The Microfoundations of the Keynesian Multiplier Process

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1 Introduction

The Keynesian multiplier process is the economist's paradigmatic positive feedback loop, in which an initial departure from full-employment equilibrium cumulates instead of being corrected. The existence of some such positive feedback loop in actual economies is attested to by the typical hump-shaped impulse response pattern of GDP to a random shock in estimated times-series models. For example, Chari, Kehoe and McGrattan (2000) report that quarterly movements in the log of detrended US GDP are well approximated by the following AR2 process:

$$y_t = 1.30y_{t-1} - 0.38y_{t-2} \tag{1}$$

according to which a negative shock that reduces GDP by 1 percent this quarter is expected to reduced it by 1.3 percent next quarter, and by 1.31 percent the following quarter.

As originally formulated by Kahn and Keynes, and as described in most undergraduate textbooks, the multiplier process involves a coordination problem arising from non-price interactions between decentralized transactors. In a world of perfect price flexibility, a drop in planned spending would cause wages and prices to adjust instantaneously so as to keep aggregate demand fully coordinated with productive capacity. But when prices are slow to adjust, one person's drop in spending causes a drop in other people's incomes, causing a drop in their spending, and so on, resulting in a cumulative increase in the gap between demand and capacity.

The theoretical foundation of this multiplier process is still not well understood. Clower (1965) showed how such a process could arise in a Walrasian general equilibrium setting if price adjustment takes place in real transaction time; when labor is in excess supply, unemployed workers will not present their notional consumption demands to the auctioneer but will instead present demands that are constrained by realized sales income. These ideas were pursued at length in the literature on disequilibrium analysis that followed Clower's original contribution and culminated in the book by Barro and Grossman (1976). But this literature raised more questions than it answered, largely because it offered no explicit account of a decentralized market economy's coordination mechanisms. Instead, it modeled price adjustment as if it takes place just the same as in the idealized world of Walrasian economics, where it is led by a fictitious centralized auctioneer, and supposed that while the auctioneer is groping towards equilibrium, transactors are constrained to trade according to rationing rules that are imposed from outside the system by a process that was never even discussed.

One of the supposed advantages of the rational-expectations-equilibrium approach that quickly displaced disequilibrium analysis from its dominant position on the frontiers of macroeconomic theory in the early 1970s was that it did not have to deal with the thorny details of disequilibrium adjustment. Instead it was based on the premise that one can restrict attention exclusively to equilibrium states, in which everyone's beliefs and actions have somehow been coordinated with the beliefs and actions of everyone else. But by adopting this premise, the approach has taken the coordination problem out of macroeconomics, and has denied the very existence of the Keynesian multiplier process, a process which has to do with disequilibrium adjustment rather than with equilibrium behavior.[1]

The present paper re-examines the foundations of the multiplier process making use of an explicit model of the coordination mechanism, one which was developed in an earlier paper with Clower (Howitt and Clower, 2000). The starting point of the analysis is the observation that in reality the job of coordinating trades is performed not by some centralized auctioneer but by commercial enterprises - - retailers, wholesalers, brokers, jobbers, etc. These "shopkeepers" are the visible counterparts of Adam Smith's invisible hand. They create facilities ("shops") in which others can trade at pre-announced prices, in convenient location and during regular hours, ensure that goods are available at times that can be chosen by their clients, and more generally take care of the various logistical details of the exchange process. Shopkeepers are the agents that bear the brunt of adjustment in the face of day-to-day imbalances between spending plans and productive capacity, and accordingly it is to shopkeepers that we should look for an understanding of an economy's coordination mechanisms.

The earlier paper with Clower showed how a coherent network of shops can emerge from competitive evolution. In that analysis, no one has any understanding of the whole economy, yet the adaptive adjustments made by shopkeepers trying to earn profits by serving their individual markets often guide the system to a fully coordinated state, a state which could be modeled as if it were an equilibrium in which everyone's beliefs were indeed based on

[1] Leijonhufvud (1968), Patinkin (1976).

common knowledge of a correct model of the economy.

In the present paper I investigate the real-time dynamics of that same model in the face of disturbances. I show that these dynamics contain within them a multiplier process that produces a hump-shaped impulse-response pattern very similar to that of equation (1) that characterizes the US economy.

The multiplier process takes place because of an institutional factor not usually considered in the literature, namely the exit of trading facilities. A shock that disrupts normal trading relationships can cause some of the businesses that coordinate trades to fail, inducing people who formerly had employment relationships with those businesses to curtail their expenditures for lack of money, which forces closing of other businesses in cumulative fashion. Although new shops will enter and eventually take the place of those that failed, this process takes time. New entrepreneurs will perceive a gain from entering to replace failed shops, but not all of them will succeed. The shakeout period that generates replacement shops will typically be a turbulent one, since no agent possesses the knowledge, authority or incentive to coordinate the coordinators. New entrants will have to base their entry decisions and prices more on animal spirits than experience until a new stable pattern of trade is established.

The analysis challenges a common belief among macroeconomists to the effect that coordination problems arise only when there are impediments to price adjustment. It shows instead that price-stickiness is not in fact critical to the operation of the multiplier process. That is, when a shop fails for lack of demand, there will be unemployed suppliers to these shops who can no longer attain the means of payment necessary to buy goods in other shops. These other shops will be in risk of failing until a replacement has been found for the original failed shop. Speeding up the process of price adjustment does not cure this problem, because what is lacking is not the ability to find market clearing prices but rather the institutional apparatus for facilitating exchange. Indeed, as we shall see, faster price adjustment can actually hinder the recovery process by adding more noise to an already confused situation, and a temporary shock does not require any price to change in the long run in order for full employment to be restored.

2 The basic model

The model sketched below is one of a truly decentralized economy, in the sense that each actor behaves according to an opportunistic and myopic rule that presupposes no knowledge of the overall economy and is predicated on purely local information. The model is identical to that of Howitt and Clower (2000), except that here I suppose that a common medium of exchange has already been determined. The reader is referred to Howitt and Clower for details of the model.

I begin with a brief overview. The model portrays an economy as a collection of individuals, each endowed with one commodity and wanting to consume another, over an infinite sequence of periods. Logistical problems prevent people from trading directly with one another. From time to time it occurs to someone to create a shop, in which one commodity can be traded for another. The shops are hard to locate. Someone that locates one can form an ongoing trading relationship with it. The shops post prices at which they will buy and sell, which they adjust according to adaptive rules aimed at achieving a desired profit level. But there are fixed costs to operating a shop, and hence a shop that fails to attract enough patrons on each side of the market will fail to cover its fixed costs, and will eventually go out of business.

To be more specific, there are N transactors and n commodities. Time is discrete, indexed by t = 1,...,T, each unit being a "week." Commodity number 1 ("wheat") has emerged as a universal medium of exchange. (See Howitt and Clower for how this might happen.) All commodities are perishable, and trading takes place strictly within the week. Each transactor can eat only one of the commodities (his "food") and is endowed each week with enough labor services to produce one unit of a particular commodity (his "manna"), which is never his food. A transactor whose manna is i and who eats j is said to be of type (i, j). For each of the $n(n-1)$ ordered pairs (i, j) of commodities, there is the same number b of transactors of type (i, j). The population of the economy is thus $N = bn(n-1)$.

Because no transactor eats his own manna, he must trade to eat. I assume that trading is such a costly affair that it can only take place through an organized facility called a "shop." Also, to trade with a shop a transactor must form a trading relationship with it. Each shop is capable of dealing in only two commodities, one of which is wheat. Each transactor may have ongoing trading relationships with at most one shop (his "employer") that deals in his manna, and at most one shop (his "store") that deals in his food. Each week each transactor delivers his unit endowment to his employer if he has one; if he is neither a wheat-eater nor a wheat-maker then he delivers the wheat received from his employer to his store in exchange for food.

Each shop posts a pair of prices for the "good" (the commodity other than wheat) that it deals in; a wholesale price, or "wage," w and a retail price. I denote by p the inverse of a shop's retail price. The shop agrees to buy all goods delivered to it at the price w and all wheat delivered at the price p.

Every week some transactors search for information about possible trading relationships. Specifically, a searcher gathers a sample of shops, some through direct observation of potential shop locations, and some through contact with other transactors. If he finds one offering a higher wage for his manna than his current employer is offering he will switch to the new employer. If he finds one with a lower retail price for his food than his current store is charging then he will switch to the new store.

Shops can be opened only by transactors that innovate - "entrepreneurs." To keep the analysis simple I suppose that only wheat-eaters can be entrepreneurs, and that the good a shop trades must be the entrepreneur's manna. Each week a certain number of wheat-eaters are randomly struck by an idea for opening a shop. There is a psychic setup cost of opening the shop, defraying which requires a weekly increment of c units in the owner's wheat consumption and a weekly consumption of s units of the good it trades. The shop also incurs a weekly fixed operating cost of f units of the good it trades. Before incurring the setup cost a prospective entrepreneur consults a small number of transactors that might adopt the newly-created shop as a store and a few others that might use it as an employer. If this market research indicates sufficient strength on both sides of the market the shop will open; otherwise the opportunity lapses.

Motivated by pursuit of gain, but lacking reliable information about the relation of price to profit, the shop posts prices that promise to cover its fixed costs, including compensation for the setup cost of operating the shop, provided that it succeeds in attracting its target delivery levels ("targets" for short). Thus it will set w and p such that:

$$w = \frac{\widehat{m} - c}{\widehat{q}} \text{ and } p = \frac{\widehat{q} - f - s}{\widehat{m}} \tag{2}$$

where $\widehat{q}$ and $\widehat{m}$ are the respective targets for goods and wheat, except that when one of these formulas is negative the corresponding price will be set equal to zero. The first equation follows from the condition that the firm's expected trading profit - - its expected revenue $\widehat{m}$ minus its expected wage bill $w\widehat{q}$ - - be just enough to compensate for the setup cost. The second equation has an analogous interpretation.

When a shop opens, the entrepreneur picks initial targets at random, from a uniform distribution over the set $\{1, 2, \ldots, xMax\}$. The outcome of these draws represents the entrepreneur's "Animal Spirits." Each period after that he adjusts his target according to a simple adaptive scheme:

$$\Delta\widehat{q} = \alpha\left(q - \widehat{q}\right), \quad \Delta\widehat{m} = \alpha\left(m - \widehat{m}\right) \tag{3}$$

where q and m are the actual deliveries and the parameter α representing the speed of adaptation lies between 0 and 1.

A firm may decide to exit when its actual operating profit becomes less than c, or when it is persistently unable to pay its fixed operating cost. More specifically, the shop's "operating surpluses" are:

$$\pi_m = m - wq \text{ and } \pi_q = q - pm - f \tag{4}$$

Any week in which either of these surpluses is negative the entrepreneur will exit with fixed probability θ. Otherwise it will remain in business for at least another week with certainty. Note that the shop always sets prices aimed at yielding operating surpluses that are large enough (c and s) to cover its annuitized setup costs.

When either operating surplus is negative the shop confronts what is, in effect, a stockout problem: whether and how to honor immediate customer demands. In actual economies, firms deal with impending stockouts by depleting inventories, producing overtime, lengthening delivery lags and making emergency purchases from competitors. Since none of these remedies fits easily into my story I evade the stockout issue at this point by supposing that entrepreneurs always honor their customers' demands, engaging when necessary in negative consumption.

2.1 Weekly timeline

The model sketched above is has been implemented as a computer program, written in the C programming language.[2] The program is almost identical to the one that implements the model of Howitt and Clower (2000), where more details are provided. It represents the economy as an algorithm. The initial conditions each week consist of a certain number of established shops, the goods they trade and their predetermined targets, and a historically given configuration of ongoing trading relationships between transactors and shops. It then proceeds to generate a set of initial conditions for the following week, in six stages, each of which represents an important component of the workings of a decentralized economy. First, each shop sets its prices for the week according to formula (2). Second, a certain number of wheat-eaters experience an opportunity to become entrepreneurs, which they do depending on the results of their market research, as indicated above; they also set their prices for the week according to (2). Third, a certain fraction of transactors are given the opportunity to search and thus form or switch trading relationships, which they do as indicated above. Fourth, they trade according to the scheme described above. Fifth, shops for whom one of the operating surpluses (4) realized in trading during the previous stage is negative randomly exit. Sixth, surviving shops revise their targets according to formula (3). The algorithm repeats for T weeks.

3 Equilibrium

As a reference point, I describe an equilibrium in which each transactor is either an entrepreneur or has an employer and a store, all prices are constant over time, and all trading relationships remain unchanged. There are $n-1$ shops in the equilibrium, one for each good. (The fixed costs imply a natural monopoly for each good). Each shop receives a weekly goods delivery equal to the total number of transactors endowed with the good it trades:

$$q^* = (n-1)\, b$$

It receives a weekly wheat delivery equal to b from its wheat-endowed customers and $w^* b\,(n-2)$ from the rest, where w^* is the common equilibrium wage. Plugging these delivery quantities

[2] The source code will be posted on my webpage: http://www.econ.brown.edu/fac/Peter_Howitt.

into (2) yields the unique equilibrium prices:[3]

$$w^* = \frac{b-c}{b} \text{ and } p^* = \frac{(n-1)\,b - f - s}{(n-1)\,b - (n-2)\,c} \tag{5}$$

As long as animal spirits ($xMax$) are not too high, the equilibrium will be stable against entry. This is because in order to pass the stage of market research a prospective entrepreneur would have to offer prices (w, p) greater than (w^*, p^*), but to do this he would have to suppose that he will receive even more deliveries, of both goods and money, than the existing monopoly. If $xMax < (n-1)\,b - (n-2)\,c$ this can never happen. I make this assumption below. An equilibrium is thus an absorbing state of the algorithm.

Computer simulation shows that this equilibrium is the algorithm's unique long run outcome. That is, even if we start in week 1 with no shops and no customer relations, the economy eventually converges each time to the equilibrium.

Moreover, it is easily verified that the equilibrium constitutes a Pareto efficient allocation of resources in the economy given that all trading must occur through shops. Aggregate GDP in the economy is total consumption. Capacity GDP is the sum of all endowments minus the operating costs of the shops. The smallest number of shops consistent with everyone trading is $n-1$, so that capacity GDP equals $N - (n-1)\,f$. This is achieved in an equilibrium because all endowments are delivered to a shop and either used to pay the operating cost, or paid out to a customer who consumes it, or consumed by the shop's owner to defray his setup cost.

Thus the network of private markets operated by entrepreneurs is self-organizing and self-adjusting. Everyone in the system is acting on purely local knowledge, implementing behavioral rules that can be formulated without any understanding of the overall system. Yet the interplay of their independently formulated actions is to bring about a state of market organization that coordinates their trading activities with no deadweight loss.

[3] I assume parameter values such that these prices are stictly positive.

4 The multiplier process

I am interested in the system's response to various shocks. In particular, I want to study what happens when people reduce their demands for some products, without immediately signalling to anyone what they are planning to demand instead of these products. This is the classic coordination problem that Keynes wrestled with. Consumers may decide to spend less than their income, but this does not amount to a specific demand for future consumption. Instead, their future demands remain latent, and entrepreneurs must somehow discover them through trial and error. Likewise, unemployed workers' notional demands remain undiscovered until some entrepreneurs find it in their interest to employ the workers and thereby provide them with the means of making their demands effective.

To portray such a shock in the above system, I suppose that at a certain date some fraction of the population switches from eating one good to another. To preserve the aggregate structure I suppose that the total number of each type remains constant, so that for every i-eater that becomes a j-eater there is a j-eater that switches to i. At the time of this shock, each switcher is suddenly without a store, and his former store loses a customer. The switcher may continue to sell his manna to his employer but he does not spend his wages. GDP falls because of the reduced goods consumption of the switchers that no longer show up to their former stores, and because of the reduced wheat consumption of the entrepreneurs whose operating surplus in wheat suddenly falls.

Because their revenues have fallen, the former stores of switchers will reduce both their wages and their retail prices, according to (2). The fall in wages will help to offset their profit shortfall, but it will spread the shortfall to other shops, some of whose customers will now deliver less wheat because their wages have fallen. Meanwhile, the fall in wages and prices will do little by itself to raise GDP, which will stay below capacity until the switchers find new stores.

During this process, the luck of the draw may result in particularly large shortfalls for some shops. A shop whose wheat surplus has fallen below zero will be at risk of failure. If that happens then all of the former suppliers of the failed shops will be without an employer, and their sudden drop of wage income will result in a sudden drop in revenues to their respective stores, who may also now become at risk of failure. In other words, the collapse of shops can be self-reinforcing, leading to a cumulative fall in GDP as in the more familiar multiplier process of textbook macroeconomics.

Of course whenever a shop fails, new entrepreneurs will start entering, and employment relations will start to form again. But because of fixed costs, and because a lot of firms may enter the same market, there will be a “shakeout period” which not all new entrants will survive. Thus the process of shop failures may continue for some time before a new stable pattern of shops re-emerges and the economy begins to recover from the cumulative downturn.

4.1 Numerical analysis

To illustrate this process I have computed thousands of numerical examples. Starting from an initial position of equilibrium I suppose that a fraction γ of non-wheat-eaters switch preferences, and then I follow the evolution of the percentage GDP gap over 250 periods. During each run of the model I retain the following parameter values:

N	number of transactors	7350
n	number of commodities	50
$xMax$	animal spirits	50
λ	fraction who search each week	1
θ	weekly exit probability of a firm at risk of failing	0.03
f	fixed operating cost	4.8
s	annuitized setup cost in goods	1.2
c	annuitized setup cost in money	2
I	number of potential innovators each week	10

In the baseline run I also set the speed of adaptation of targets α equal to 0.75.

No attempt has been made to calibrate these parameter values to real world data. The values were chosen arbitrarily, with some brief trial and error to ensure asymptotic convergence. (Ongoing research is aimed at calibrating the model and testing for sensitivity.) Yet, as we shall see, the model generates a hump-shaped impulse response pattern very similar to the equation (1) that was fitted to actual U.S. data.

Because the algorithm described above involves many random elements (where to look for shops, which person is given an opportunity to form a shop each period, etc.) there is considerable variation from one run to another even when parameter values are fixed. Thus I ran 10,000 simulations, each time using the same parameter values as listed above, three different times, for γ equal to 0.04, 0.08 and 0.12 respectively. Figure 1 shows what happens on average each time to GDP, relative to its capacity level.

As Figure 1 indicates, the average trajectory following a shock has the hump-shaped pattern characteristic of real-world data. GDP starts to fall, reaching a trough after about 30 weeks, before returning monotonically to its capacity level. Figure 1 also depicts what would happen according to the fitted equation (1). In all cases the model agrees closely with this fitted equation.

Figure 1 shows no evidence of a corridor effect.[4] That is, the response seems to mimic the same linear process (1) no matter how large or small the initial shock, as measured by γ. It is also independent of the nature of the initial shock, as measured by the variance or skewness of the distribution of initial demand reduction across the shops in the economy.

What seems to matter most for the response pattern is the incidence of business failures. When I shut down the exit process by reducing the exit probability θ to zero there is no discernible drop in GDP and no multiplier process. Across the runs depicted in Figure 1 there was a strong correlation between business failures and the magnitude of recession as measured by the maximal GDP gap during a run. This correlation is showed in Figure 2 below. Runs in which there were no failures had a maximal GDP gap very close to zero. As failures rose so did the maximal gap. Figure 2 also shows that this relationship between failures and amplitude is invariant to the size of shocks.

As Figure 2 indicates, the average impulse response pattern of Figure 1 hides a great deal of variance. When few failures occurred the macro-economic effect was negligible, but when a lot of failures occurred the recession was deep and prolonged.

Finally, to test for the effects of price flexibility, I varied the speed of adaptation α of a shop's targets, from 0.25 to 1.0. The results are shown in Figure 3. There is no monotonic

[4] Leijonhufvud (1973).

relationship between α and either the average number of failures or the average size of the maximal percentage GDP gap. When α reaches its upper limit of 1 the number of failures increases rapidly, because it implies that new entrants will be impatient; unless lucky they learn quickly that they have so few customers they are unable to cover their costs at any price, and therefore drop out. Nevertheless the maximal gap is smallest when $\alpha = 1$ because firms that aren't destined to fail adjust most rapidly then.

5 Summary and Conclusion

The main point of this paper has been to illustrate the important but much neglected role that firms play in economic life by virtue of their activities as market makers rather than as producers. I sketched an idealized economic system in which all trade takes place through the intermediary of shops, and in which these shops behave according to simple adaptive rules rather than according to elaborate maximization procedures. In such a system one can see at work a version of the Keynesian multiplier process, which arises because of the induced increase in business failures and resulting disruption of trading relationships.

The degree of price flexibility has little to do with the amplitude of the economy's reaction to a negative demand shock. This is a result that Keynes emphasized in his *General Theory*.[5] Here it occurs for reasons unrelated to those given by Keynes. That is, the business failures at the heart of the multiplier process arise not so much because businesses are charging the wrong price as because they have too few paying customers. Restoring equilibrium involves restoring severed trading relationships, a process that involves search, entry and exit. Price adjustment does little to hasten this process.

[5] See also Tobin (1975), DeLong and Summers (1986) and Howitt (1986).

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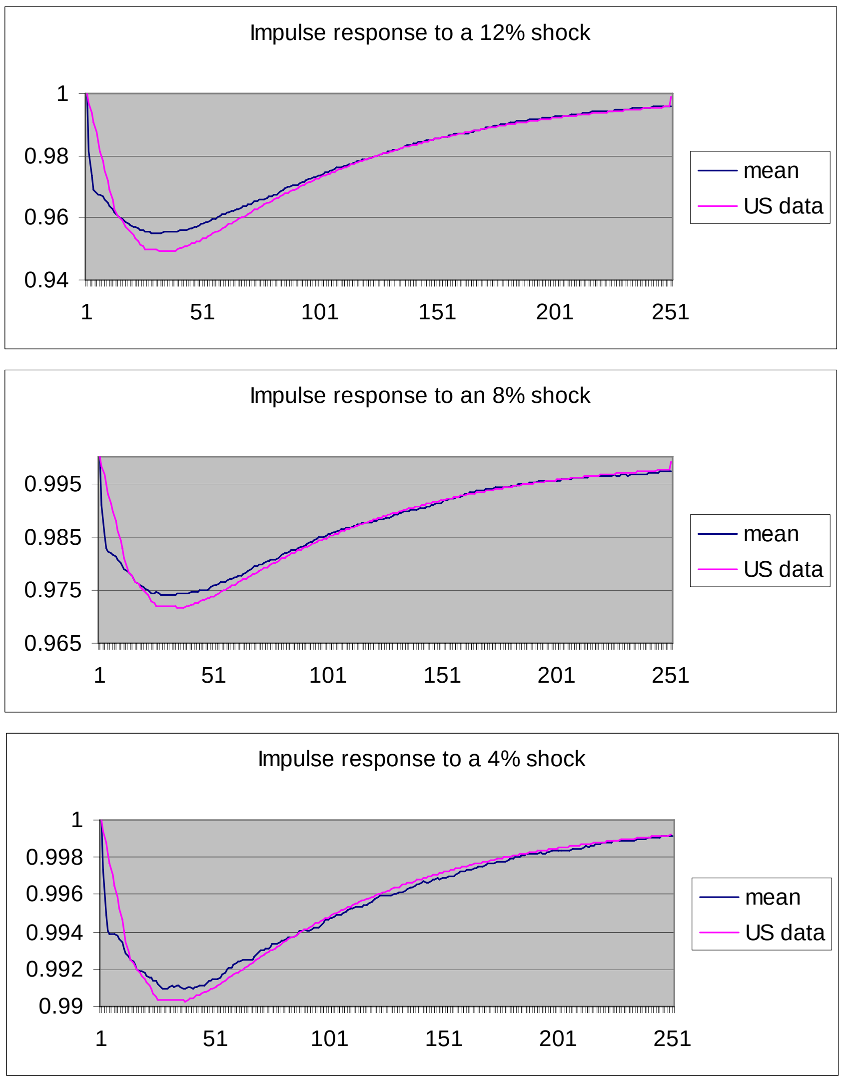

Figure 1: Response to a negative demand shock

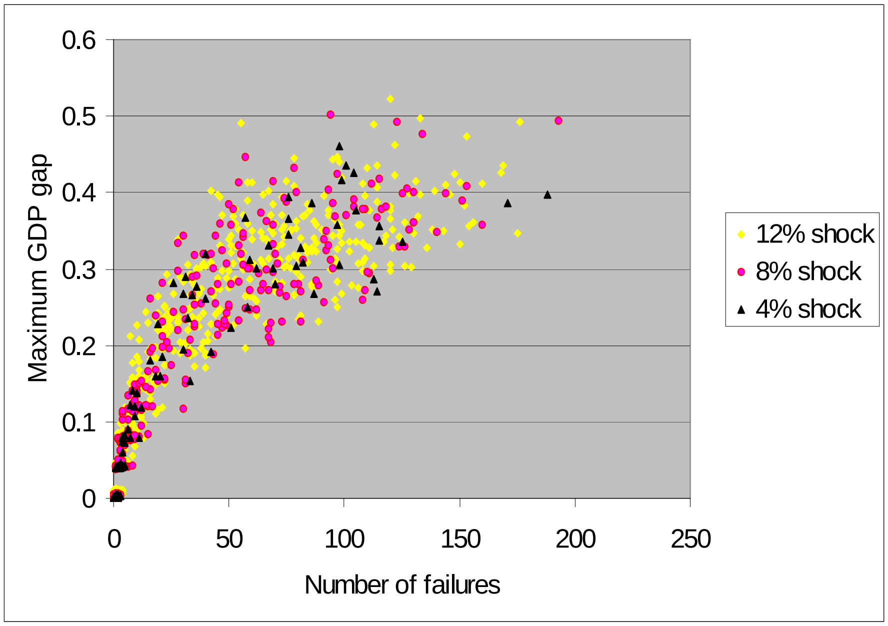

Figure 2: Business failures and depth of recession

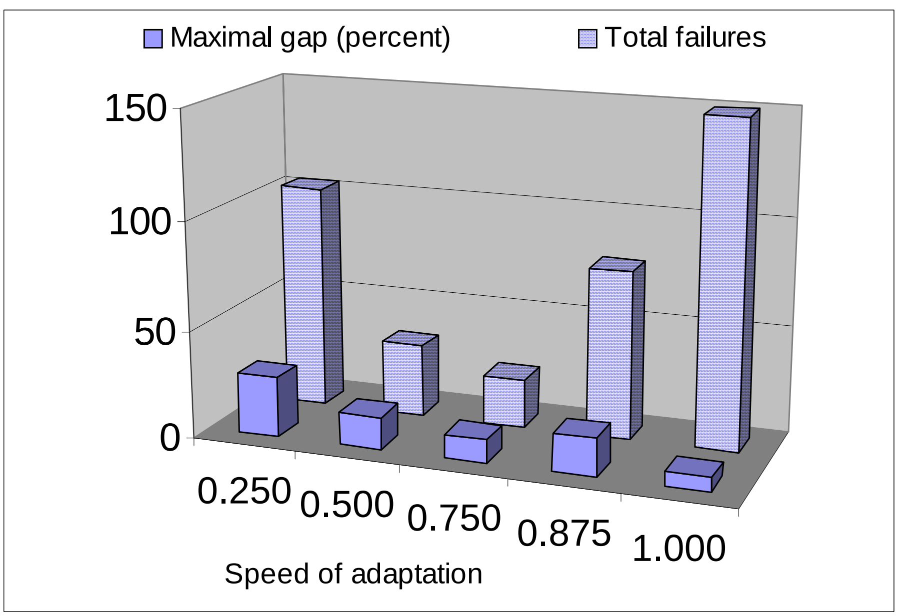

Figure 3: The effect of price flexibility on failures and depth of recession