Comment on the Aruoba-Wright paper

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Despite efforts by some of the best minds in our profession, monetary theory at the start of the 21st Century remains flawed by the same fundamental shortcoming that Clower (1967) pointed out in the 1960s, namely the absence of any useful model of money in which widespread monetary exchange - - the use of a common money in almost all transactions - - can be shown to arise endogenously. The models of money that have proven most useful for empirical work are almost all based, to the extent that they have any conceptual basis at all, on the money-in-the-utility-function (MIU) approach developed by Patinkin (1965). But as many writers have observed, MIU starts from the presumption of widespread monetary exchange, a presumption that seems at odds with other aspects of the approach. Although search theory has provided models in which widespread monetary exchange can be shown to exist as an equilibrium phenomenon, a large gap still separates these models from those that have proven useful.

The paper by Aruoba and Wright (AW) narrows the gap a little, although not by as much as it claims. Ironically, it narrows the gap not just by making search theory more realistic but also by providing a better foundation to the MIU approach, the approach that search theory was originally intended to supplant. That is, the model AW present is actually a special case of MIU, a case in which the indirect utility of money is rationalized by search theory rather than by Patinkin's "stochastic payments process." Monetary exchange in this special case, instead of being widespread, is limited to a shadow economy involving only a subset of all transactions.

The paper strengthens search theory by dropping some of its most egregiously counterfactual assumptions, namely the indivisibility of money and the absence of organized exchange. Divisibility has been recognized since Jevons as one of money's most important attributes, but it has proven remarkably difficult to incorporate into search theory. Technically the problem is that the randomness of the matching process gives rise to a distribution of money holdings across people, and this distribution influences the prices that result from bilateral bargains. Keeping track of the distribution is relatively easy if people can only hold one unit of money or none. But keeping track

of it in previous models with divisible money has created technical complications so great as to obscure any substantive message.

The paper provides a neat finesse to the technicalities of divisibility. Specifically, after each day of random matching there is an organized market in which people can rebalance their money holdings, and under various linearity assumptions they will all choose to return to search the next day holding the same amount of money. Thus the pre-search distribution remains constant, and degenerate. Nothing could be simpler, although the restrictive linearity assumptions prevent this contribution from being anything more than an interesting example.

The absence of organized exchange is another glaring defect of previous search models. In the canonical search model, someone wanting a pair of shoes wanders aimlessly through space encountering people at random, hoping to meet someone who happens to be carrying a spare pair that would fit and would suit the searcher's tastes, and who also happens to want something that the searcher is carrying. In reality, no one would waste time in such aimless activity; instead, people wanting to buy shoes in any economy since the dawn of civilization have gone directly to a shoe store, a cobbler's shop, a Walmart, or some other specialized trading facility. The absence of such facilities makes search theory look more like trade in prehistoric times than in any economy of record.

AW go some way towards remedying this defect by introducing an organized trading sector, which they call "centralized", that functions alongside the unorganized sector, which they call "decentralized". The introduction of the organized sector plays a dual role in the paper because the presence of this sector is also what allows the authors to finesse the divisibility problem. Trading frictions being assumed absent entirely from the organized sector, everyone adjusts his or her money balances every period all the way to the common target value, thus undoing any redistribution that occurred in the unorganized sector.

The authors claim that their model provides an integration of neoclassical growth theory with micro-founded monetary theory. Even ignoring the special linearity assumptions on which their model rests, this claim deserves a large discount factor. It is true that the model with capital gives rise to exactly the same equations for accumulation and consumption of the "general" good traded in the organized sector as one finds in the canonical discrete-time Ramsey model. It is also true that the model supports valued token money and monetary exchange in the unorganized sector through a standard search-theoretic argument. But these two sectors, rather than being integrated, are assumed to operate in parallel, with almost no interconnection at all.

More specifically, the only link between the two sectors is provided by the fact that money can be traded in both sectors. Nothing produced in either sector can be traded in the other. Moreover, money is not *used* in the organized sector where the equations of Ramsey are satisfied, and nothing like neoclassical growth is taking place in the sector where money *is* used. Money has value in the organized sector, but only because it can be used in the parallel unorganized sector. Monetary exchange is not widespread.

The authors' claim is similar to the one implicit in Patinkin's (1965) subtitle, "An Integration of Monetary and Value Theory". More specifically, Patinkin also analyzed a neoclassical economy of the sort found in standard general equilibrium theory except that people valued money because of its usefulness in a parallel activity that did not in any other way interact with the standard economy. In his analysis the parallel activity was a "stochastic payments process" which he assumed took place each week after the Walrasian tâtonnement. People valued money carried out of the tâtonnement and into this process because it helped them avoid the embarrassment of being unable to pay when randomly matched with a potential supplier. In AW's analysis people value money carried out of the organized sector and into the unorganized sector because it helps them realize more gains from trade when randomly matched with a potential supplier. AW's account is more explicit and theoretically more satisfactory than Patinkin's, but in neither case can we really say that the theory integrates the use of money into the conceptual foundations of neoclassical value theory or growth theory.

Indeed, because it provides a Patinkinesque rationale for valuing money in an otherwise standard neoclassical economy, AW's model can be interpreted as an updated and improved model of MIU, in which search theory has taken over from Patinkin's stochastic payment process as the reason for having money into the utility function. To see this formally note that in a standard[1] MIU model with capital, the representative household solves an intertemporal optimization problem defined by the Bellman equation:

$$V_t\left(\frac{M_t}{P_t}, k_t\right) = \max_{\{x,m',k'\}} \left\{ v\left(x, \frac{M_t}{P_t}\right) + \beta V_{t+1}\left(\frac{m' + \tau M_t}{P_{t+1}}, k'\right) \right\}$$

$$\text{subj. to} \quad x + \frac{m'}{P_t} + k' = \frac{M_t}{P_t} + f\left(k_t\right) + \left(1 - \delta\right) k_t$$

where P_t is the price level, $v\left(\right)$ is the period utility function, and the other variable have the same meaning as in AW. It is straightforward to verify that exactly the same Bellman equation is satisfied

[1] That is, standard in the 21st Century, not in 1965.

by the value function of the representative household entering the organized market in AW's model with capital (section 3), with the period utility function:

$$v\left(x, \frac{M_t}{P_t}\right) = x + \alpha\sigma\left[u\left(q\left(\frac{M_t}{P_t}\right)\right) - g\left(q\left(\frac{M_t}{P_t}\right)\right)\right] + \alpha V_0$$

where I have written the function $q(s, \widetilde{s}) = q\left(\frac{M_t}{P_t}\right)$ determining the amount traded in a single-coincidence meeting as depending on real balances in accordance with the equations of AW's Lemma 2, and I have expressed the value functions also in terms of real balances, with the time-dependence coming only through the given time path of the price level.[2]

It is a testimony to the lasting strength of Patinkin's contribution that search theorists who originally set out to overthrow his MIU approach have ended up embracing it. The result of their round trip is a somewhat improved version of "practical" MIU monetary theory, but one that still suffers from the disconnect between the activities underlying the demand for money and the other activities described by the theory. A more satisfying integration of money into growth or value theory would take into account that the infrastructure of market organization is not costless even in the most organized sectors. The estimates of Wallis and North (1986) for the United States show that transaction costs as a fraction of GDP are large, and that they rise as market development proceeds. Thus organized trade mitigates but does not eliminate the costs of trading that underlie the use of money, and money continues to be used even today in all sectors of the US economy. Existing monetary theory fails to take this into account, and as a result we are still missing a coherent theory of money as it is used in a modern organized economy. What AW have presented is not this but a hybrid model, with money being used in a prehistoric unorganized sector and not being used at all in a futuristic, costlessly organized sector.

References

Clower, Robert W. (1967). "A Reconsideration of the Microfoundations of Monetary Theory." *Western Economic Journal* 1, 1-9.

Patinkin, Don. (1965). *Money, Interest and Prices: An Integration of Monetary and Value Theory.* 2nd ed. New York: Harper-Row.

Wallis, John J., and Douglass C. North. (1986). "Measuring the Transaction Sector in the American Economy, 1870-1970." In *Long-Term Factors in American Economic Growth*, edited by Stanley Engerman and Robert Gallman. Chicago: University of Chicago Press.

[2] To verify, use AW's equations (9), (23) and (24), and note that, from their Lemma 2, $\phi d\left(\frac{M_t}{P_t}\right) = g\left(q\left(\frac{M_t}{P_t}\right)\right)$.