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Tacit collusion under interest rate fluctuations

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Previous literature has shown that demand fluctuations affect the scope for tacit collusion. I study whether discount factor fluctuations can have similar effects. I find that collusion depends not only on the level of the discount factor but also, and more surprisingly, on its volatility. Collusive prices and profits increase with a higher discount factor level, but decrease with its volatility. These results have important implications for empirical studies of collusive pricing, the role that collusive pricing may play in economic cycles and the study of cooperation in repeated games.

1. Introduction

■ It is well known that oligopolies can use the threat of future price wars to sustain prices above competitive levels if firms care enough about the future (Friedman, 1971). The extent to which firms care about the future depends primarily on the interest rate if the firms' objective is to maximize the present value of profits. The firms' discount factor may also depend on other forces, such as the probability that the product may become obsolete and the time needed for cheating to be detected. Given that the interest rate and other variables that affect the discount factor are constantly changing, it is important to study tacit collusion under discount factor fluctuations.

I characterize collusive prices and profits when the discount factor changes over time and show that collusive prices and profits increase with both present and future levels of the discount factor but decrease with its volatility. These results have important implications not only for the study of collusion but also for repeated game theory in general.

Oligopoly games are one example among many of an environment in which it is natural to assume that the discount factor changes over time. Another example would be that of a partnership, where the probability that the partnership might end varies over time. Thus, the volatility of the discount factor may be an important determinant of cooperation for many kinds of repeated games, not just oligopoly.

The environments I study and the specific results I find are as follows. I consider the case in which the discount factor, identical for all firms, is randomly and independently drawn every period. I characterize the maximum symmetric tacit collusion prices and profits that can be

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supported in an environment in which firms are identical and compete repeatedly on price. The three main results derived from this characterization are as follows. [1]

First and more interestingly, I show that the higher the volatility of the discount factor, the lower the collusive prices and profits that can be supported in equilibrium. The reason for this is twofold. First, given that the combination of the incentive compatibility and feasibility constraints results in a concave collusive profit function (as a function of the discount factor), an increase in volatility leads to a decrease in expected profits. Second, this decrease in expected profits reduces the size of future punishment and hence results in a decrease in equilibrium profits and prices.

Second, the higher the discount factor in a given period, the higher the collusive prices and profits that can be supported in equilibrium in that period. The intuition behind this is straightforward: the higher the discount factor, the stronger the threat of future price wars and the higher prices and profits can be without firms deviating.

Third, a shift in the distribution function toward higher discount factors would result in an increase in the profits and prices that can be supported in equilibrium for each discount factor. Again, the intuition is straightforward. From the previous result, we know that the higher the realization of the discount factor, the higher collusive prices and profits will be. Hence, a shift in the distribution function toward higher discount factors would result in an increase in the expected value of collusive profits and an increase in the threat of future punishment, allowing higher equilibrium prices and profits.

The rest of the article is organized as follows. In Section 2, I relate this article to the previous literature. In Section 3, I study the optimal tacit collusion solution and provide comparative statics results. In Section 4, I conclude.

2. Related literature

■ To my knowledge, thereis only one article that considers a fluctuating discount factor. This is Baye and Jansen (1996), which provides folk theorem results for repeated games with stochastic discount factors.

The well-known article by Rotemberg and Saloner (1986) offers interesting results with respect to tacit collusion that follow, as do the results in this article, from changes in the relative importance of present and future profits. In their article, however, those changes are driven by changes in demand, not the discount factor. This difference is not trivial and leads to significantly different results.

First, in this article, an increase in the discount factor always has a nonnegative effect on the equilibrium price, while in Rotemberg and Saloner, an increase in demand may result in either a decrease or an increase in price (depending on whether the incentive compatibility restriction is binding or not). In addition, the effect of an increase of demand on prices may not be robust to assuming quantity competition instead of price competition, as Rotemberg and Saloner note, or to the existence of capacity constraints, as Staiger and Wolak (1992) note. The effect of an increase in the discount factor is robust. Second, while in this article an increase in the volatility of the discount factor always results in a decrease in profits and prices, in Rotemberg and Saloner's model, an increase in the volatility of demand is again ambiguous. Third, under discount factor fluctuations, the issue of serial positive correlation of the shocks is less important than under demand fluctuations. The fact that high demand today makes it difficult to support collusion, while high demand in the future makes it easy has led a number of authors to study the consequences of demand correlation on collusion (see Kandori, 1991; Haltiwanger and Harrington, 1991; Bagwell and Staiger, 1997). In contrast, both high discount factors today and in the future facilitate collusion. Therefore, positive correlation per se does not affect the positive effect of an increase of the discount factor on collusive prices.

Finally, the literature on customer markets also relates oligopoly prices with the discount factor (see Phelps and Winter, 1970; Gottfries, 1991; Klemperer, 1995; Chevalier and Scharfstein, 1996). In those models, an increase in the discount factor increases the incentives to invest in new customers and results in lower prices, contrary to the results of this article.

[1] The same results hold under quantity competition, see Dal Bó (2002).

3. Model and results

■ The model is as follows. Consider a market with N identical firms with a constant marginal cost of c and facing a demand function $D(p)$ for $p \in \Re$. Assume that $D(p)$ is bounded, continuous and decreasing in p and that there exists a price $\overline{p} > c$ such that $D(p) = 0$ for any $p > \overline{p}$. Firms compete repeatedly on price and, in each period, demand is divided equally among those firms charging the lowest price. Firms care only about profits and are risk neutral and, hence, their objective is to maximize the discounted stream of profits. The distinctive feature of this model is that the discount factor, δ_t, which discounts earnings from $t+1$ to t, is a continuous, independent and identically distributed random variable between a and b, with p.d.f. $f(\delta_t)$ and c.d.f. $F(\delta_t)$.

The timing of the game in a given period t is as follows: the firms observe the realization of the discount factor, δ_t, *then* they choose the price for that period and finally they observe all the chosen prices, quantities and payoffs. All characteristics of the environment are common knowledge.

Given that firms cannot commit to charge a given price, or sign contracts among themselves or with third parties regarding prices, any equilibrium of the model must be a subgame perfect equilibrium of the infinitely repeated oligopoly game. I restrict my attention to equilibria in which all the firms charge the same price, p. In this symmetric case, I can write the profits of each firm as $\pi(p) = (p - c)D(p)/N$. Given the assumptions regarding the demand function, it is straightforward to show that $\pi(p)$ is continuous and attains a maximum. Denote as π^m the maximum (monopoly or perfect collusion) profit per firm. Assume that there is a unique price, p^m, that results in profits π^m (a sufficient condition is for the demand function not to be too convex relative to its slope: $d^2D(p)/dp^2 < -[2/(p-c)](dD(p)/dp)$ for every p).

□ **Optimal tacit collusion with a random discount factor.** In this section, I characterize the optimal symmetric tacit collusion solution. First, I characterize the profits that can be supported by subgame perfect equilibria. Second, I define and prove existence and uniqueness of the optimal tacit collusion solution. Finally, I characterize this solution.

I start by calculating the expected value of profit per firm if firms agree to set prices to achieve profits of $\pi(\delta)$ when the discount factor is δ. Using the recursiveness of the problem, the present value at t of a stream of profits to a firm can be written as

$$V(\delta_t) = \pi(\delta_t) + \delta_t \int_a^b V(\delta_{t+1}) f(\delta_{t+1}) d\delta_{t+1}, \tag{1}$$

where $\pi(\delta_t)$ denotes the profits that the firms receive at time t if the discount factor is δ_t. Integrating both sides of equation (1) and rearranging, we have

$$\int_a^b V(\delta_t) f(\delta_t) d\delta_t = \frac{1}{1-\overline{\delta}} \int_a^b \pi(\delta_t) f(\delta_t) d\delta_t,$$

where $\overline{\delta}$ is the expected value of δ_t. Plugging this into (1), the present value of profits can be written as

$$V(\delta_t) = \pi(\delta_t) + \frac{\delta_t}{1-\overline{\delta}} \int_a^b \pi(\delta_{t+1}) f(\delta_{t+1}) d\delta_{t+1}. \tag{2}$$

The following lemma characterizes the profit functions that can be supported by subgame perfect equilibria.

Lemma 1. A profit function $\pi(\delta)$ can be supported by a subgame perfect equilibrium if and only if

$$\pi(\delta) \geq -\frac{\delta}{(1-\overline{\delta})} \int_a^b \pi(\delta') f(\delta') d\delta' \qquad \text{for all } \delta \tag{3}$$

and

$$\pi(\delta) \leq \min\left\{\frac{\delta}{(N-1)(1-\overline{\delta})}\int_a^b \pi(\delta')f(\delta')d\delta', \pi^m\right\} \qquad \text{for all } \delta. \tag{4}$$

Proof. Firms can secure a zero profit in every period by setting $p = c$; this implies that, for any δ, $V(\delta)$ is nonnegative, which proves (3).

By just undercutting the collusive price, a firm can increase its profits in the current period by $(N-1)\pi(\delta)$; in any future period, its profits will be equal to zero and therefore we must have

$$(N-1)\pi(\delta) \leq \frac{\delta}{(1-\overline{\delta})}\int_a^b \pi(\delta')f(\delta')d\delta' \text{ for all } \delta. \tag{5}$$

In addition, the profits per firm cannot be larger than the profits under monopoly. Together with (5), this proves (4).

We have proven the "only if" part of the lemma. To prove the "if" part, assume that a function $\pi(\delta)$ satisfies (3) and (4) and let $\phi(\pi)$ be the smallest price that yields a profit equal to π, so that any price $p < \phi(\pi)$ yields profits smaller than π. Then, $\pi(\delta)$ can be supported by the following strategy: if there has been no deviation, choose $p(\delta) = \phi(\pi(\delta))$; otherwise, choose $p = c$. *Q.E.D.*

Now I turn my attention to optimal tacit collusion profit functions defined as follows.

Definition 1. An optimal symmetric tacit collusion profit function $\pi^*(\delta)$ maximizes the expected present value of profits $V(\delta)$ for every δ among the subgame perfect equilibrium profit functions.

Proposition 1. There exists a unique optimal tacit collusion profit function, $\pi^*(\delta)$. It satisfies

$$\pi^*(\delta) = \min\left\{\frac{\delta}{(N-1)(1-\overline{\delta})}\int_a^b \pi^*(\delta')f(\delta')d\delta', \pi^m\right\}. \tag{6}$$

Proof. For any δ, the set of profits that can be supported by a subgame perfect equilibrium is bounded above by π^m. It therefore has a least upper bound, which we call $\pi^*(\delta)$.

It is clear that π^* satisfies (3), as replacing any profit function that is supported by a subgame perfect equilibrium by π^* increases the left-hand side of (3) and decreases its right-hand side.

To show that π^* satisfies (4), note that, for any δ and any $\varepsilon > 0$, there exists a supported π such that

$$\begin{aligned}
\pi^*(\delta) &\leq \pi(\delta) + \varepsilon \\
&\leq \min\left\{\frac{\delta}{(N-1)\left(1-\overline{\delta}\right)}\int_a^b \pi(\delta')f(\delta')d\delta', \pi^m\right\} + \varepsilon && \text{(by (4))} \\
&\leq \min\left\{\frac{\delta}{(N-1)\left(1-\overline{\delta}\right)}\int_a^b \pi^*(\delta')f(\delta')d\delta', \pi^m\right\} + \varepsilon, && (\text{as } \pi(\delta') \leq \pi^*(\delta') \text{ for all } \delta').
\end{aligned}$$

We now must prove that π^* satisfies (6). Assume that this is not true and that, for some $\widetilde{\delta}$ and some $\eta > 0$, we have

$$\pi^*(\widetilde{\delta}) = \min\left\{\frac{\widetilde{\delta}}{(N-1)(1-\overline{\delta})}\int_a^b \pi^*(\delta')f(\delta')d\delta', \pi^m\right\} - \eta.$$

Then the function equal to $\pi^* + \eta$ for $\widetilde{\delta}$ and to π^* everywhere else satisfies (3) and (4) and hence can be supported by a subgame perfect equilibrium, which contradict the definition of π^*. *Q.E.D.*

Define the optimal tacit collusion price function $p^*(\delta)$ as the equilibrium prices that result in $\pi^*(\delta)$.

Proposition 2. The optimal symmetric tacit collusion price function, $p^*(\delta)$, exists and is unique.

Proof. By Lemma 1, $\pi^*(\delta) \in [0, \pi^m]$ for every δ. If $\pi^*(\delta) = \pi^m$, then $p^*(\delta) = p^m$. If $\pi^*(\delta) < \pi^m$, then the incentive-compatibility constraint (5) is binding and the price must be equal to $\phi(\pi^*(\delta))$ for firms not to deviate, where ϕ is defined as in the proof of Lemma 1. *Q.E.D.*

Note that it is straightforward to prove that the function ϕ is increasing and, therefore, equilibrium profits and prices are positively related.

The following proposition characterizes the optimal symmetric tacit collusion profit function.

Proposition 3. The optimal symmetric tacit collusion profit function $\pi^*(\delta)$ depends on $f(\delta)$ and N in the following way:

(i) if $\overline{\delta} \geq 1 - [a/(N-1)]$, $\pi^*(\delta) = \pi^m$;

(ii) if $(N-1)/N \leq \overline{\delta} < 1 - [a/(N-1)]$, $\pi^*(\delta) = \pi^m$ for $\delta \geq \widehat{\delta}$ and $\pi^*(\delta) = (\delta/\widehat{\delta})\pi^m$ for $\delta < \widehat{\delta}$, for a number $\widehat{\delta} \in (a, b]$ that solves the following equation:

$$\widehat{\delta} = (N-1)(1-\overline{\delta}) + \int_a^{\widehat{\delta}} F(\delta)d\delta; \tag{7}$$

(iii) if $\overline{\delta} < (N-1)/N$, $\pi^*(\delta) = 0$.

Proof. Taking expectations over both sides of equation (6) and denoting the expected value of profits as $\overline{\pi}$, equation (6) implies that $\overline{\pi} \leq [\overline{\delta}/(N-1)(1-\overline{\delta})]\overline{\pi}$. If $\overline{\pi} > 0$, this is possible only if $\overline{\delta} \geq (N-1)/N$. If $\overline{\delta} < (N-1)/N$, then $\pi^*(\delta) = 0$.

If $\overline{\delta} \geq 1 - [a/(N-1)]$, $\pi^m \leq [a/(N-1)(1-\overline{\delta})]\pi^m \leq [\delta/(N-1)(1-\overline{\delta})]\pi^m$ for all δ and, by equation (6), perfect collusion, $\pi^*(\delta) = \pi^m$, can be supported for every discount factor.

Finally, I show that, for $(N-1)/N \leq \overline{\delta} < 1 - a/(N-1)$, there is a unique solution to equation (6) with positive expected profits. In this solution, the two terms inside the brackets in equation (6) are binding for different ranges of δ. Given that the first term is increasing in δ, this term is binding for $\delta < \widehat{\delta}$, the second term is binding for $\delta > \widehat{\delta}$ and both terms are equal and binding for $\delta = \widehat{\delta}$, where $\widehat{\delta} \in [a, b]$. This implies that $\overline{\pi}$ satisfies

$$\overline{\pi} = \frac{\pi^m(N-1)(1-\overline{\delta})}{\widehat{\delta}}.$$

By (6), we also have

$$\overline{\pi} = \int_a^{\widehat{\delta}} \frac{\delta}{(N-1)(1-\overline{\delta})}\overline{\pi} f(\delta)d\delta + (1 - F(\widehat{\delta}))\pi^m.$$

From these two equations (and integrating by parts), we obtain equation (7).

The difference between the right-hand side and the left-hand side of equation (7) is a continuous, decreasing function of $\widehat{\delta}$, which is strictly positive for $\widehat{\delta} = a$ and negative for $\widehat{\delta} = a$, which proves the result. *Q.E.D.*

Two characteristics of the optimal tacit collusion solution can be easily derived from Proposition 3. First, collusive profits and prices are decreasing in the number of firms. In fact, if the number of firms is large enough, the optimal tacit collusion solution coincides with Bertrand's outcome. Second, the characterization of optimal tacit collusion under discount factor fluctuations includes the case of a fixed discount factor. For the fixed discount factor case, $a = b$, Proposition 3 coincides with the textbook solution: perfect collusion if $\delta \geq (N-1)/N$ and no collusion otherwise.

Finally, from Proposition 3, it is straightforward to show that an increase in the discount factor results in an increase in equilibrium profits and prices.

Corollary 1. Optimal collusive period profits $\pi^*(\delta)$ and prices $p^*(\delta)$ are increasing in δ.

□ **The effects of changes in $f(\delta)$.** In this section, I study how changes in the distribution of the discount factor affect collusive profits and prices.

I study, as a first step, how changes in the distribution function modify the range of perfect collusion under case 2 of Proposition 3. For a cumulative distribution function F, define $\overline{\delta}_F$ as the expected discount factor and $\widehat{\delta}_F$ as the limit to perfect collusion under case 2 of Proposition 3.

Lemma 2. Consider two cumulative distribution functions, F and G. If $(N-1)/N \leq \overline{\delta}_i < 1 - [a/(N-1)]$, $i = F, G$, and F second-order stochastically dominates[2] G, then $\widehat{\delta}_F \leq \widehat{\delta}_G$.

Proof. By second-order dominance, $\int_a^{\delta'} F(\delta)d\delta \leq \int_a^{\delta'} G(\delta)d\delta$ for all $\delta' \in [a, b]$ and, furthermore, $\overline{\delta}_F \geq \overline{\delta}_G$; hence, by (7),

$$\begin{aligned}\widehat{\delta}_F &= (N-1)(1-\overline{\delta}_F) + \int_a^{\widehat{\delta}_F} F(\delta)d\delta \\ &\leq (N-1)(1-\overline{\delta}_G) + \int_a^{\widehat{\delta}_F} G(\delta)d\delta \\ &= \widehat{\delta}_G + \int_{\widehat{\delta}_G}^{\widehat{\delta}_F} G(\delta)d\delta.\end{aligned}$$

If we had $\widehat{\delta}_F > \widehat{\delta}_G$, because $G(\delta) < 1$ for $\delta < b$, this would imply $\widehat{\delta}_F < \widehat{\delta}_G + (\widehat{\delta}_F - \widehat{\delta}_G) = \widehat{\delta}_F$, a contradiction. *Q.E.D.*

Denote $\pi_F^*(\delta)$, $\overline{\pi}_F^*$ and $p_F^*(\delta)$ as the optimal tacit collusion profit, its expected value and optimal collusion prices under F, respectively.

Proposition 4. Consider two cumulative distribution functions, F and G. If F second-order stochastically dominates G, then $\pi_F^*(\delta) \geq \pi_G^*(\delta)$ and $p_F^*(\delta) \geq p_G^*(\delta)$ for every δ. In addition, $\overline{\pi}_F^* \geq \overline{\pi}_G^*$.

Proof. By second-order stochastic dominance, $\overline{\delta}_F \geq \overline{\delta}_G$. It follows that, if the solution under F belongs to case 1 from Proposition 3, the solution under G can belong to any of the three cases; if the solution under F belongs to case 2, the solution under G can belong to case 2 or 3; and if F belongs to case 3, the solution under G also belongs to case 3.

If the solutions under both F and G belong to case 2, then $\widehat{\delta}_F \leq \widehat{\delta}_G$, by Lemma 2. Thus, $\pi_F^*(\delta) = (\delta/\widehat{\delta}_F)\pi^m \geq \pi_G^*(\delta) = (\delta/\widehat{\delta}_G)\pi^m$ if $\delta \leq \widehat{\delta}_F$; $\pi_F^*(\delta) = \pi^m > \pi_G^*(\delta) = (\delta/\widehat{\delta}_G)\pi^m$ if $\widehat{\delta}_F < \delta < \widehat{\delta}_G$; and $\pi_F^*(\delta) = \pi_G^*(\delta) = \pi^m$ if $\widehat{\delta}_G \leq \delta$. It follows that $\pi_F^*(\delta) \geq \pi_G^*(\delta)$. For the other combinations of solution cases, the result is straightforward. The result with respect to prices follows directly from the positive relationship between profits and prices.

Note that $\pi_F^*(\delta)$ is increasing and concave; hence, by second-order stochastic dominance and $\pi_F^*(\delta) \geq \pi_G^*(\delta)$ for every δ, we have that $\overline{\pi}_F^* \geq \int_a^b \pi_F^*(\delta)g(\delta)d\delta \geq \overline{\pi}_G^*$. *Q.E.D.*

The intuition of this result becomes clear if we consider two particular cases of second-order stochastic dominance: when F first-order stochastically dominates[3] G and when G is a mean-preserving spread of F.

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[2] For two cumulative distribution functions $F(\delta)$ and $G(\delta)$, F second-order stochastic dominates G if for any r, $a \leq r \leq b$, $\int_a^r F(\delta)d\delta \leq \int_a^r G(\delta)d\delta$, and the inequality is strict in some range. Then, $\overline{\delta}_F \geq \overline{\delta}_G$ and $\int_a^b u(\delta)f(\delta)d\delta \geq \int_a^b u(\delta)g(\delta)d\delta$, for any increasing concave twice-piecewise-differentiable function $u(\delta)$. See Hirshleifer and Riley (1992).

[3] For two cumulative distribution functions $F(\delta)$ and $G(\delta)$, F first-order stochastically dominates G if, for all r, $a \leq r \leq b$, $F(r) \leq G(r)$, and the inequality is strict in some range. Then F second-order stochastically dominates G

From Corollary 1, we know that, given a distribution of the discount factor, say G, equilibrium prices and profits are increasing in the realization of the discount factor. Then a shift in the distribution function to higher values (which yields a cumulative distribution function F that first-order stochastically dominates G), would result in an increase in expected profits. This, in turn, increases the threat of future punishments and increases equilibrium prices and profits.

Corollary 2. If F first-order stochastically dominates G, then $\pi_F^*(\delta) \geq \pi_G^*(\delta)$ and $p_F^*(\delta) \geq p_G^*(\delta)$ for every δ. In addition, $\overline{\pi}_F^* \geq \overline{\pi}_G^*$.

From Proposition 3, it follows that given a distribution function, say F, the optimal tacit collusion profit function is concave in the discount factor. Therefore, a mean-preserving spread (which yields G) would result in a reduction in expected profits. This, in turn, reduces the threat of future punishment and results in lower equilibrium prices and profits.

Corollary 3. If G is a mean-preserving spread of F, then $\pi_F^*(\delta) \geq \pi_G^*(\delta)$ and $p_F^*(\delta) \geq p_G^*(\delta)$ for every δ. In addition, $\overline{\pi}_F^* \geq \overline{\pi}_G^*$.

Therefore, the volatility of the discount factor is inversely related to the firms' profits. The combination of the incentive-compatibility constraint with the feasibility constraint yields a profit function that is concave in the discount factor even when firms are risk neutral. Hence, an increase in volatility reduces expected profits, reducing the threat of future punishment and lowering equilibrium prices and profits.

Note that this result does not depend on the firms not having access to insurance against discount-factor fluctuations. Even if they could buy actuarially fair insurance, an increase in the volatility of the discount factor would reduce the preinsurance expectation of profits and, hence, the fixed amount that a firm could earn with insurance.

4. Conclusions

■ In a repeated oligopoly, I characterized the optimal symmetric collusion under discount factor fluctuations and found that collusive prices and profits increase with both present and future discount factor levels and decrease with discount factor volatility. These results show the importance of discount factor levels in repeated games and introduce a new element to the literature: the volatility of the discount factor.

This work has several important implications for future study. While most of the existing empirical literature on collusive pricing has largely ignored the role of the interest rate (see, for example, Porter, 1983; Rotemberg and Saloner, 1986; Domowitz, Hubbard, and Petersen, 1986; Slade, 1987; Ellison, 1994; Borenstein and Shepard, 1996), this article suggests that both the level and the volatility of the interest rate are important determinants of collusive pricing and should be considered in future empirical work.

This article also has implications for the study of aggregate fluctuations.[4] I show that any change in policy, preferences or technology may have an impact on the aggregate level of activity through changes in collusive behavior, not only by affecting the real interest rate but also by affecting its volatility.

It is important to note that the results of this article may not necessarily hold under general stochastic processes of the discount factor—see Dal Bó (2002) for an example. Determining more general conditions under which the results presented here hold remains for future work.

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and $\int_a^b u(\delta)f(\delta)d\delta \geq \int_a^b u(\delta)g(\delta)d\delta$, for any increasing piecewise differential function $u(\delta)$. See Hirshleifer and Riley (1992).

[4] See Rotemberg and Saloner (1986) and Rotemberg and Woodford (1992) for the role of tacit collusion under demand fluctuation in aggregate fluctuations.

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