empirical measure of the firm response. Recognizing that the observed elasticity is

$$\hat{\varepsilon}_{TR} = \frac{\Delta \log h_L^*}{\Delta \log (1 - \tau)} = (\delta + (1 - \delta) (1 - \zeta)) \varepsilon$$

the analogs of predictions 2 and 3 follow immediately.

**Prediction A2:** Search costs interact with hours constraints to generate firm responses to tax reforms. The size of the firm response and observed elasticity rises with the fraction of workers who face a tax reform:

$$\Delta \log h_F = \Delta \log h_{NL} > 0 \text{ iff } \zeta < 1$$

$$\frac{\partial \Delta \log h_F}{\partial (1 - \zeta)} > 0, \quad \frac{\partial \hat{\varepsilon}_{TR}}{\partial (1 - \zeta)} > 0.$$  

**Prediction A3:** Firm and individual responses to a tax reform are positively correlated across occupations:

$$\text{cov}(\Delta \log h^q_I, \Delta \log h^q_F) > 0$$
Non-Constant Structural Elasticities. Suppose agents have quasi-linear utilities of the form \( u_i(c,h) = c - \frac{1}{\alpha_i} \psi(h) \). This utility permits the structural elasticity of labor supply \( \varepsilon = \frac{\partial \ln h^*}{\sigma \ln (1-\tau_1) w} \) to vary arbitrarily with the net-of-tax rate depending upon \( \psi''(h) \). In the frictionless model, workers who face an increase in their marginal tax rates from \( \tau_1 \) to \( \tau_2 \) at an earnings level of \( K \) bunch at the kink iff \( \alpha_i \in [\underline{\alpha}(\tau_1), \overline{\alpha}(\tau_2)] \), where \( \underline{\alpha}(\tau_1) = \psi'(h_K)/(1 - \tau_1) w \) and \( \overline{\alpha}(\tau_2) = \psi'(h_K)/(1 - \tau_2) w \). The amount of bunching at the kink is therefore \( B_{NL}^*(\tau_1, \tau_2) = \int_{\underline{\alpha}(\tau_1)}^{\overline{\alpha}(\tau_2)} F(\alpha_i) \) d\( \alpha_i \). It follows that for any tax rates \( \tau_1 < \tau_2 < \tau_3 \), the amount of bunching created from two smaller kinks is exactly equal to the bunching created at one larger kink:

\[
B_{NL}^*(\tau_1, \tau_3) = B_{NL}^*(\tau_1, \tau_2) + B_{NL}^*(\tau_2, \tau_3)
\]

Now consider special case 2 of the model with frictions, where agents pay a fixed cost \( \phi \) to search. Here, the amount of bunching is

\[
B_{NL}(\tau_1, \tau_2) = \theta (\tau_2 - \tau_1) \int_{\underline{\alpha}(\tau_1)}^{\overline{\alpha}(\tau_2)} F(\alpha_i) \]

where the fraction of workers who pay the search cost to locate at the kink (\( \theta \)) increases with the change in tax rates at the kink (\( \tau_2 - \tau_1 \)). Therefore the model with frictions instead implies that bunching at one large kink is greater than the sum of bunching at two smaller kinks:

\[
B_{NL}(\tau_1, \tau_3) > B_{NL}(\tau_1, \tau_2) + B_{NL}(\tau_2, \tau_3)
\]

Appendix B: Data

We merge selected variables from the following registers available at the Center for Applied Microeconometrics at University of Copenhagen through Statistics Denmark: a) the Income Statistics Register, which covers everyone who is tax liable in Denmark, b) the Population Register, which covers the entire population on December 31st of a given year and provides basic demographic information such as age and gender, and c) the Integrated Database for Labour Market Research (IDA), which contains information on labor market experience, occupation, employment status, education, family status, etc. For every gender-age cell of the individuals between the ages of 16 and 70, we have tax records for between 99.96 and 100% of the population. We do not have tax records for people over 70 years of age, and 83% of 15 year olds have records in the tax register.

Statistics Denmark’s Employment Classification Module combines several administrative records to assign every observation in the IDA database one of eight employment codes, contained in the variable \( \text{beskst} \) (employment status). The employment status code distinguishes individuals who are wage earners, wage earners with unemployment income, wage earners with self employment income, and five categories of non-wage earners (self-employed, pensioners, etc.). To form our primary analysis dataset, we keep only individuals with \( \text{beskst}=4 \), thereby excluding all non wage earners, wage earners with self employment income, and wage earners with unemployment. Broadening this definition to include all non-self employment categories (\( \text{beskst}=4,5,7, \) or 8) does not affect the results; for instance, we find excess mass at the top kink of \( b = 0.83 \) in the broader sample compared with \( b = 0.81 \) for the narrower sample used in Figure 3.

To calculate marginal tax rates and income relative to the tax bracket cutoffs, we develop a tax simulator for Denmark analogous to the NBER TAXSIM. Denmark has essentially an individual tax system, but there are some joint aspects, so the tax simulator uses as inputs both income related to the social security number associated with a given tax record (\( \text{pnr} \)) as well as that of the
spouse for tax purposes (henv). The municipality of residence in the previous year (glskkmnr) is used to determine what tax rates the individual faces. For the tax payer and his or her spouse, the variables used in the tax simulator are primarily the personal exemption (pfrdst, berfrdst), personal income (perindkp, berpi), capital income (kapindkp, berkap), special deductions (lignfrdp). We also make use of some other more disaggregated variables in the tax records to account for transitional schemes and special adjustments to the tax bases. These include deductions in personal income for individual contributions to pension schemes (kappens, fosfjfrd), employer contributions to capital pension schemes (arbpen14, arbpen15), and alimony paid (underhol). We calculate marginal tax rates holding each individual’s personal deduction fixed at the statutory level.

We define the marginal tax rate as the change in tax liability for an individual from an additional 1 DKr of wage earnings. For married individuals, we take the jointness of the tax system into account but compute individual marginal tax rates. We do so by holding the spouse’s income and the deduction transfer (bundfradrag) relevant for the middle tax at their observed values. We then compute the change in tax liability for the individual from earning one more DKr. For example, in a couple where one spouse is earning DKr 50,000 above the individual middle tax cutoff and the other is earning DKr 50,000 below the cutoff, we code the higher earner as bunching at the middle tax kink because his tax liability increases by the middle tax rate when he earns one more DKr. However, the lower earner is not coded as bunching at the middle tax cutoff because if she earns one more DKr, only her husband’s tax liability is affected. Note that under this method, the marginal tax rate for the higher earner in a couple always coincides with the household marginal tax rate because an increase in the higher earner’s income only affects his own tax liability and not his spouse’s.

We assess the accuracy of the tax calculator using data from the tax register on the exact amount of municipal, regional, bottom, middle and top tax paid by each individual. Our tax calculator is correct to within +/- 5DKr ($1) of the actual amount paid for all of these taxes for 95% of the observations in the data. It is accurate to within +/-1,000DKr ($167) for 98% of the observations. The discrepancies arise from our inability to fully model complex capital income transfer rules that apply to some spouses as well as unusual circumstances such as individuals who die during the year or those working both in Denmark and abroad who are subject to special tax treaties. Since we do not have tax records for people aged less than 15 or more than 70, we also cannot fully account for the joint aspects of the tax system for people with spouses aged less than 15 or over 70.

In addition to the variables described above used to compute taxable income and pension contributions, we also use the following source variables in our empirical analysis: wage earnings (qlontmp2), self-employment profits and retained earnings (govskvir, virkordind), labor market experience (erhver, erhver79), and occupational code (discok). We define an individual’s net deductions in the top tax base as the level of wage earnings he/she would need to start paying the top tax minus the statutory top tax cutoff (i.e. the level of total personal income at which individuals must start paying the top tax).

The STATA code and tax simulator are available from the authors and have been posted on the servers at the Center for Applied Microeconometrics.
## TABLE A.1
ISCO Occupation Codes and Employment Levels

<table>
<thead>
<tr>
<th>ISCO Code</th>
<th>Occupation Description</th>
<th>Avg No. of Workers 1995-2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Military</td>
<td>24,451</td>
</tr>
<tr>
<td>11</td>
<td>Legislators and senior officials</td>
<td>4,329</td>
</tr>
<tr>
<td>12</td>
<td>Corporate managers</td>
<td>53,802</td>
</tr>
<tr>
<td>13</td>
<td>General managers</td>
<td>3,029</td>
</tr>
<tr>
<td>21</td>
<td>Physical, mathematical and engineering science professionals</td>
<td>41,704</td>
</tr>
<tr>
<td>22</td>
<td>Life science and health professionals</td>
<td>30,043</td>
</tr>
<tr>
<td>23</td>
<td>Teaching professionals</td>
<td>105,257</td>
</tr>
<tr>
<td>24</td>
<td>Other professionals</td>
<td>71,594</td>
</tr>
<tr>
<td>31</td>
<td>Physical and engineering science associate professionals</td>
<td>68,622</td>
</tr>
<tr>
<td>32</td>
<td>Life science and health associate professionals</td>
<td>73,777</td>
</tr>
<tr>
<td>33</td>
<td>Teaching associate professionals</td>
<td>75,422</td>
</tr>
<tr>
<td>34</td>
<td>Other associate professionals</td>
<td>147,759</td>
</tr>
<tr>
<td>41</td>
<td>Office clerks</td>
<td>231,329</td>
</tr>
<tr>
<td>42</td>
<td>Customer service clerks</td>
<td>32,575</td>
</tr>
<tr>
<td>51</td>
<td>Personal and protective service workers</td>
<td>226,129</td>
</tr>
<tr>
<td>52</td>
<td>Models, sales persons, and demonstrators</td>
<td>73,818</td>
</tr>
<tr>
<td>61</td>
<td>Skilled agricultural and fishery workers</td>
<td>13,156</td>
</tr>
<tr>
<td>71</td>
<td>Exaction and related trades workers</td>
<td>95,270</td>
</tr>
<tr>
<td>72</td>
<td>Metal, machinery and related trades workers</td>
<td>110,705</td>
</tr>
<tr>
<td>73</td>
<td>Precision, handicraft, printing and related trades workers</td>
<td>11,475</td>
</tr>
<tr>
<td>74</td>
<td>Other craft and related trades workers</td>
<td>19,718</td>
</tr>
<tr>
<td>81</td>
<td>Stationary plant and related operators</td>
<td>10,905</td>
</tr>
<tr>
<td>82</td>
<td>Machine operators and assemblers</td>
<td>106,391</td>
</tr>
<tr>
<td>83</td>
<td>Drivers and mobile plant operators</td>
<td>35,991</td>
</tr>
<tr>
<td>91</td>
<td>Sales and services elementary occupations</td>
<td>99,307</td>
</tr>
<tr>
<td>92</td>
<td>Agricultural, fishery and related labourers</td>
<td>9,207</td>
</tr>
<tr>
<td>93</td>
<td>Mining, construction, manufacturing, and transport</td>
<td>72,843</td>
</tr>
</tbody>
</table>

Notes: This table lists the two digit International Labour Organization's ISCO codes that are used in Figure XII along with employment levels (mean number of wage earners in each ISCO between 1995 and 2001).
FIGURE A.1
Distinguishing Changes in Tax Incentives from Inflation and Wage Growth

Notes: This figure replicates the income distribution in Figure IVd, zooming in around the top tax bracket cutoff. The location of the bracket cutoff in 1997 is marked with the solid line. The dashed green line shows the level of the 1994 top bracket cutoff adjusted for inflation. The dashed blue line shows the 1994 bracket adjusted for average wage growth.
Notes: This figure plots the empirical distribution of wage earnings and broad income around the statutory top tax cutoff (which applies to individuals with 0 net deductions) for the population of wage earners from 1995-2001, the years in which pension contribution data are available. Broad income is defined as taxable income plus contributions to tax-deductible pension accounts. The figure also shows the counterfactual distributions and excess masses, computed as in Figure IIIa.
FIGURE A.3
Bunching for Individuals who Switch Between Top and Middle Kinks

Notes: This figure restricts attention to wage earners who earned within DKr 50,000 of the top tax bracket cutoff in a given year $t$ and within DKr 50,000 of the middle tax bracket cutoff in year $t + 2$. For this fixed group of individuals, we plot the empirical distribution of taxable income in year $t$ around the top bracket cutoff and the distribution of taxable income around the middle tax cutoff in year $t + 2$. The figure also shows the counterfactual distributions and excess masses, computed as in Figure IIIa.
FIGURE A.4
Bunching at Top vs. Middle Kink for Highest Earners in Households

Notes: This figure plots taxable income distributions around the middle and top tax cutoffs. It replicates Figures IIIa and VIa, restricting the sample to only the highest earner within a household (and including all single individuals). The figure also shows the counterfactual distributions and excess masses, computed as in Figure IIIa. See notes to Figures IIIa and VIa for additional details.
FIGURE A.5
Survey Evidence: Knowledge about Middle and Top Tax Cutoffs

Notes: This figure plots the distribution of perceived middle and top tax cutoffs from an internet survey of 3,299 members of a union representing public and financial sector employees. Individuals were asked to report the income levels at which they would have to begin paying the middle and top taxes in the 2008 Danish tax code. The figure shows a histogram of the responses for the top tax (solid red line) and middle tax (dashed blue line) cutoffs using bins of DKr 30,000 in width. The bins are centered on the true cutoffs, so that the mode of each distribution represents the fraction of people whose perception of the tax bracket cutoff was within DKr 15,000 of the correct value.
FIGURE A.6
Distribution of Net Deductions

(a) Unconditional Distribution

(b) Conditional Distribution Given Deductions > DKr 20,000

Notes: Panel (a) plots a histogram of net deductions, defined as deductions minus non-wage income relevant for the top tax base. Panel (b) plots a histogram of net deductions between DKr 20,000 and DKr 50,000. To identify bunching in deductions at the pension kink, in Panel (b) we recenter deductions in each year so that the pension contribution limit in that year equals the average pension contribution limit across the years (DKr 33,000).
FIGURE A.7
Individual Bunching at Top vs. Middle Pension Kinks

Notes: This figure plots the distribution of wage earnings relative to the top and middle pension kinks (demarcated by the green vertical line), for wage earners with greater than DKK 20,000 of deductions. The pension kink is defined as the top or middle bracket cutoff plus the maximum tax-deductible pension contribution in each year. For the middle tax, our definition of “deductions” includes the transferable spouse exemption (bundfradrag) This figure also shows the counterfactual distributions and excess masses, computed as in Figure IIIa.
FIGURE A.8
Male vs. Female Wage Earners: Effects of Occupational Heterogeneity

(a) Female Wage Earners

Excess mass (b) = 1.37
Standard error = 0.08

(b) Male Wage Earners

DFL Reweighted Excess mass (b) = 0.85
Standard error = 0.09

Unweighted Excess mass (b) = 0.46
Standard error = 0.03

Notes: These figures plot the empirical distributions of taxable income around the top tax cutoff for (a) female wage earners and (b) male wage earners. The series in grey squares in Panel B shows the raw distribution of taxable income for men. The series in blue circles shows reweights the observations for men to match the occupational distribution of women (defined by 4 digit ISCO codes). Following DiNardo, Fortin, and Lemieux (1996), we reweight an observation in occupation $i$ by $\frac{p_f}{1-p_f}$, where $p_f$ is the probability that a wage earner in occupation $i$ is female. The figure also shows the counterfactual distributions and excess masses, computed as in Figure IIIa.