Beyond Search: Fiat Money in Organized Exchange*

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Abstract: A model of fiat money is constructed in which spatial separation and the logistics of communication are made explicit as in search theory, but in which exchange is organized by profit-seeking business enterprises as in all market economies. Firms mitigate search costs by opening shops that are easily located. Equilibria may exist in which fiat money is used as a universal medium of exchange. When a monetary equilibrium exists, fiat money is essential. The model provides a foundation to cash-in-advance theory, without specifying in advance that one object will be used as the universal medium of exchange

1   Introduction

One of the objectives of monetary theory is to produce models of endogenous fiat money; that is, models that do not assign to fiat money any *a priori* role in the transactions process, but in which fiat money is nonetheless used as a universal medium of exchange. (Wallace 1998). In recent years, such models have been produced using search theory (Kiyotaki and Wright 1989, and Aiyagari and Wallace 1991), according to which exchange takes place only when traders with complementary tastes and endowments are randomly matched with one another. These models have the virtue that they specify explicit environments in which spatial separation limits the ability to communicate and to trade. As Kocherlakota (1998) argues, fiat money is a record-keeping device that helps overcome these limitations.

In contrast to what happens in search models, exchanges in actual market economies are organized by specialist traders, who mitigate search costs by providing facilities that are easy to locate. Thus when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer; when desiring to sell their labor services they go to firms known to offer employment. Few people would think of planning their economic lives on the basis of random encounters with non-specialists. The purpose of this paper is to provide a model of endogenous fiat money that specifies explicitly the spatial limitations to communication and trade as in search theory, but in which trade is organized by specialist traders, through facilities called "shops."

The model is based on recent work on *commodity* money, in which exchange is organized by an endogenous network of shops. Starr and Stinchcombe (1999) showed that under a variety of circumstances an optimal network of shops for trading n commodities has a monetary structure; i.e., that it has $n - 1$ shops and there is one commodity c such that for every other commodity j there is one shop trading j and c. Starr and Stinchcombe (1998) show that such an arrangement characterizes some equilibria in an environment with free entry of shops. Howitt and Clower (2000) show by means of computer simulation that this is the only equilibrium that can be reached by a particular decentralized mechanism in which

people act according to simple adaptive rules.

Two features of the technology of operating a shop are invoked to generate a shop network with a monetary structure in these theories. One is that at least part of the cost is fixed independently of the volume of trade. The other is that each shop can trade only a limited variety of commodities. In the model below these features are critical to the existence of an equilibrium with valued fiat money. A shop that did not trade fiat money would be unable to operate on a large enough scale to cover its fixed cost when competing with shops that do trade money, because its clientele would be limited to people that satisfy the double condition of being endowed with one of the objects traded in the shop and having a taste for consuming another of them. If fiat money is expected to be traded in all shops then this restriction does not apply to a shop that trades it, and the expectation will be self-fulfilling.[2]

The present paper goes beyond existing work on organized commodity exchange, by paying attention to spatial separation and communication costs at the same level as modern search theory. The paper also specifies that people cannot transfer objects from one shop to another within a period, thus requiring material balance constraints to apply at all stages of a multi-step exchange. As I argue in section 4.2 below, this no-cross-hauling restriction is crucial for supporting fiat money, as opposed to the commodity money studied in the earlier papers. Finally, in order to deal with fiat money I study an intertemporal economy with an infinite horizon rather than the atemporal one-period model of earlier studies.

The analysis is also related to the "trading-post" models of Shapley and Shubik (1977), dynamic versions of which have been studied by Hayashi and Matsui (1996) and Alonzo (1999), both of whom discuss conditions under which fiat money will have value and will be used.[3] Those models assume that trade can take place in specific locations, in each of which a specific pair of objects can be traded for each other. The exchange rate at each trading post is determined as the ratio of quantities delivered on either side of the market. These models are characterized by a coordination problem giving rise to multiple equilibria. Specifically, if no one delivers commodities to a particular trading post then no one will want

to do so. Thus in Alonzo's model, for any given choice of trading posts, there will be an equilibrium in which those trading posts are inactive. Hayashi and Matsui eliminate this multiplicity by assuming that prices are posted at all trading posts, and that all participants assume they can trade all they want at those prices, even if some trading posts are inactive, but this begs the questions of who posts the prices at the inactive trading posts and why people think they can trade all they want when there is no one else to trade with.

The present paper can be seen as providing a micro-foundation to the trading-post story, by allowing profit-seeking business firms to create and operate them, and by making explicit the constraints that limit the objects that can be traded in them and that determine the way people can visit them. I also suppose, as indicated above, that there is a fixed cost to operating a trading post; this allows fiat money to be used and to have value under circumstances where it would not in the other dynamic trading-post models.[4]

The model shares several features with search-theoretic models. It starts from the same premises of spatial separation and costly communication. Also, the above-mentioned "double condition" that handicaps a shop that does not trade a generally accepted medium of exchange is nothing but the double coincidence of wants, the unlikelihood of which underlies the use of money in search theory. However, the model deals easily with perfectly divisible money, which has led to considerable difficulties in search theory.[5] Also, unlike most search models (and the dynamic trading-post models discussed above) the model analyzed below does not depend on special assumptions regarding the joint distribution of tastes and endowments (such as the well-known "Wicksellian triangle") that rule out the possibility of a double coincidence of wants. Indeed I assume a symmetric distribution under which everyone could potentially find a double-coincidence trading partner at the same time.

Moreover, although the model starts from the same premises as search theory, it bears a stronger resemblance to existing "cash-in-advance" models of money than to search-theoretic models. It has long been recognized (Howitt 1974, and Kohn 1981) that cash-in-advance models can be rationalized by assuming that people trade exclusively with shops that will

accept only money in exchange for commodities and will only pay only money for commodities. These rationalizations do not however provide models of endogenous money, because they specify *a priori* the unwillingness of shops to exchange commodities directly for other commodities. The present paper provides a cash-in-advance theory that does away with this *a priori* specification. Rather the specification emerges endogenously from the underlying assumptions of the model.

I show below that a monetary equilibrium in this model is efficient whenever it exists. Moreover, fiat money is "essential," (Hahn 1973), in the sense that whenever a monetary equilibrium exists fiat money is the universal medium of exchange in any efficient network of shops. The reason for this is the saving in transaction costs; that is, in the fixed costs of operating shops. As others have noted, a monetary pattern of shops allows everyone to trade using the least possible number of shops, and hence paying the least possible fixed cost.

Money is even essential in cases where a robust monetary equilibrium does not exist. The reason is related to the wedge between the private and social costs of holding money, which underlies Friedman's (1969) argument regarding the optimum quantity of money. Specifically, from the private point of view there is an extra time cost to trading using money, because it takes two sequential transactions instead of one. Thus when the cost of setting up a shop is low enough, barter merchants can break a monetary equilibrium, despite their suboptimal scale of operation, by offering immediate gratification. But from a social point of view the time cost of using money is non-existent; in equilibrium, current consumption equals current endowment minus the fixed costs of operating shops, regardless of the method by which commodities are distributed among people.[6]

2 Basics

2.1 Preferences and endowments

Time is discrete, with an infinite sequence of periods indexed by $t = 0, 1, 2, ...$ There are $n \geqq 4$ distinct commodities, all perfectly divisible and none storable from one period to another. In addition to commodities there is a perfectly divisible and perfectly durable object called "money." There are two classes of transactors, a continuum of "households," each of whom lives forever, and a finite number of "merchants," each of whom lives for only one period. The number of transactors in each class is constant over time, dying merchants being replaced each period by newborn merchants.

Each household has a "type," identified by an ordered pair of distinct commodities (i, j), meaning that the household is endowed only with commodity i (is an "i-maker") and consumes only commodity j (is a "j-eater"), a utility function:

$$\sum_{t=0}^{\infty} \beta^t x_t, \ 0 < \beta < 1$$

where x_t is the quantity of j consumed at t, a constant endowment flow equal to $y \ (> 0)$ units per period, and an initial holding of $m \ (> 0)$ units of money, where m and y are both constant across households of different types. There is a unit measure of each type of household.

Each merchant has a type, identified by an *unordered* pair of distinct tradeable objects (i, j), where object $i = 0$ is money and object $i > 0$ is commodity i. A merchant of type (i, j) can set up a shop in which objects i and j can be traded for each other. There are always two merchants of each type, indexed by $k = 1, 2$. The merchant is a "monetary merchant" if $i = 0$ or $j = 0$, and a "barter merchant" otherwise. The merchant has no commodity endowment, and has lexicographic preferences in which the first element is π, its total consumption of commodities i and j (in the case of a barter merchant) or its consumption of j (in the case of

a monetary merchant), and the second element is its activity level, equal to 1 if the merchant is "active" (accepts deliveries from households) and 0 if inactive.[7]

2.2 Locations and shops

The logistical constraints imposed on people are similar to those of search theory. Each household begins each period in a private home not accessible to anyone else. This is where endowments are received. There is a continuum of locations at which traders can meet to exchange objects. Each household consists of two traders, each of whom can visit at most one location per period.[8] One trader can deliver money to a location and the other can deliver commodity endowment to a location. A trader can trade only with others who visit the same location that period. No record can be kept of who visited what locations, or who traded what amounts, in previous periods.

A merchant who visits a location can create a shop there. Only one shop can operate on any given location. Any number of traders can visit a location with a shop, but only to trade with the merchant who created the shop (the "shopkeeper"). As discussed in the introduction above, it is critical to recognize that each shop can trade only a limited variety of objects. As Alchian (1977) argued, if there were no such limitation then there would be no need for anyone to trade indirectly or to hold a temporary abode of purchasing power. Such a limitation is empirically plausible, given the casual observation that no retail outlet (even Walmart) in any economy of record trades more than a small fraction of all tradeable objects. Moreover, the limitation can be rationalized by the costs of acquiring the specialized knowledge needed to assess commodities, as Alchian also argued.[9] Accordingly, I suppose that only two objects can be traded at a given shop, the two identified by the shopkeeper's type.

If no shop exists on a location, then no one can distinguish it from any other location that lacks a shop, without first visiting it. In particular, no one can tell if anyone else has chosen to visit it.[10] Thus anyone who visits a location without a shop must choose one at

random. A shop, however, can be seen from anyone's home. Thus a trader can choose to visit any given location on which a shop has been created that period. When choosing what location to visit, the trader can see what objects are traded in each shop, and can also see the terms at which each shopkeeper offers to trade.

Merchants compete for customers, à la Bertrand, by posting these terms. Specifically, each monetary merchant posts a pair of offer prices (w_0, w_1). By doing so, it offers to pay w_0 units of money for each unit of its tradable commodity delivered by a trader, and w_1 units of the commodity for each unit of money delivered. Thus w_0 is the merchant's wholesale price for the commodity and w_1 is the inverse of its retail price. Each barter merchant posts a uniform offer price w_b, thus promising to pay w_b units of one commodity for each unit of the other commodity delivered to it.[11]

As mentioned in the introduction, it is crucial to assume a fixed cost of operating a shop. Suppose accordingly that an active merchant must give up the amount $\sigma > 0$ of the commodities it trades. More specifically, an active monetary merchant must give up σ of the only commodity it trades, whereas an active barter merchant must give up the amount $\sigma/2$ of each commodity it trades. A merchant that is unable thus to defray its operating cost must refuse all deliveries.

Assume that the set of possible locations is so large that the probability of encountering another household through random search is zero. Thus households will always trade with shopkeepers, whom they can find costlessly, rather than undergo the (pointless) inconvenience of random search.[12]

2.3   The sequencing of actions

Events transpire in a fixed sequence each period, with three stages. In stage 1, endowments are received, and all merchants visit random locations to establish their shops and post their prices. In stage 2, each trader can visit a single location with some amount of money or of newly received commodity. In stage 3, merchants pay for these deliveries, transactors

consume the commodities acquired through trade, and money holdings are stored until the beginning of the next period.

3   Trading within each period

Formally I model the choices made each period as a two-stage game with observable actions. In stage 1, each merchant chooses what prices to post. At stage 2 each household chooses what quantity of commodity endowment to deliver to a merchant, which merchant to deliver it to, what quantity of money to deliver to a merchant and which merchant to deliver it to.

3.1   Designated merchants and conventional trading modes

The second stage game will generally have multiple equilibria, for reasons that are related to the "zero-activity" problem of trading-post models (see Alonzo 1999). That is, there is a strong strategic complementarity in the choice of which merchants to patronize. More specifically, a household will have no incentive to deliver anything to a merchant if no other household chooses to deliver anything to that merchant, even if the merchant is offering the most advantageous prices, because the merchant, which begins the period with no inventories, will be unable to pay anything for the delivery. Thus in general it is possible to have equilibria of the second-stage delivery subgame in which for example each i-maker delivers all its endowment to the $(0, i)$ merchant offering the lowest wholesale price and each i-eater delivers money only to the same $(0, i)$ merchant, or equilibria in which all households of type (i, j) visit only the (i, j) merchant with the highest posted offer price, and so forth.

To circumvent this particular version of the zero-activity problem, I restrict the equilibria to be considered in the following two ways. First, the only equilibria I will consider to the second-stage game are those in which there is at most one active merchant of each type. Specifically, I assume that there is a convention according to which one of the two merchants of each type will be "designated," and no one will visit the other merchant. The convention

is based on who is offering the highest prices, with ties going to merchant number 1. It works as follows.

Let w_{b1} and w_{b2} be the respective offer prices of the two barter merchants of a given type. The designated merchant of that type is:

$$
d_b = \left\{ \begin{array}{l} 1 \text{ if } w_{b1} \geqq w_{b2} \\ 2 \text{ if } w_{b2} > w_{b1} \end{array} \right\}
$$

Let (w_{01}, w_{11}) and (w_{02}, w_{12}) be the respective pairs of offer prices of the two monetary merchants of a given type. The designated merchant of that type is:[13]

$$
d_m = \left\{ \begin{array}{l} 1 \text{ if } (w_{01}, w_{11}) \not\leq (w_{02}, w_{12}) \\ 2 \text{ if } (w_{02}, w_{12}) \geq (w_{01}, w_{11}) \end{array} \right\}
$$

Thus the designated merchant of each type is merchant 1 unless its offer prices are dominated by merchant 2's.

The convention of never visiting an undesignated merchant is self-sustaining because it takes at least some positive measure of households to visit a merchant delivering commodity in order for the merchant, who has no endowments of its own, to defray its operating cost. Thus there is nothing to be gained from being the only visitor to a merchant, who must in that case refuse all deliveries. I assume therefore, without violating any equilibrium conditions, that all households obey this convention in stage 2.

The second sense in which I restrict the class of equilibria to be considered is by imposing a uniform trading mode on all households. That is, there is a convention according to which either every household delivers all its commodity endowment to its designated monetary merchant or else every household delivers all its commodity endowment to its designated barter merchant. I assume that whether it is monetary or barter merchants that are thus favored depends on the maximal price $w_b^{\max}$ offered by any barter merchant in the economy, according to a trading function $\widetilde{z}(\ )$ that takes on the value 0 or 1. Specifically, the convention

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is for each household of type (i, j) to deliver all its endowment to the designated $(0, i)$ merchant if $\widetilde{z}\left(w_b^{\max}\right) = 0$ or the designated (i, j) merchant if $\widetilde{z}\left(w_b^{\max}\right) = 1$.

This convention is also self-sustaining because there is nothing to be gained from being the only visitor to a merchant and because no merchant of any other type is offering to pay any amount of commodity j or money for the household's endowment of i. I assume therefore, again without violating any equilibrium conditions, that all households obey this convention in stage 2. Different equilibria will be defined by different trading-mode functions $\widetilde{z}(\ )$, as discussed in more detail below.

3.2   Admissible offer prices

In stage 1, each merchant is competing to become the designated merchant of its type. It is promising to pay its offer prices for every unit delivered, if it becomes designated and is not excluded by the trading-mode convention. For this promise to be admissible the merchant must be able to honor it in the event that it receives the maximum possible deliveries.

Thus for a barter merchant of any type (i, j) the set of admissible offer prices is all $w_b \geqq 0$ such that:

(1)
$$(1 - w_b) y - \sigma/2 \geqq 0$$

if such a price exists, or else $w_b = 0$. The left-hand side of (1) is the merchant's trading surplus in commodity i in the event that it receives maximum deliveries; that is, the delivery of y from each of the unit mass of type (i, j) households, minus the amount $w_b y$ the merchant must pay for the maximum delivery of y from each of the unit mass of type (j, i) households, minus the amount $\sigma/2$ used up in defraying the setup cost of activating the shop. Thus (1) is the merchant's material balance constraint for commodity i in the event that it becomes the designated merchant and receives the maximum deliveries. By symmetry it is also the merchant's material balance constraint for commodity j in the same event.

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The offer price:

(2)
$$\widehat{w}_b \equiv 1 - \sigma/2y.$$

is the "breakeven price" of a barter merchant - the offer price that just satisfies its material balance constraint (1) with equality. Accordingly, the set of admissible prices of each barter merchant can be expressed as:

$$A_b = \begin{cases} [0, \widehat{w}_b] & \text{if } \widehat{w}_b \geqq 0 \\ \{0\} & \text{otherwise} \end{cases}$$

For a monetary merchant of any type $(0, i)$, the set of admissible offer prices is all $(w_0, w_1) > (0, 0)$ such that:

(3)
$$\begin{cases} (n-1) m - w_0 (n-1) y & \geqq 0 \text{ and} \\ (n-1) y - w_1 (n-1) m - \sigma & \geqq 0 \end{cases}$$

if such a pair of prices exists, or else $(w_0, w_1) = (0, 0)$. The first inequality in (3) is the material balance constraint for money in the event the merchant receives maximum deliveries; the left-hand side the money it will receive if all of the mass $(n-1)$ of money-holding i-eaters deliver all their money minus what the merchant must pay for commodity deliveries if all of the mass $(n-1)$ of i-makers deliver y to it. Similarly the second inequality is the material balance constraint for commodity i under the same circumstances, in which the left-hand side is the merchant's trading surplus in commodity.

Define:

(4)
$$\widehat{w}_0 \equiv \frac{m}{y}$$

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and:

$$(5) \qquad \widehat{w}_1 \equiv \frac{y - \sigma/(n-1)}{m}.$$

Then $\widehat{w}_0$ is the breakeven wholesale price of a monetary merchant - the one that just satisfies the first of the material balance constraints (3) with equality, and $\widehat{w}_1$ is the breakeven inverse-retail price of a monetary merchant - the one that just satisfies the second material balance constraint with equality. Accordingly, the set of admissible prices for each monetary merchant is:

$$A_m = \left\{ \begin{array}{ll} [0, \widehat{w}_0] \times [0, \widehat{w}_1] & \text{if } \widehat{w}_1 \geqq 0 \\ \{(0,0)\} & \text{otherwise} \end{array} \right\}.$$

3.3 Stage 2 - household behavior

The conventions described in section 3.1 severely restrict each household's choices in stage 2. Each household of type (i, j) will deliver its endowment to the designated $(0, i)$ merchant if $\widetilde{z}(w_b^{\max}) = 0$ or to the designated (i, j) merchant if $\widetilde{z}(w_b^{\max}) = 1$, and there is no merchant to whom it would make sense for the household to deliver money other than the designated $(0, j)$ merchant.

Whether or not the household delivers money to this merchant depends on the merchant's inverse-retail price w_1 and on the household's expectations concerning next period, which I assume are consistent with the equilibrium conditions to be specified in section 4 below. There I will restrict attention to stationary symmetric equilibria in which there is a triple of offer prices: $(w_b^*, w_0^*, w_1^*) \geqq (0, 0, 0)$ such that all barter merchants post w_b^* each period, all monetary merchants post (w_0^*, w_1^*) each period, and each period begins with the amount m of money in the possession of each household. Therefore the trading mode $\widetilde{z}(w_b^*)$ will be the same each period. Thus if $w_1 < \beta w_1^*$ and $\widetilde{z}(w_b^*) = 0$ the household will save all of its money and plan to spend it next period, since therefore each unit of money will yield a utility gain[14] of βw_1^* instead of w_1. Otherwise it will deliver all m units to the designated $(0, j)$ merchant

this period, which is at least as good as any other use of the money.

3.4 Stage 1 - merchant behavior

Consider a barter merchant of index $k \in \{1, 2\}$. Let w_{bk} denote its offer price, and δ_{bk} its activity level in the event that the conventional trading mode is barter, with $\delta_{bk} = 1$ (resp. 0) indicating that the merchant would be active (resp. inactive) in that event. In a symmetric stationary equilibrium the merchant's choice of w_{bk} and δ_{bk} will be governed by the decision problem:

$$
(\mathrm{D}_{bk}) \quad \left\{
\begin{array}{l}
\displaystyle \operatorname*{maximize}_{\{w_{bk}, \delta_{bk}\}} \pi_{bk} = \delta_{bk} \cdot [2\,(1 - w_{bk})\,y - \sigma] \cdot \widetilde{z}\,(\max\{w_{bk}, w_b^*\}) \\[2ex]
\text{subject to:} \\[1ex]
w_{bk} \in A_b \text{ and} \\[1ex]
\delta_{bk} = \left\{
\begin{array}{ll}
1 & \text{if } k = 1 \text{ and } w_{b1} \geqq w_b^* \\
1 & \text{if } k = 2 \text{ and } w_{b2} > w_b^* \\
0 & \text{otherwise}
\end{array}
\right\}
\end{array}
\right\}
$$

The objective function in (D_{bk}) is the sum of the two trading surpluses of a merchant receiving maximum deliveries, according to (1). Each of the surpluses is the amount of the commodity available for the merchant to consume after paying for its deliveries of the other commodity and defraying its operating cost. The first constraint indicates that the posted offer price must be admissible and the second indicates that in order to be active the merchant must be designated. Note that the barter merchant takes into account that its own offer price might affect the economy's conventional choice of trading mode.

Because of the second-order lexicographic preference for being active, the merchant's

choice must also satisfy:

$$(\mathrm{E}_{bk}) \qquad \begin{cases} \delta_{bk} = 1 \text{ if there exists a } w'_{bk} \text{ such that:} \\[2mm] \text{(a) } \widetilde{z}\left(\max\left\{w'_{bk}, w^*_b\right\}\right) = 1 \text{ and} \\[2mm] \text{(a) } (w'_{bk}, 1) \text{ solves } (\mathrm{D}_{bk}). \end{cases}$$

Next consider a monetary merchant of index $k \in \{1, 2\}$. Let (w_{0k}, w_{1k}) denote its offer prices, and δ_{mk} its activity level in the event that the conventional trading mode is monetary. In a symmetric stationary equilibrium the merchant's choice of (w_{0k}, w_{1k}) and δ_{mk} will be governed by the decision problem:

$$(\mathrm{D}_{mk}) \qquad \begin{cases} \displaystyle\max_{\{w_{0k}, w_{1k}, \delta_{mk}\}} \pi_{mk} = \delta_{mk} \cdot \left[(n-1)(y - w_{1k}m) - \sigma\right] \cdot \left[1 - \widetilde{z}(w^*_b)\right] \\[2mm] \text{subject to:} \\[2mm] (w_{0k}, w_{1k}) \in A_m \text{ and} \\[2mm] \delta_{mk} = \begin{cases} 1 & \text{if } k = 1,\ w_{11} \geqq \beta w^*_1 \text{ and } (w_{01}, w_{11}) \not\leq (w^*_0, w^*_1) \\ 1 & \text{if } k = 2,\ w_{12} \geqq \beta w^*_1 \text{ and } (w_{02}, w_{12}) \geq (w^*_0, w^*_1) \\ 0 & \text{otherwise} \end{cases} \end{cases}$$

The objective function in (D_{mk}) is the commodity trading surplus of a monetary merchant receiving maximum deliveries, according to (3). The second constraint indicates that in order for the merchant to be active, its inverse retail price must be no less than its customers' choke price βw^*_1. (Otherwise the merchant will not have the means to pay for any commodity deliveries.) The monetary merchant, unlike the barter merchant, takes the conventional trading mode as given.

Because of the second-order lexicographic preference for being active, the merchant's

choice must also satisfy:

$$(\text{E}_{mk}) \quad \left\{ \begin{array}{l} \delta_{mk} = 1 \text{ if:} \\[1em] \text{(a) } \widetilde{z}\left(w_b^*\right) = 0 \text{ and} \\[1em] \text{(a) there exists a } \left(w_{0k}', w_{1k}'\right) \text{ such that } \left(w_{0k}', w_{1k}', 1\right) \text{ solves } (\text{D}_{mk}). \end{array} \right\}$$

4   Equilibrium

As indicated above, what constitutes an equilibrium will depend on the function $\widetilde{z}(\ )$ governing the convention with respect to trading modes. Consider any such $\widetilde{z}(\ )$.

Definition *A Stationary Symmetric Equilibrium with respect to* $\widetilde{z}(\ )$ *is a triple* $(w_b^*, w_0^*, w_1^*) \geqq (0, 0, 0)$ *such that for each* $k = 1, 2$: *(i)* $(w_{bk}, \delta_{bk}) = (w_b^*, 2 - k)$ *solves the problem* (D_{bk}) *and satisfies condition* (E_{bk}), *and (ii)* $(w_{0k}, w_{1k}, \delta_{mk}) = (w_0^*, w_1^*, 2 - k)$ *solves the problem* (D_{mk}) *and satisfies condition* (E_{mk}).

Note that a Stationary Symmetrical Equilibrium requires $\delta_{bk} = \delta_{mk} = 2 - k$ for each k because the rule for designating merchants always designates merchant 1 in the event of a tie. This definition corresponds to the usual notion of subgame perfect equilibrium in a repeated two-stage game. However, the assumption that merchants honour their posted prices in stage 3 whenever they are able to requires some commitment because otherwise the merchants would keep all commodity deliveries (net of operating costs) for their own consumption rather than using them to honour their posted offers.

Stationarity requires that each household carry over the same amount m of money each period as held initially. This will happen in any Stationary Symmetric Equilibrium if the designated monetary merchants are inactive in the equilibrium because in that case each household will neither receive nor give up money. It will also happen if the designated monetary merchants are active, because in that case the merchants' second-order preference for activity implies that w_0^* must equal the breakeven wholesale price $\widehat{w}_0$,[15] so a household's net acquisition of money each period will be the sales receipt $\widehat{w}_0 y$ minus the expenditure m,

which equals zero by (4).

The conventional trading mode is always barter, independently of all merchants' prices, if $\widetilde{z}(\ )$ is given by the function:

$$z_b \left(w_b^{\text{max}}\right) \equiv 1 \text{ for all } w_b^{\text{max}}.$$

Definition *A Barter Equilibrium is an offer price $w_b^* > 0$ such that, for some pair $(w_0^*, w_1^*) \geqq (0,0)$, (w_b^*, w_0^*, w_1^*) is a Stationary Symmetric Equilibrium with respect to $z_b(\ )$.*

Note the requirement that $w_b^* > 0$ implies that in any Barter Equilibrium households receive a positive benefit from trading with barter shops. It is easy to show that there exists a unique Barter Equilibrium, in which the offer price is the breakeven price (2), provided only that this breakeven price is positive. The following proposition is proved in the Appendix:

Proposition 1 *If*

(6)
$$1/2 < y/\sigma$$

then

$$\widehat{w}_b \equiv 1 - \sigma/2y$$

constitutes the unique Barter Equilibrium. Otherwise there does not exist a Barter Equilibrium.

The existence of this equilibrium is quite intuitive. That the designated barter merchants should just break even is a standard consequence of Bertrand price competition. The existence condition (6) requires just that a barter merchant that received the maximum possible delivery y of each commodity traded would have enough to defray the operating cost $\sigma/2$ in that commodity and to pay something positive for the other commodity. If (6) were violated then clearly the economy would be unable to operate barter shops paying positive prices.

The existence of this equilibrium even when money exists is like the existence of an equilibrium with valueless fiat money in other standard monetary models such as cash-in-advance or the overlapping-generations model. The difference is that in this case people are still able to trade even without using money, provided condition (6) is satisfied.

Next we show conditions under which there is an equilibrium in which all traders visit monetary merchants and trade profitably using money. The conventional trading mode is always monetary, independently of all merchants' prices, if $\widetilde{z}(\ )$ is given by the function:

$$z_m\left(w_b^{\max}\right) \equiv 0 \text{ for all } w_b^{\max}.$$

Definition *A Monetary Equilibrium is a pair of offer prices* $(w_0^*, w_1^*) > (0,0)$ *such that, for some* $w_b^* \geqq 0$, (w_b^*, w_0^*, w_1^*) *is a Stationary Symmetric Equilibrium with respect to* $z_m(\ )$.

It is straightforward to show that there exists a unique Monetary Equilibrium, in which the offer prices are the two breakeven prices (4) and (5), provided only that these breakeven prices are both positive. In the appendix we prove:

Proposition 2 *If*

(7) $$1/\left(n-1\right) < y/\sigma$$

then

$$(\widehat{w}_0, \widehat{w}_1) = \left(\frac{m}{y}, \frac{y - \sigma/\left(n-1\right)}{m}\right)$$

constitutes the unique Monetary Equilibrium. Otherwise there does not exist a Monetary Equilibrium.

That the designated monetary merchants should just break even is again a standard consequence of Bertrand price competition. The existence condition (7) requires just that a monetary merchant that received the maximum possible delivery $(n-1)\,y$ of the commodity it trades would have enough to defray the operating cost σ and to sell positive amounts to

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its customers. If (7) were violated then clearly the economy would be unable to operate monetary shops with finite retail prices.

4.1 Robust Monetary Equilibrium

The zero-activity problem makes the possibility of monetary exchange almost automatic because it makes the convention of never visiting barter merchants, regardless of their prices, self-sustaining. But to explain the ubiquitous use of money on the basis of such reasoning is less than compelling. That households would not consider seriously the possibility of a barter merchant being able to accept deliveries at competitive prices may seem a reasonable approximation to reality in any economy of record, but it remains to be seen under what conditions a Monetary Equilibrium would exist even if people did take barter alternatives seriously, as they presumably would learn to do, at least in some places and times, if advantageous barter alternatives were potentially available.

Thus it is useful to consider another conventional rule $\widetilde{z}(\ )$ that would not ostracize barter merchants offering better deals than monetary merchants, in order to ensure that fiat money will be used even when people take barter alternatives seriously. To this end, consider the following class of rules. Let $(\widetilde{w}_0, \widetilde{w}_1) \geqq (0, 0)$ be any given pair of offer prices, and define:

$$z_r\left(w_b^{\max}; \widetilde{w}_0, \widetilde{w}_1\right) \equiv \left\{ \begin{array}{l} 1 \text{ if } w_b^{\max} \geqq \beta \widetilde{w}_0 \widetilde{w}_1 \\ 0 \text{ otherwise} \end{array} \right\}$$

If $\widetilde{z}(\ ) = z_r(\ ; \widetilde{w}_0, \widetilde{w}_1)$ then people would all deliver their endowments to barter merchants whenever there was a single barter merchant offering at least as good a deal as a network of monetary merchants all offering the constant offer prices $(\widetilde{w}_0, \widetilde{w}_1)$. This is because in such a network a household that delivered a unit of endowment to a monetary merchant would receive in exchange $\widetilde{w}_0$ units of money, which could be exchanged next period for $\widetilde{w}_0 \widetilde{w}_1$ units of the household's consumption commodity, yielding a utility gain of $\beta \widetilde{w}_0 \widetilde{w}_1$, whereas delivering the same unit to a barter merchant offering $w_b^{\max}$ would yield a utility gain of

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$w_b^{\max}$.

Definition *A Robust Monetary Equilibrium is a pair of offer prices $(w_0^*, w_1^*) > (0,0)$ such that, for some $w_b^* \geqq 0$, (w_b^*, w_0^*, w_1^*) is a Stationary Symmetric Equilibrium with respect to $z_r(\ ; w_0^*, w_1^*)$, and $w_b^* < \beta w_0^* w_1^*$.*

In a Robust Monetary Equilibrium households all trade exclusively with monetary merchants even though they are collectively so eager to consider barter alternatives that they would all revert to barter if a single barter merchant in the economy offered a better deal than the existing network of monetary merchants. A necessary condition for a Robust Monetary Equilibrium to exist is that $\beta \widehat{w}_0 \widehat{w}_1 > \widehat{w}_b$. That is, when all merchants charge their breakeven prices and people expect the same breakeven prices to prevail in the future then monetary trade will be more advantageous than barter. This necessary condition can be expressed as:

$$(8) \qquad y/\sigma < \left(1 + \frac{n-3}{2(1-\beta)}\right) / (n-1).$$

Condition (7) is also still necessary because otherwise the economy cannot support monetary shops with finite retail prices. In the Appendix we prove:

Proposition 3 *If*

$$1/(n-1) < y/\sigma < \left(1 + \frac{n-3}{2(1-\beta)}\right) / (n-1)$$

then

$$(\widehat{w}_0, \widehat{w}_1) = \left(\frac{m}{y}, \frac{y - \sigma/(n-1)}{m}\right)$$

constitutes the unique Robust Monetary Equilibrium. Otherwise there does not exist a Robust Monetary Equilibrium.

4.2 Commentary

Proposition 2 outlines necessary and sufficient conditions for fiat money to potentially play the role of universal medium of exchange. This requires only that the fixed cost of operating a shop be less than the economy's total endowment of each commodity. However, by Proposition 3 a monetary equilibrium will be fragile if this fixed cost is too small, for if households are confident that others will visit barter merchants that offer favorable prices, then a monetary merchant will not be able to compete effectively with barter merchants.

The fragility of monetary merchants when operating costs are small can be explained as follows. A household that sells to a monetary merchant must wait until another transaction is completed before consuming the proceeds of the sale, whereas no such wait is required if the household sells instead to a barter merchant. So to compete against barter merchants, a monetary merchant must offer an advantage in the form of a relative wholesale price $w_0 w_1^*$ that is sufficiently larger than any barter merchant's offer price. The reason why it can offer this advantage is the economy of scale that arises from the fixed cost of operating a shop. That is, because a monetary merchant of type $(0, j)$ can service all j-makers, not just those who also consume some particular commodity i, it can have a smaller average cost than any barter merchant. Without the fixed cost, however, there would be no economy of scale, and monetary exchange would offer no advantage over barter.[16]

According to Proposition 3, fiat money is more robust the more patient people are. That is, an increase in patience, represented by an increase in the discount factor β, would raise the right-hand side of the necessary condition (8), making it more likely that a Robust Monetary Equilibrium will exist. In the limit, if people did not discount the future at all ($\beta = 1$) then condition (8) would always be satisfied, so a Robust Monetary Equilibrium would exist whenever a Monetary Equilibrium existed. This is because from the household's point of view the only disadvantage of monetary exchange relative to barter is that it requires one period to convert endowments into consumption. The less people discount the future the less is this disadvantage. By the same token if people discounted the future at an infinite

rate ($\beta = 0$) then the necessary condition (8) would imply the failure of the condition (6) for Barter Equilibrium to exist, so a Robust Monetary Equilibrium would never exist whenever the economy was able to support barter shops paying positive prices.

Proposition 3 also shows that fiat money is more robust, the larger is the variety n of commodities in the economy, given the aggregate supply $(n-1)y$ of each commodity. This is because as variety increases, the market that can be served by a barter merchant becomes increasingly fragmented, making it less likely that such a merchant can operate on a large enough scale to break a monetary equilibrium by stealing the customers of monetary merchants. Thus as economies become increasingly complex the likelihood increases that fiat money will be used. Since economic development tends to be characterized by increasing product variety, this result accords with the observation[17] that the degree of monetization rises as economic development proceeds.

Proposition 3 implies further that fiat money is less likely to be robust as output per commodity $(n-1)y$ increases, ceteris paribus. This is because in the limit as the market for any given commodity becomes infinitely large, the economy of scale implied by a fixed operating cost vanishes, and hence the advantage of monetary merchants vanishes. Thus with economic growth either an economy eventually reaches the point where fiat money becomes unstable or the effects of increasing output per commodity are continually offset by the effects of increasing product variety. Asymptotically, if product variety grows faster than output per commodity then beyond some point a robust monetary equilibrium will always exist.

The spatial/logistical constraints imposed in section 2.2 above are also important for monetary exchange to be viable, in a sense that cannot be seen immediately in the statement of Propositions 2 and 3. For suppose that each trader could visit two locations during any given period,[18] trading in one shop the object acquired in another, instead of being restricted to one location per period. Then a stationary monetary equilibrium could not exist. For in this case each household would choose to sell the proceeds of its current sale this period

rather than incur a waiting cost; but then there would always exist an excess supply of money, with no one willing to hold it at the end of the period. This excess supply of money would correspond to an excess demand for commodities, which would prevent a stationary equilibrium from existing at any finite price level.

Note that a Monetary Equilibrium and a Barter Equilibrium must coexist whenever a Barter Equilibrium exists. Indeed even a Robust Monetary Equilibrium must coexist with a Barter Equilibrium over some range, because the upper limit on y/σ specified by the robustness condition (8) exceeds the lower limit specified by the existence condition (6) for a Barter Equilibrium. That is, condition (6) simply requires that the breakeven price of a barter merchant be positive, whereas in order to break a Robust Monetary Equilibrium a barter merchant must be capable of offering a price that is not just positive but large enough to steal customers from the designated monetary merchants.

Of course it would be surprising not to find this kind of multiplicity in a model of fiat money. To analyze whether the economy might achieve a barter equilibrium or a monetary equilibrium when both exist would take us beyond the scope of the present paper, for it would require an adaptive analysis like that of Howitt and Clower (2000) intended to portray an economy's coordination mechanisms. In particular, Howitt and Clower take into account that entrepreneurship and the building of a customer base are not instantaneous, costless activities as in the present analysis. The analysis of that paper suggests that in cases of multiple equilibria only the monetary equilibrium would be reachable. For the equilibria of Propositions 2 and 3 above corresponds to the unique absorbing state of the adaptive mechanism studied by Howitt and Clower, except that the universal medium of exchange that emerges there is a commodity money rather than fiat money.

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5 Efficiency

To address the question of social efficiency, define the utilitarian welfare criterion:

$$W = \Sigma_{t=0}^{\infty}\beta^t W_t = \Sigma_{t=0}^{\infty}\beta^t \Sigma_{i=1}^{n} C_{it}$$

where C_{it} is total consumption of commodity i, by i-eaters and merchants that trade i, in period t. Then W is the sum of all household lifetime utilities plus the discounted sum of all merchants' first-order utilities.

Next, note that in order for anyone to consume commodity i there must exist at least one active shop trading i to which an i-maker can make a delivery. Assume in addition that a household of type (i,j) will deliver i only to an (i,j) shop or a $(0,i)$ shop.[19]

The level of $W_t = \Sigma_{i=1}^{n} C_{it}$ attainable in period t depends on what shops are active that period. Suppose there are N^m active monetary shops and $N^b/2$ active barter shops. Assume with no loss of generality that there are no two active shops of the same type. Let $N^{b/m}$ be the number of distinct household types (i,j) such that an (i,j) shop is active but not a $(0,i)$ shop; each one of these types can be induced to deliver endowments to a barter shop. In addition, there are N^m commodities i for which an $(0,i)$ shop is active, and for each of these commodities there are $(n-1)$ household types that can be induced to deliver to the $(0,i)$ shop. Since each shop incurs a total setup cost of σ, and each household type can deliver at most y, therefore the maximal level of W_t is:

(9)
$$U = \left[(n-1)\,N^m + N^{b/m}\right] y - \sigma\left[N^m + N^b/2\right]$$

Since this maximal level is independent of the configuration of active shops in any other period, maximizing W is equivalent to maximizing U period by period. The following con-

straints apply. First, $N^{b/m}$ is no greater than twice the number of active barter shops:

(10)
$$N^{b/m} \leqq N^b,$$

and no greater than the number of types (i, j) such that a $(0, i)$ shop is not active:

(11)
$$N^{b/m} \leqq (n-1)(n-N^m).$$

Next the number of active monetary shops is no greater than the number of commodities:

(12)
$$N^m \leqq n$$

and the number of active barter shops is no greater than half the number of distinct household types:

(13)
$$N^b \leqq n(n-1)$$

A "feasible allocation" is a collection $\left(N^b, N^{b/m}, N^m\right)$ of three non-negative integers satisfying (10) $\sim$ (13). An "efficient allocation" is a feasible allocation that maximizes U among all feasible allocations.

Suppose that the existence condition (7) for a Monetary Equilibrium is satisfied. A Monetary Equilibrium corresponds to the allocation $(0, 0, n)$, and yields a value of U equal to:

$$U^m = n(n-1)y - n\sigma > 0.$$

Moreover, any other feasible allocation yields a strictly lower value of U. To see this note that (9) can be written as:

$$U = [(n-1)y - \sigma]\left[N^m + \frac{N^{b/m}}{n-1}\right] + \sigma\left[\frac{N^{b/m}}{n-1} - \frac{N^b}{2}\right]$$

This and the inequality (11) imply:

$$U \leqq U^m + \sigma \left[\frac{N^{b/m}}{n-1} - \frac{N^b}{2} \right]$$

This and the inequality (10) imply:

$$U \leqq U^m + \sigma \left[\frac{1}{n-1} - \frac{1}{2} \right] N^b$$

From this and the assumption that $n > 3$ it follows that $U < U^m$ if there are any barter shops at all. Moreover, it follows directly from (9) that if there are no barter shops then $U = (N^m/n) U^m$. Therefore U is uniquely maximized by setting $N^b = 0$ and $N^m = n$, which yields the configuration of the Monetary Equilibrium. In summary:

Proposition 4 *If a Monetary Equilibrium exists then the unique efficient allocation according to the welfare criterion W has one active monetary shop for each commodity and no active barter shops, and is achieved by the Monetary Equilibrium.*

Intuitively, monetary exchange is required for social optimality because it minimizes the number of shops, and hence the total operating cost, needed to allow all of the economy's potential GDP to be consumed each period.

It does not follow that a Robust Monetary Equilibrium always exists when it is needed for efficiency. On the contrary, as we have seen, if the transaction cost σ is too low then a Barter Equilibrium exists but no Robust Monetary Equilibrium exists, even though Proposition 4 implies that the Barter Equilibrium is not efficient. The reason is related to Friedman's (1969) argument regarding the optimal quantity of money. Specifically, from the private point of view there is a time cost to trading using money, because it takes two transactions instead of one. This is why, as we have seen, when the cost of setting up a shop is low enough barter merchants can break a monetary equilibrium. But from a social point of view this time cost is non-existent; in equilibrium, current consumption equals current endowment

minus the fixed costs of operating shops, regardless of the method by which commodities are redistributed. Thus the attempt to avoid a private cost for which there is no corresponding social cost can destroy the only mechanism by which an efficient allocation of resources can be achieved. The problem could presumably be solved either by following Friedman's proposal to engineer a deflation through lump sum taxes that contracted the money supply over time, or by making barter transactions illegal as suggested by Engineer and Bernhardt (1991).

6 Conclusion

One of the oldest problems of monetary economics is that the models of money that are most useful in applied work are far removed from those in which the institution of money can be shown to arise spontaneously. This gap undermines the credibility of useful models, by raising the question of the extent to which their conclusions rest on *ad hoc* assumptions needed to fit money into a scheme where it doesn't really belong. My aim in this paper was to take a step toward reducing the gap between useful and well-founded models, by showing that competition between firms that create facilities for trading limited subsets of objects can spontaneously generate a network of markets in which, to use Clower's (1967) aphorism, money buys goods and goods buy money but goods do not buy goods. The resulting equilibrium bears a strong resemblance to that of the cash-in-advance model that has been one of the profession's useful workhorses. The existence of this equilibrium does not depend on any special configuration of tastes and endowments.

According to the argument developed in the paper, fiat money rests on the same spatial/logistical foundations as search theory, but those foundations result in organized trade, through trade facilities that are easily located, as is the case in most economies of record, rather than the disorganized random exchange depicted by search theory. In reality, most people trade using money as a universal medium of exchange because the enterprises that

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organize trade give them no other option. The paper proposed an account of how such a restrictive pattern of market organization might exist as an equilibrium phenomenon.

The model is simple but could be generalized in a number of useful directions. Allowing a reservation demand for endowments (or, equivalently, a psychological cost of producing the endowments) would permit a more meaningful efficiency analysis; to do this it would make sense to allow merchants to set two-part tariffs, with a lump-sum entry fee and a positive marginal cost of trading, since otherwise the equilibrium marginal cost of producing sold consumption goods would be strictly less than the retail price (as it is in the Monetary Equilibrium above) and therefore the merchants would generally not be allowing their customers fully to exploit all potential gains from trade. This would result in a general-equilibrium version of the Baumol-Tobin model of money-demand[20], in which households must incur a lumpy setup cost with each transaction regardless of the size of the transaction.

Another useful extension would be to generalize the model so that a merchant can trade more than two objects. Preliminary work on the above model that makes only this change indicates that analogues to Propositions $1 \sim 3$ can be derived. But there will be barter trades even in a monetary equilibrium, because there will be household types (i, j) for which both i and j can be traded in the same shop. There is clearly no need for such a household to hold or use money in a monetary equilibrium. However, as long as the maximum number of objects traded in a shop is small relative to the total number of commodities n, most trades will be monetary in a monetary equilibrium.

A more interesting way to allow for a merchant to trade more than one object would be to suppose that each household has only one endowment good, which it must sell to a producer of a corresponding consumption good, and that it has a taste for consuming all possible consumption goods, as in the analysis of Laing, Li and Wang (1997). Then if households were faced with the same logistical constraints as assumed above they would like to visit "supermarket" merchants that trade a large variety of consumption goods. We could allow such supermarkets to form by assuming that merchants have the possibility to visit many

producers and to sell a large variety of consumption goods (but not to trade all endowment goods).

Another extension would be to allow for durable goods. Preliminary work indicates that analogues of propositions $1 \sim 3$ go through under perfect durability. Stationary equilibria in which fiat money is used and has value continue to exist, because in a stationary equilibrium the storage option is not used. Indeed this would even be true if storage were a productive activity, because a demand for money would continue to be supported by the cash-in-advance constraint implied by the basic logistical assumptions of the model. However there will also exist various commodity-money equilibria, in which fiat money is not used but one of the goods emerges as the universal medium of exchange.

Yet another extension would be to include credit markets. Merchants could be allowed, for example, to set up banks, which are facilities in which money can be exchanged for promises to deliver money in the future. Households who decide to enter the credit market would have to forego the opportunity to buy or sell goods that period, because they have decided instead to visit a location on which a bank is situated. The result would be another variant of Baumol-Tobin, in which the setup cost of visiting a bank is the foregone opportunity to visit another shop that period.

Likewise, by adding stochastic endowment shocks, we could introduce out-of-steady-state dynamics, along with precautionary savings and precautionary money demand.

By putting business firms at the heart of a theory of money, this approach is also well suited for studying an economy's adjustment dynamics. The coordination mechanisms that allow a decentralized economy to regulate itself cannot be studied by conventional equilibrium methods, because to assume that the economy is in an equilibrium presupposes an unspecified coordination mechanism, involving unspecified agents. In reality, the agents of coordination are the enterprises that organize trade, for they are typically the agents that set prices, arrange for goods to be available at predictable times in known locations, and more generally undertake to coordinate supply and demand. By putting such agents at the

center of monetary theory, the approach of this paper aims at laying the foundation not only for equilibrium monetary theories but also for disequilibrium theories. The eventual goal is to study the behavior of a stochastic decentralized market system in which competing merchants must decide on prices, inventory holdings, advertising, entry and exit, without the full system-wide information that would be needed to carry out equilibrium actions.[21]

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Appendix

Proof of Proposition 1 (Existence of Barter Equilibrium)

(a) First I show that $\widehat{w}_b$ is a Barter Equilibrium if (6) holds. To this end, assume (6) is satisfied. Then $\widehat{w}_b > 0$, so it suffices to show that $(\widehat{w}_b, 0, 0)$ is a Stationary Symmetric Equilibrium with respect to $z_b(\ )$. Let $(w_b^*, w_0^*, w_1^*) = (\widehat{w}_b, 0, 0)$ and $\widetilde{z}(\ ) = z_b(\ )$. Then we just need to show that for $k = 1, 2$; (i) $(\widehat{w}_b, 2 - k)$ solves (D_{bk}) and satisfies (E_{bk}), and (ii) $(0, 0, 2 - k)$ solves (D_{mk}) and satisfies (E_{mk}). (i) By inspection, $(w_{bk}, \delta_{bk}) = (\widehat{w}_b, 2 - k)$ is feasible (i.e., it satisfies all the constraints of (D_{bk})) for each k. For all feasible (w_{b2}, δ_{b2}), $w_{b2} \leqq \widehat{w}_b = w_b^*$ (by the definition of A_b) so $\delta_{b2} = 0$ and $\pi_{b2} = 0$. Therefore (D_{b2}) is solved, and (E_{b2}) is satisfied, by all feasible (w_{b2}, δ_{b2}), including $(\widehat{w}_b, 0)$. When $(w_{b1}, \delta_{b1}) = (\widehat{w}_b, 1)$, $\pi_{b1} = 0$; whereas for any other feasible (w_{b1}, δ_{b1}), $w_{b1} < \widehat{w}_b = w_b^*$ so $\delta_{b1} = 0$ and $\pi_{b1} = 0$. Therefore (D_{b1}) is solved, and (E_{b1}) satisfied, by $(\widehat{w}_b, 1)$. (ii) For each k, $\pi_{mk} = 0$ for all feasible $(w_{0k}, w_{1k}, \delta_{mk})$ since $\widetilde{z}(w_b^*) = 1$, and therefore (D_{mk}) is solved by all feasible $(w_{0k}, w_{1k}, \delta_{mk})$, including $(0, 0, 2 - k)$. Likewise each (E_{mk}) is satisfied by $(0, 0, 2 - k)$ because $\widetilde{z}(w_b^*) = 1$.

(b) Next I show that $\widehat{w}_b$ is the only Barter Equilibrium if (6) holds. Assume again that (6) holds. Suppose, to the contrary, that w_b^* is a Barter Equilibrium and $w_b^* \neq \widehat{w}_b$. Let $\widetilde{z}(\ ) = z_b(\ )$. Since $(w_b^*, 1)$ solves (D_{b1}) therefore $w_b^* \in A_b$. By (6), $\widehat{w}_b > 0$. Therefore by the definition of A_b, $w_b^* < \widehat{w}_b$. It follows that $(w_{b2}, \delta_{b2}) = ((w_b^* + \widehat{w}_b)/2, 1)$ is feasible and yields $\pi_{b2} = 2(1 - (w_b^* + \widehat{w}_b)/2)y - \sigma = (\widehat{w}_b - w_b^*)y > 0$, whereas $(w_{b2}, \delta_{b2}) = (w_b^*, 0)$ yields $\pi_{b2} = 0$. Therefore $(w_b^*, 0)$ does not solve (D_{b2}). Therefore w_b^* is not a Barter Equilibrium.

(c) If w_b^* is a Barter Equilibrium then, by definition, $w_b^* > 0$ and, since $(w_b^*, 1)$ solves (D_{b1}), $w_b^* \in A_b$. If (6) fails then $\widehat{w}_b \leqq 0$, so by the definition of A_b there is no $w_b^* \in A_b$ with $w_b^* > 0$. Therefore if (6) fails there is no Barter Equilibrium. ||

Proof of Proposition 2 (Existence of Monetary Equilibrium)

(a) First I show that $(\widehat{w}_0, \widehat{w}_1)$ is a Monetary Equilibrium if (7) holds. To this end, assume (7) is satisfied. Then $(\widehat{w}_0, \widehat{w}_1) > (0, 0)$, so it suffices to show that $(0, \widehat{w}_0, \widehat{w}_1)$ is a Stationary Symmetric Equilibrium with respect to $z_m(\ )$. Let $(w_b^*, w_0^*, w_1^*) = (0, \widehat{w}_0, \widehat{w}_1)$ and $\widetilde{z}(\ ) = z_m(\ )$. Then we just need to show that for $k = 1, 2$; (i) $(0, 2 - k)$ solves (D_{bk}) and satisfies (E_{bk}), and (ii) $(\widehat{w}_0, \widehat{w}_1, 2 - k)$ solves (D_{mk}) and satisfies (E_{mk}). (i) For each k, $\pi_{bk} = 0$ for all feasible (w_{bk}, δ_{bk}) since $z_m(\ ) \equiv 0$, and therefore (D_{bk}) is solved by any feasible choice, including $(0, 2 - k)$. Since $z_m(\ ) \equiv 0$ therefore $(0, 2 - k)$ also satisfies each (E_{bk}). (ii) By inspection, $(w_{0k}, w_{1k}, \delta_{mk}) = (\widehat{w}_0, \widehat{w}_1, 2 - k)$ is feasible for each k. For all feasible $(w_{02}, w_{12}, \delta_{m2})$, $(w_{02}, w_{12}) \ngeqq (\widehat{w}_0, \widehat{w}_1) = (w_0^*, w_1^*)$ (by the definition of A_m), so $\delta_{m2} = 0$ and $\pi_{m2} = 0$. Therefore (D_{m2}) is solved, and (E_{m2}) is satisfied, by all feasible $(w_{02}, w_{12}, \delta_{m2})$, including $(\widehat{w}_0, \widehat{w}_1, 0)$. When $(w_{01}, w_{11}, \delta_{m1}) = (\widehat{w}_0, \widehat{w}_1, 1)$, $\pi_{m1} = 0$; whereas for any other feasible $(w_{01}, w_{11}, \delta_{m1})$, $(w_{01}, w_{11}) \leq (\widehat{w}_0, \widehat{w}_1) = (w_0^*, w_1^*)$, so $\delta_{m1} = 0$ and $\pi_{m1} = 0$. Therefore (D_{m1}) is solved by $(\widehat{w}_0, \widehat{w}_1, 1)$. Since the third element is unity, $(\widehat{w}_0, \widehat{w}_1, 1)$ also satisfies (E_{m1}).

(b) Next I show that $(\widehat{w}_0, \widehat{w}_1)$ is the only Monetary Equilibrium if (7) holds. Assume again that (7) holds. Suppose, to the contrary, that (w_0^*, w_1^*) is a Monetary Equilibrium and $(w_0^*, w_1^*) \neq (\widehat{w}_0, \widehat{w}_1)$. Since $(w_0^*, w_1^*, 1)$ solves (D_{m1}) therefore $(w_0^*, w_1^*) \in A_m$. By (7), $(\widehat{w}_0, \widehat{w}_1) > 0$. Therefore, by the definition of A_m, $(w_0^*, w_1^*) \leq (\widehat{w}_0, \widehat{w}_1)$. It follows that $(w_{02}, w_{12}, \delta_{b2}) = ((w_0^* + \widehat{w}_0)/2, (w_1^* + \widehat{w}_1)/2, 1)$ is feasible for (D_{m2}) and hence yields $\pi_{m2} \geqq 0$ (by the definition of A_m), whereas $(w_{02}, w_{12}, \delta_{b2}) = (w_0^*, w_1^*, 0)$ yields $\pi_{m2} = 0$. Therefore $(w_0^*, w_1^*, 0)$ cannot both solve (D_{m2}) and satisfy (E_{m2}). Therefore (w_0^*, w_1^*) is not a Monetary Equilibrium.

(c) If (w_0^*, w_1^*) is a Monetary Equilibrium then, by definition, $(w_0^*, w_1^*) > (0, 0)$ and, since $(w_0^*, w_1^*, 1)$ solves (D_{m1}), $(w_0^*, w_1^*) \in A_m$. If (7) fails then $\widehat{w}_1 \leqq 0$, so by the definition of A_m

there is no $(w_0^*, w_1^*) \in A_m$ with $(w_0^*, w_1^*) > (0, 0)$. Therefore if (7) fails there is no Monetary Equilibrium. ||

Proof of Proposition 3 (Existence of Robust Monetary Equilibrium)

The two inequalities of the premise are (7) and (8) in the text.

(a) First I show that $(\widehat{w}_0, \widehat{w}_1)$ is a Robust Monetary Equilibrium if (7) and (8) are satisfied. To this end, assume (7) and (8) are satisfied. Then $(\widehat{w}_0, \widehat{w}_1) > (0, 0)$, so it suffices to show that $(0, \widehat{w}_0, \widehat{w}_1)$ is a Stationary Symmetric Equilibrium with respect to z_r ($; \widehat{w}_0, \widehat{w}_1$). Let $(w_b^*, w_0^*, w_1^*) = (0, \widehat{w}_0, \widehat{w}_1)$ and $\widetilde{z}( ) = z_r$ ($; \widehat{w}_0, \widehat{w}_1$). Then we just need to show that for $k = 1, 2$; (i) $(0, 2 - k)$ solves (D_{bk}) and satisfies (E_{bk}), and (ii) $(\widehat{w}_0, \widehat{w}_1, 2 - k)$ solves (D_{mk}) and satisfies (E_{mk}). (i) For each k, for all feasible (w_{bk}, δ_{bk}), $w_{bk} \leqq \max\{\widehat{w}_b, 0\}$ by the definition of A_b, and $\max\{\widehat{w}_b, 0\} < \beta \widehat{w}_0 \widehat{w}_1$ by (7) and (8); so $\max\{w_{bk}, w_b^*\} = \max\{w_{bk}, \widehat{w}_b\} < \beta \widehat{w}_0 \widehat{w}_1$, z_r ($\max\{w_{bk}, w_b^*\}$; $\widehat{w}_0, \widehat{w}_1$) $= 0$ and $\pi_{bk} = 0$. Therefore each (D_{bk}) is solved, and each (E_{bk}) satisfied, by any feasible choice, including $(0, 2 - k)$. (ii) Since $z_r (0; \widehat{w}_0, \widehat{w}_1) = 0 = z_m (0)$, therefore the demonstration (in the proof of Proposition 2) that $(\widehat{w}_0, \widehat{w}_1, 2 - k)$ solves (D_{mk}) and satisfies (E_{mk}) for each k when $\widetilde{z}(w_b^*) = z_m (0)$ implies that it also does so when $\widetilde{z}(w_b^*) = z_r (0; \widehat{w}_0, \widehat{w}_1)$.

(b) Next I show that $(\widehat{w}_0, \widehat{w}_1)$ is the only Robust Monetary Equilibrium if (7) and (8) are satisfied. Assume again that (7) and (8) are satisfied. Suppose, to the contrary, that (w_0^*, w_1^*) is a Robust Monetary Equilibrium and $(w_0^*, w_1^*) \neq (\widehat{w}_0, \widehat{w}_1)$. Then there is a w_b^* such that $w_b^* < \beta w_0^* w_1^*$ and (w_b^*, w_0^*, w_1^*) is a Stationary Symmetric Equilibrium with respect to z_r ($; w_0^*, w_1^*$). Let $\widetilde{z}( ) = z_r( )$. Since $(w_0^*, w_1^*, 0)$ solves (D_{m2}) therefore $(w_0^*, w_1^*) \in A_m$. By (7), $(\widehat{w}_0, \widehat{w}_1) > (0, 0)$. Therefore by the definition of A_m, $(w_0^*, w_1^*) \leq (\widehat{w}_0, \widehat{w}_1)$. It follows that $(w_{02}, w_{12}, \delta_{b2}) = ((w_0^* + \widehat{w}_0)/2, (w_1^* + \widehat{w}_1)/2, 1)$ is feasible for (D_{m2}) and hence yields $\pi_{m2} \geqq 0$, whereas $(w_{02}, w_{12}, \delta_{b2}) = (w_0^*, w_1^*, 0)$ yields $\pi_{m2} = 0$. Therefore either $(w_0^*, w_1^*, 0)$ does not solve the problem (D_{m2}) or, since $w_b^* < \beta w_0^* w_1^*$ and hence $\widetilde{z}(w_b^*) = 0$, it does not satisfy (E_{m2}). Therefore (w_0^*, w_1^*) is not a Robust Monetary Equilibrium.

(c) If (w_0^*, w_1^*) is a Robust Monetary Equilibrium then, by definition, $(w_0^*, w_1^*) > (0, 0)$

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and, since $(w_0^*, w_1^*, 1)$ solves (D_{m1}), $(w_0^*, w_1^*) \in A_m$. If (7) fails then $\widehat{w}_1 \leqq 0$, so by the definition of A_m there is no $(w_0^*, w_1^*) \in A_m$ with $(w_0^*, w_1^*) > (0, 0)$. Therefore if (7) fails there is no Robust Monetary Equilibrium. Now suppose that (8) fails. Then $\widehat{w}_b > 0$. To show that there is no Robust Monetary Equilibrium, suppose on the contrary that (w_0^*, w_1^*) is a Robust Monetary Equilibrium. Then there is a w_b^* such that $w_b^* < \beta w_0^* w_1^*$ and (w_b^*, w_0^*, w_1^*) is a Stationary Symmetric Equilibrium with respect to $z_r (\ ; w_0^*, w_1^*)$. Therefore $z_r (w_b^*; w_0^*, w_1^*) = 0$ and the choice of $(w_{b2}, \delta_{b2}) = (w_b^*, 0)$ yields $\pi_{b2} = 0$. However, since the failure of (8) implies $\widehat{w}_b \geqq \beta \widehat{w}_0 \widehat{w}_1$ and the fact that $(w_0^*, w_1^*) \in A_m$ implies $\widehat{w}_0 \widehat{w}_1 \geqq w_0^* w_1^*$; therefore the choice of $(w_{b2}, \delta_{b2}) = (\widehat{w}_b, 1)$ is feasible and $z_r (\widehat{w}_b; w_0^*, w_1^*) = 1$. Therefore $(w_b^*, 0)$ cannot both solve (D_{b2}) and satisfy (E_{b2}). Therefore (w_0^*, w_1^*) is not a Robust Monetary Equilibrium. $\|$

Notes

[2]Rey's (2001) analysis of international currency markets also assumes increasing returns in the transaction technology, coming from a Marshallian externality whereby all traders' costs fall when there is a larger number of traders, rather than from the cost of setting up a shop. In Rey's model the transaction technology looks linear from the point of view of any individual trader, rather than having a discontinuity at zero as in the other papers mentioned above.

[3]The shops of the present paper are also like the "trading zones" of Iwai (1996) except that in Iwai's analysis someone who visits a trading zone must continue to search for a trading partner in that zone, whereas in the present analysis the shops obviate the need for search. Iwai also assumes there is no fixed cost to operating a trade facility.

[4]Several other papers are related. Banerjee and Maskin (1996) is discussed below in footnote 9. Camera (2000) portrays organized exchange as an alternative to monetary exchange. Corbae, Temzelides and Wright (2003) replace the stochastic pairwise matching of search theory with deterministic pairwise matching, but without intermediaries or specialist trading facilities. Hellwig (2000) analyzes equilibria with intermediation under stochastic matching. Laing, Li and Wang (1997) assume that each worker is matched randomly with an infinity of different shops.

[5]Lagos and Wright (2002) provide a possible resolution of these difficulties by assuming that trade takes place partly through unorganized search and partly through organized markets. In their model, unlike in the present model, no medium of exchange is needed in the organized markets.

[6]Engineer and Bernhardt (1991) make a similar argument and note that this implies a rationale for legal restrictions against barter transactions.

[7]The role of this second-order preference for activity is to yield the uniqueness result of Propositions 2 and 3 below. Without it there could exist monetary equilibria in which $w_0^* < \widehat{w}_0$, because a merchant's monetary surplus would not be payoff-relevant. (See footnote 15 below.) This is an artifact of the assumption that merchants live for one period only, and hence have no use for end-of-period money holdings, an assumption I make to avoid strategic complications that arise when merchants can influence next period's prices by carrying over a positive measure of money.

[8]The device of assuming a "divided household" is borrowed from Lucas (1990).

[9]Banerjee and Maskin (1996) present a formal argument along the same lines, in which one good emerges as the universal medium of exchange in a competitive equilibrium with asymmetric information concerning the quality of specific commodities. They assume no fixed cost of operating a shop. As a result, a monetary equilibrium would not exist in their analysis if they were to assume a symmetric distribution of types as in our analysis. Instead they assume the Wicksellian triangle (A-makers eat B, B-makers eat C and C-makers eat A) that many authors have shown leads to monetary exchange by eliminating all double coincidence of wants.

[10]If the space of locations were small enough that there was a positive probability of two households choosing the same location at random, then this assumption would be a way of rationalizing the standard assumption in search theory to the effect that the arrival and identity of a potential trading partner are both random.

[11]Nothing substantive is lost by resticting barter merchants to a uniform offer price. Relaxing this restriction would force obvious modifications to the definitions of equilibrium in section 4 below, but Propositions $1 \sim 4$ would go through unchanged.

[12]This is not to deny that search is important in real-world monetary economies, just that it is necessary for understanding why money is used. A fuller analysis would reintroduce search as it was initially conceived by Stigler (1961), with people searching not for random encounters with other searchers but for shops. The assumption above that each worker can costlessly observe each shop's location, objects and prices rules out this kind of search. The dynamic analysis of Howitt and Clower (2000), which models the emergence of commodity money, depends very much on the cost of searching for shops.

[13]If x and x' are conformable vectors, $x > x'$ denotes $x_i > x_i'$ for all i, $x \geqq x'$ denotes $x_i \geqq x_i'$ for all i, and $x \geq x'$ denotes $x_i \geqq x_i'$ for all i with at least one strict inequality. The reverse inequalities $<, \leqq$ and $\leq$ are defined analogously.

[14]This assumes that the designated monetary merchants will be active next period, which they will be in equilibrium because of their second-order lexicographic preference for activity.

[15]Otherwise, by the definition of A_m, $w_0^* < \widehat{w}_0$, so a monetary merchant of index 2 could choose $(w_{02}, w_{12}, \delta_{m2}) = (\widehat{w}_0, w_1^*, 1)$, which would yield $\pi_{m2} \geqq 0$, whereas in equilibrium it receives $\pi_{m2} = 0$ with $\delta_{m2} = 0$, so its equilibrium actions could not both solve (D_{m2}) and satisfy (E_{m2}).

[16]This is why in the dynamic trading-post model of Hayashi and Matsui (1996), which has no costs of operating a trading post, there does not exist a monetary equilibrium under the configuration of tastes and endowments considered here. Likewise, in the related model of Alonzo (1999), which also does not have any costs of operating trading posts, a monetary equilibrium exists under such a configuration, but only what she calls a "cash-in-advance" equilibrium, in which no one ever considers visiting a barter trading post.

[17]See, for example, Bordo and Jonung (1987).

[18]As is assumed by Starr and Stinchcombe (1998, 1999) and by Howitt and Clower (2000) in their models of commodity money.

[19]A refusal to deliver to any other shop is not privately costly because given the absence of any record-keeping device other than money there is no way the household could be rewarded in the form of increased consumption, now or in the future, for making such a delivery. In the absence of such refusal W would be maximized by having $n/2$ active barter shops ($+1$ if n is odd), which is the minimum number needed for all commodities to be deliverable.

[20]More precisely, a general-equilibrium Clower-Howitt (1978) model, since the lumpy transactions costs faced by households are costs of trading commodities, not financial assets.

[21]Howitt and Clower (2000) represents another step in this same direction.

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