Value of Life, Value of Time, and Constant Relative Risk Aversion Utility

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Abstract

This paper develops two straightforward value of life models; one is a probabilistic value of life model and the second is a deterministic value of time model. Simplifying assumptions allow both models to be solved analytically. Constant relative risk aversion utility functions are used, and both value of life and value of time are solved for as functions of the relative risk-aversion parameter.

Keywords

Value of life, value of time, constant relative risk aversion utility

Classi¯cation Code

D11, D61, D6, D91

I. Introduction

The probabilistic willingness-to-pay-to-avoid-fatal-risk approach has become the standard method for valuing human lives in economic theory. This approach starts with following essential question: Suppose a consumer has an opporunity to buy a reduction ² in her probability of dying. What is the maximum C she would pay for the extra survival probability? If she would pay at most C for ²; then we say she \values her life" at C=²: (For major contributions and useful surveys, see, e.g., Jones-Lee (1974), Jones-Lee (1976), Mishan (1971), Mishan (1982), Schelling (1968), Schelling (1987), Viscusi (1992), and Viscusi (1993).)

The basic form of the probabilistic value of life model has two periods (time zero, when the plan is made; time one, when the outcome is known) and two possible states in period one (alive; dead). This form has been extended by a number of authors to either multiple periods or continuous time. (See, for example, Jones-Lee (1976), Kenyon and McCandless (1984), and Moore and Viscusi (1988).) In reality, of course, the decisions consumers make at time zero often a®ect survival probabilities at every instant of time over many future years. However, modeling this type of dependency becomes di±cult or impossible without major sympli¯ng assumptions.

An alternative to the probabilistic value of life model is developed in Ehrlich and Chuma (1990). The Ehrlich/Chuma model is a complex dynamic optimization certainty model in which a consumer chooses consumption, health, work time, and several other °ow variables, as well as length of life, so as to maximize discounted lifetime utility from consumption and health.

Reduced to one of its core parts, the Ehrlich/Chuma model becomes a deterministic value of time model, with the following essential question: Suppose a consumer has an opportunity to buy an increase ² in her (certain) length of life. What is the maximum C she would pay for the extra longevity? If she would pay at most C for ², then we say she

\values her time alive" at C=^{2}:

Of course, what people are willing to pay for additional survival probability (in an uncertainty context) or additional years of life (in a certainty context), depends crucially on the nature of their utility functions. Perhaps the utility function characteristic that is of greatest importance is risk aversion. In fact, Jones-Lee (1980) relates the probabilistic value of life model to a risk-aversion measure he calls asymptotic risk-aversion, which is in turn related to the Arrow-Pratt measure of absolute risk aversion.

The purpose of this paper is to formulate two simple models | a probabilistic value of life model, and a deterministic value of time model, to structure the models in su±ciently simple ways that they can be easily solved for the value of life and the value of time, and to relate the value of life and the value of time to Arrow's measure of relative risk-aversion (see Arrow (1971), Chapter 3).

The models are elementary enough that both value of life and value of time will be solved as simple functions of the relative risk-aversion measure, as well as a few other crucial variables and parameters. At the end of the paper I will present some computed values for value of life and value of time, contingent on relative risk aversion, and I will show how the value of life and the value of time compare to each other.

II. The Probabilistic Willingness-to-Pay Value of Life Model

This is a two-period model, with a \before" and an \after", or ex ante and ex post. A two-period model is clearly less realistic than a multi-period model or a continuous time model, but the lack of realism is compensated for by tractability. \Before" is called time zero; \after" is called time one.

At time zero, an individual is contemplating her state at time one. In that future period, she will be either alive or dead. If alive, her utility (as contemplated from time zero) will depend on the amount of money she has to spend. If dead, her utility (as contemplated from time zero) will depend on the amount of money in her estate, and on a factor that represents her "fear of death."

Our individual is endowed with a given sum of money at time zero. She may use it in three ways: (1) She may hold it to spend on consumption at time one, or to bequeath, as the case may be. The money she uses this way is called consumption. (2) She may spend it at time zero on precaution. By spending it on precaution, she increases the probability that she will be alive at time one. (3) She may spend it at time zero one on life insurance. By spending it on life insurance, she increases her bequest by the face value of whatever life insurance policy she buys.

I assume that the individual has an expected utility function which she wants to maximize. Her maximization problem is to choose the sums of money used in the three ways detailed above, so as to achieve the highest ex ante expected utility.

For this model I use the following notation:

x	=	amount of money to be spent on consumption (or bequeathed if dead) at time one
y	=	money spent on precaution at time zero
z	=	money spent to purchase life insurance at time zero
$\bar{x}$	=	$x + y + z$ = cash endowment at time zero
$p(y)$	=	probability of life at time one
$1 - p(y)$	=	probability of death at time one
$V(y; z)$	=	face value of life insurance policy
$v(y)$	=	price of insurance, per dollar of face value
$f(x)$	=	ex ante utility contingent on life
$g(x + V)$	=	ex ante utility contingent on death
A	=	bequest parameter, or "altruism"
K	=	"fear of death" parameter

As in a typical probabilisitic value of life model, I assume the subject chooses $x; y$ and z so as to maximize expected utility

$$Eu = p(y)f(x) + (1 - p(y))g(x + V(y; z));$$

subject to the constraints

$$\bar{x} = x + y + z; \quad x, y \geq 0; \quad \text{and} \quad x + V \geq 0.$$

I don't require that z be nonnegative because it is analytically simpler to allow individuals to buy negative quantities of life insurance (i.e., rather like annuities, translated into the structure of this model). I do require that consumption, precaution, and the estate $x + V$ be non-negative.

To solve the model, I make the following special assumptions:

<u>Assumption 1</u> (Actuarially fair life insurance).

$$V = \frac{z}{1 - p(y)}.$$

That is, the premium for the life insurance policy z equals the expected payout, $V \cdot [1 - p(y)]$. To put it slightly differently, the price for a dollar's worth of insurance is $v = 1/(1 - p(y))$.

There are two possible alternative assumptions about insurance companies' pricing policies: They may be able to observe each individual's degree of precaution y, and price accordingly (offering each individual a schedule of rates, depending on y). Or they may not (offering an individual one price, $1/(1 - p)$). The latter assumption produces a slightly simpler model, and I will follow it here. It implies that, when choosing x, y, and z, the utility-maximizing individual assumes $\frac{\partial V}{\partial y} = 0$, or, to say the same thing, she views the price v of a unit of insurance as a constant, rather than as a function of y.

<u>Assumption 2</u> (Utility if dead).

$$g(x + V) = Af(x + V) - K,$$

where A and K are non-negative constants. That is, the utility from the dead state is comprised of (a) a scaled up (or down) version of utility if alive, minus (b) a fear of death parameter K.

Since the formulation of assumption 2 is somewhat arbitrary, it warrants some comments. First, it is reasonable to assume that an individual cares about the size of her bequest $x + V$; because she is concerned about her heirs. Her concern might in theory be reflected by any type of function of $x + V$; but assuming it is given by a scaled version of her utility if alive function seems to me to be a plausible simple specification, and perhaps the least arbitrary specification. Second, the scale factor A is now a measure of the individual's concern for her bequest. If she is very concerned about the welfare of her heirs, A might be larger than 1. If she places the same weight on the financial well-being of her heirs as she places on her own financial well-being, she might have $A = 1$: If she has no dependents and no interest in a bequest, $A = 0$: Therefore, it is reasonable to call A the altruism or the bequest parameter. Third, it is reasonable to assume that there is another aspect to utility if dead that does not depend on the size of the bequest. Her heirs get the money, but she is dead and she can't take it with her! There is an undesirable property of deadness that should be independent of the size of the bequest. The simplest way to model this, it seems to me, is to assume a negative constant $-K$ in the utility if dead function.

Assumption 3 (Constant relative risk aversion utility).

$$f(x) = \frac{x^{1-\sigma} - 1}{1 - \sigma} \quad \text{for some constant } \sigma > 0:$$

Note that σ is Arrow's relative risk aversion measure.

A few comments about assumption 3 are appropriate. First, note that for $\sigma < 1$; $f(x)$ is simply an exponential utility function. Second, for $\sigma = 1$ the assumption 3 formula makes $f(x)$ undefined. However, for any $x > 0$; $\lim_{\sigma \to 1} f(x) = \ln x$: Throughout the paper I will allow consideration of $f(x)$ with $\sigma = 1$; and by this I mean the limiting value, namely $\ln x$: Third, the common constant relative risk aversion formulation $f(x) = \frac{x^{1-\sigma}}{1-\sigma}$ is not used here for two reasons: (a) for any fixed x; it has no limit as $\sigma \to 1$; and (b) if $\sigma > 1$ it is negative for all $x > 0$:

Negative utility is problematic in the buying time model developed in the next section. (Why buy a longer life if life is painful?) The function $f(x)$ of assumption 3 has the virtue that, at least for $x > 1$, $f(x) > 0$ for any $\sigma > 0$.

When assumption 3 is not made, I assume that the function $f(\cdot)$ satisfies the following regularity conditions: For $x \geq 0$, $f(\cdot)$ is differentiable, strictly increasing and strictly concave. For $x \geq 1$, $f(\cdot)$ is non-negative.

<u>Assumption 4</u> (Survival probability).

The function $p(\cdot)$ satisfies the following regularity conditions: For $y \geq 0$, $p(y)$ is positive, differentiable, strictly increasing and strictly concave.

Note that $p(0) > 0$ is assumed. This means if subject spends nothing on precaution, her survival probability is positive. Also, by assuming that $p(\cdot)$ is only a function of y, I am excluding a conscious choice of suicide, which would complicate assumption 1. Also, since $p(\cdot)$ is a probability, it is bounded above by 1.

III. Solving the Probabilistic Willingness-to-Pay Value of Life Model

In general terms, our individual wants to maximize expected utility subject to certain constraints. Using only assumptions 1 and 2, the problem is to maximize:

$$E(u) = p(y)f(x) + (1 - p(y))[Af(x + vz) - K], \text{ subject to}$$

$$x + y + z = \bar{x}, \quad x, y \geq 0, \quad \text{and} \quad x + vz \geq 0.$$

First-order conditions for maximization lead to the following equations, assuming an interior maximum:

$$(1) \qquad f'(x) = Af'(x + vz).$$

$$p'[f(x) - (Af(x + vz) - K)] = f'(x) = \lambda: \tag{2}$$

Here the primes denote derivatives, and λ is the Lagrange multiplier associated with the budget constraint.

Note that the first half of equation 2 can be rewritten:

$$\frac{1}{p'} = \frac{1}{f'(x)}[f(x) - (Af(x + vz) - K)]:$$

The intuitive interpretations of p' and $(p')^{-1}$ are important: p' is the increase in survival probability resulting from an incremental dollar of precaution expenditure y; for the utility maximizing individual who is choosing y along with x and z: Its inverse is the increase in dollars of precaution expenditure per incremental unit of survival probability. But this is the standard economics notion of the probabilistic value of life, which is written VOL in what follows. Hence,

$$VOL = \frac{1}{f'(x)}[f(x) - (Af(x + vz) - K)]: \tag{3}$$

Note that equation (3) has a simple intuitive interpretation:

$$VOL = \frac{\text{Utility advantage from being alive over being dead}}{\text{Marginal utility of money spent on consumption}}: \tag{4}$$

Let me briefly consider the possibility of a non-interior solution to the expected utility maximization problem, which would occur if any of the inequality constraints are binding. I will show below that the $x + vz \geq 0$ constraint will be satisfied under assumption 3 of the model; and the $x \geq 0$ constraint should not be binding for relevant variable and parameter ranges. However, the $y \geq 0$ constraint may be binding. In the case where $y = 0$ at an expected utility maximum, equation (1) remains as is, but equation (2) is replaced by

$$p' \left[f(x) - (Af(x+vz) - K) \right] \leq f'(x) = \lambda$$

where the derivative p' is evaluated at $y = 0$.

It follows that when the optimal $y = 0$, it is possible to have

$$\frac{1}{f'(x)} \left[f(x) - (Af(x+vz) - K) \right] < \frac{1}{p'}.$$

In this case I will continue to interpret the left-hand side of the inequality as VOL; and I will interpret the inequality as saying VOL is less than the incremental cost (in terms of precaution expenditure) per incremental unit of survival probability. This is consistent with a choice of $y = 0$. This usage, in fact, allows for the possibility of negative VOL's (for individuals whose ex ante utility if alive is less than ex ante utility if dead).

At this point I use assumption 3. When combined with equation (1) above, it gives:

(5) $$x + vz = A^{\frac{1}{\sigma}} x, \text{ or } V = vz = (A^{\frac{1}{\sigma}} - 1)\, x.$$

It follows that the non-negative estate constraint $(x + vz \geq 0)$ is automatically satisfied as long as $x \geq 0$, because of the $A \geq 0$ assumption.

Next, substituting for $f(x)$ and for $x + vz$ in equation (3) and rearranging terms gives:

(6) $$VOL = x^{\sigma} \left[\frac{A - 1}{1 - \sigma} + K \right] + x\, \frac{1 - A^{\frac{1}{\sigma}}}{1 - \sigma}.$$

In the event that $A = 0$ (no bequest motive),

(7) $$VOL = x^{\sigma} K + \frac{x - x^{\sigma}}{1 - \sigma}.$$

Note that when $\sigma \to 1$, equation (6) approaches:

$$VOL = x\,[(1 - A)\ln x - A\ln A + K]. \tag{8}$$

If $A = 0$, equation (8) becomes:

$$VOL = x\,[\ln x + K]. \tag{9}$$

IV. The Deterministic Willingness-to-Pay Value of Time Model

There are several features of the probabilistic value of life model that are discomforting. First, it is based on an ex ante evaluation of a state — the dead state — for which there is no ex post evaluation. The utility if dead function is inescapably peculiar; many people have a hard time grasping what it means. It is not a good empirical concept. By the same token, the fear of death parameter K is intrinsically vague and odd. To paraphrase the ancient Greek philosopher Epicurus, we fear death awfully when we are alive, but when we fear it we are alive, and when we are dead we will not fear it anymore. Second, the VOL measure makes the value of life a function of parameters A (altruism) and K (fear of death). It is quite possibly for $\partial VOL/\partial A$ (x constant) to be negative, an odd result, and it is certainly the case that $\partial VOL/\partial K$ (x constant) is positive. So VOL may fall as altruism rises, and VOL definitely rises as fear of death rises. It may be undesirable to base social policies on a welfare measure that rises as fear rises and falls as altruism rises.

An alternative approach that avoids some of the troubling aspects of the probabilistic value of life model is the deterministic buying time or value of time model.

The simple model below is based in part on Jones-Lee (1976), Kenyon and McCandless (1984), and Moore and Viscusi (1988), but mostly on Ehrlich and Chuma (1990). The subject decides on consumption, life extension and bequest. Life extension expenditures, instead of increasing survival probability as in the probabilistic model, increase the length of her life. So she reveals her willingness-to-pay for years of life.

At this point, I will modify the notation and definitions of section II. I will use the following notation and definitions:

Lu	=	lifetime utility
x	=	rate of consumption per unit time, assumed constant over the lifetime
y	=	money spent by the subject on life extension, assumed spent in one lump sum at time zero, when the plan is made
z	=	bequest, assumed set aside in one lump sum at time of zero, when the plan is made.
$\bar{x}$	=	cash endowment
A	=	the bequest or altruism parameter. $A = 0$ indicates the individual has no interest in the size of her bequest, whereas $A > 0$ indicates the she gets utility from her bequest.
T	=	length of life.
u(x)	=	instantaneous utility function.

Assume for simplicity that the instantaneous utility function $u(\cdot)$ is constant over the lifetime, and satisfies the usual regularity assumptions; assume further that the individual's discount rate and the market interest rate are zero. (It is possible to assume a positive individual discount rate and a positive market interest rate, but this produces a significantly more complex model, the conclusions of which are generally similar to what is shown below.) To make the model easy to solve, suppose utility from the bequest z is $Au(z)$. (This is parallel to the bequest part of assumption 2 in section II.) Note that with no uncertainty there is no place for life insurance.

Now the subject chooses $x, y, z \geq 0$ so as to maximize her certain lifetime utility:

$$(10) \qquad Lu = \int_0^T u(x)dt + Au(z) = Tu(x) + Au(z).$$

There are two equality constraints. First is a more-or-less standard budget constraint. Consumption takes place at a rate x over a lifetime of length T. Life extension expenditure y and bequest z are made at one moment. There is no discounting in the budget. Therefore, the budget constraint is:

$$\bar{x} = xT + y + z. \tag{11}$$

The second constraint is crucial. In the probabilistic model, the more that is spent on precaution, the greater is the survival probability in the next period. In this deterministic model, the more that is spent on life extension, the longer is the lifespan. That is:

$$T = \ell(y), \tag{12}$$

I assume the function $\ell(\cdot)$ satisfies the following regularity conditions:

> Assumption 5 (Length of life). For $y \geq 0$, $\ell(y)$ is positive, differentiable, strictly increasing, and strictly concave.

It is easy to show that first-order conditions for an interior maximum for this model include the following equations:

$$u'(x) = Au'(z), \quad \text{and} \tag{13}$$

$$\frac{1}{\ell'(y)} = \frac{u(x)}{u'(x)} - x. \tag{14}$$

Note the similarity between equation (14) and equation (1) of the probabilistic model, and between equation (15) and equation (2) of the probabilistic model.

The interpretations of $\ell'(y)$ and $(\ell'(y))^{-1}$ are important: $\ell'(y)$ is the increase in units of lifespan per incremental dollar of precaution expenditure, while $(\ell'(y))^{-1}$ is the increase in dollars of precaution expenditure per incremental unit of lifespan. The latter is interpreted as the value of time or VOT. That is, for an individual choosing $y > 0$; we have:

$$\text{(15)} \qquad VOT = \frac{1}{\ell'(y)} = \frac{\text{Extra expenditure on life extension}}{\text{Extra year of life}}.$$

Note that by equation (14) we must have:

$$\text{(16)} \qquad VOT = \frac{u(x)}{u'(x)} - x; \quad \text{when } y > 0.$$

Let me briefly consider the possibility of a non-interior solution to the lifetime utility maximization problem. An optimal $x = 0$ is implausible for relevant variable and parameter ranges. An optimal $z = 0$ is possible (this simply means no bequest), in which case equation (13) is replaced with the inequality

$$Au'(z) \le u'(x).$$

An optimal $y = 0$ is also possible (this simply means no expenditure on life extension). In this case equation (14) is replaced with the inequality

$$\frac{u(x)}{u'(x)} - x \le \frac{1}{\ell'(y)}.$$

In case the optimal $y = 0$; and the strict inequality holds, I will continue to interpret $u(x)/u'(x) - x$ as VOT; and say the individual is buying no extra lifespan because the value of an extra time unit is less than the cost.

At this point I reintroduce the constant relative risk aversion utility function with:

Assumption 6 (Constant relative risk aversion instantaneous utility).

$$u(x) = \frac{x^{1-\sigma} - 1}{1 - \sigma} \quad \text{for some constant } \sigma > 0.$$

As before, for $\sigma = 1$ I take the limiting value, namely $u(x) = \ln x$.

Incorporating assumption 6 into equation (16) gives:

(17) $$VOT = \frac{x^{¾}}{1 ¡ ¾} ¡ \frac{x^{¾}}{1 ¡ ¾}:$$

When $¾ = 1;$ the equation becomes:

(18) $$VOT = x\,(\ln x ¡ 1):$$

V. Some Computed Values for Probabilistic VOL and Deterministic VOT

In this section I will present some computed values for VOL and VOT: Obviously the values depend on assumptions about underlying parameters $A; K$ and $¾:$ I will focus on the dependency of VOL and VOT on $¾;$ and I will try to compare the behavior of VOL and VOT as the underlying parameter values change.

It will ease matters somewhat to look separately at certain ranges of the constant relative risk aversion parameter $¾:$ Case 1 will be the lower risk aversion range, $0 < ¾ < 1;$ i.e., the range for which the utility functions $f(x)$ and $u(x)$ are straightforward exponential functions. Case 2 will be the $¾ = 1$ case, that is, the log utility case. Case 3 will be the higher risk aversion range, with $1 < ¾:$

In each of these cases it will simplify matters further to make particular assumptions about $A;$ the altruism or bequest parameter. Note that by equation (5) in the VOL model, the size of the bequest is $A^{1=¾}x:$ It is easy to show in the VOT model, by equation (13) and assumption 6, the size of the bequest is again $A^{1=¾}x:$

Therefore, in the VOL model $A^{1=¾}$ equals the ratio of the planned bequest to planned consumption at time one, and in the VOT model $A^{1=¾}$ equals the ratio of the chosen bequest to the rate of consumption per unit time. To illustrate the possibilities, I will look at three: (a) $A = 0;$ in which case, in either model, a bequest of 0 is chosen. (b) $A = 1;$ in which case, in either model, the individual plans on a bequest equal in magnitude to planned consumption at time one (V OL model), or the per unit time rate of consumption (V OT model). (c) A = $3^{¾}$; in which case the planned bequest is three times the size of planned consumption. (According to the 1996 Statistical Abstract of the United States, in 1994 average life insurance in force per household in the U.S. was $118,700, while average disposable income per household was $51,700. Using equation 5 of the probabilistic model would then imply A = $(3:3)^{¾}$.)

Intuition suggests, and equations (6) and (17) establish, that both V OL and V OT depend crucially on x; the consumption rate. I ¯nd it convenient in discussing the results to examine the ratios of values of life (or value of time) to consumption, or V OL=x and V OT=x: These ratios will be computed below. I also assume for purposes of the illustrations that x = 100; and K = 100: (The assumption that K = 100 is plausible. For instance, if ¾ = 1 (log utility), A = 1 (planned bequest equals planned consumption), and x = 100; we might ask the subject a John Broome (1978) style question: How much would you require be added to your planned bequest, so you would be indi®erent ex ante between the live state and the dead state? The answer would be the solution X to the equation ln 100 = ln (100 + X) ¡ 100; which gives X = 2:7 £ 10^{45}; an appropriately huge number.)

Case 1. 0 < ¾ < 1: Graph 1 shows some of the computed values. Note that V OT=x is quite sensitive to ¾; rising from around 0:1 at ¾ = 0:1; to around 3:6 at ¾ = :97: Note that when ¾ is close to 0:5 (square root utility function), V OT=x is close to 1:0: (In fact, when ¾ = 1=2; V OT=x = 1 ¡ 2=$x^{1=2}$; which equals .8 when x = 100; and approaches 1 as x gets large.) Having V OT=x close to 1 is an outcome that is rather similar to the traditional human capital model of the value of life, in which a life is valued at lifetime earning capacity, subject to various adjustments. Also note that the V OT=x function is independent of A by equation (17).

Now consider V OL=x: This is much more variable (as a function of ¾) than is V OT=x: In the A = 0 case it rises from around 2:7 at ¾ = 0:1; to around 105 at ¾ = :97:

In the A = 1 case it rises from around 1:6 at ¾ = 0:1 to around 100 at ¾ = :97: In the A = $3^{¾}$ case it rises from ¡0:6 at ¾ = 0:1 to around 88 at ¾ = :97: Observe that higher A's result in lower V OL=x's. In the A = $3^{¾}$ case, note the negative V OL=x ¯gures, all the way up to approximately ¾ = 0:2:

Negative V OL=x ¯gures mean, by the way, that the subject places a higher ex ante utility on the dead state than on the live state. With a su±cient high altruism parameter this is possible, and not irrational. Subject spends no money on increasing her probability of life, and accepts the zero-expenditure survival probability p(0); which is positive by assumption 4. The model does not allow suicide as an option. Allowing suicide would require a modi¯cation of the life insurance assumption.

Case 2. ¾ = 1: This is the log utility case. By equation (8),

V OL=x = (1 ¡ A) lnx ¡ A ln A + K; and

by equation (18)

V OT=x = ln x ¡ 1:

When A = 0; V OL=x becomes ln x + K and V OT=x remains ln x ¡ 1: The two formulas are very similar, and there is a constant (with respect to x) di®erence between V OL=x and V OT=x; of K + 1: When A = 1; V OL=x is simply equal to K; and V OT=x remains ln x ¡ 1: When A = $3^{¾}$ = 3^{1} = 3; V OL=x becomes ¡2 ln x ¡ 3 ln 3 + K; while V OT=x remains ln x ¡ 1: The two formulas are almost mirror images (as functions of x); one being ln x displaced by a constant and the other being ¡2 ln x displaced by another constant.

Case 3. ¾ > 1: Now consider higher degrees of relative risk aversion. Graph 2 shows the calculated values. Note that graph 2, unlike graph 1, is exponentially scaled on the vertical axis. Again there is one V OT=x function, but three V OL=x functions, depending on whether A = 0; A = 1; or A = $3^{¾}$: Graph 2 shows near-exponential growth of V OL=x and V OT=x as ¾ increases. Comparison of the three V OL=x functions reveals that A doesn't matter much any more. In the analytically simple case $A = 1$, $VOL/x = x^{\sigma-1}K$, and $VOT/x \approx x^{\sigma-1}/(\sigma - 1)$. Both VOL/x and VOT/x are growing exponentially with σ (VOT/x approximately), and the ratio of VOL/x to VOT/x rapidly approaches $K(\sigma - 1)$.

The conclusion is that when $\sigma > 1$, VOL/x and VOT/x are both growing very rapidly in σ, VOL/x more so than VOT/x. For, e.g., $\sigma = 2.5$, VOL is around 100,000 times consumption x, and VOT is around 600 times x. These numbers are approximately correct for $A = 0.1$ or $3^{2.5}$. Note that graph 2 shows calculated values of VOL/x and VOT/x for σ up to around 3.0. For larger values of σ, the limiting values $VOL/x = x^{\sigma-1}K$ and $VOT/x = x^{\sigma-1}/(\sigma - 1)$ are approximately correct for any plausible value of A.

VI. Conclusions

In this paper I have derived simple formulas for value of life and value of time:

(6)
$$VOL = x^{\sigma}\left[\frac{A - 1}{1 - \sigma} + K\right] + x\,\frac{1 - A^{\frac{1}{\sigma}}}{1 - \sigma}.$$

(17)
$$VOT = \frac{x^{\sigma}}{1 - \sigma} - \frac{x^{\sigma}}{1 - \sigma}$$

For the exponential utility, lower risk relative aversion range, when $0 < \sigma < 1$, both VOL and VOT grow markedly with σ, but the VOT function is more stable. For the log utility case, where $\sigma = 1$, the VOL and VOT equations are rather similar (equations (8) and (18)), but the VOT formula has the virtue of being independent of what I call altruism, A, and fear of death, K. For the higher relative risk aversion range, when $1 < \sigma$, both VOL and VOT rapidly approach exponential functions of σ and the significance of the altruism parameter rapidly disappears. However, the VOT formula continues to have the virtue of being independent of the fear of death parameter K.

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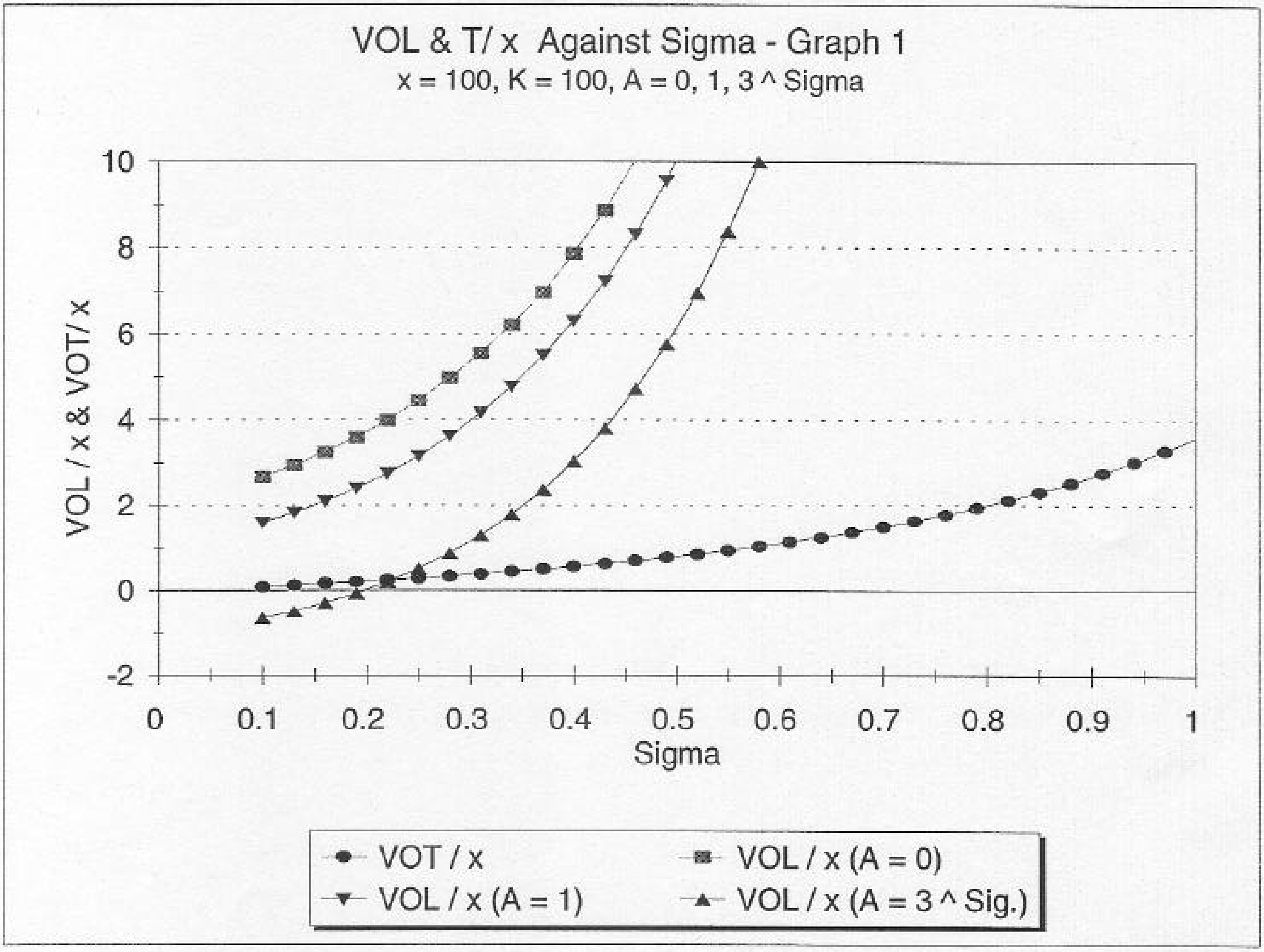
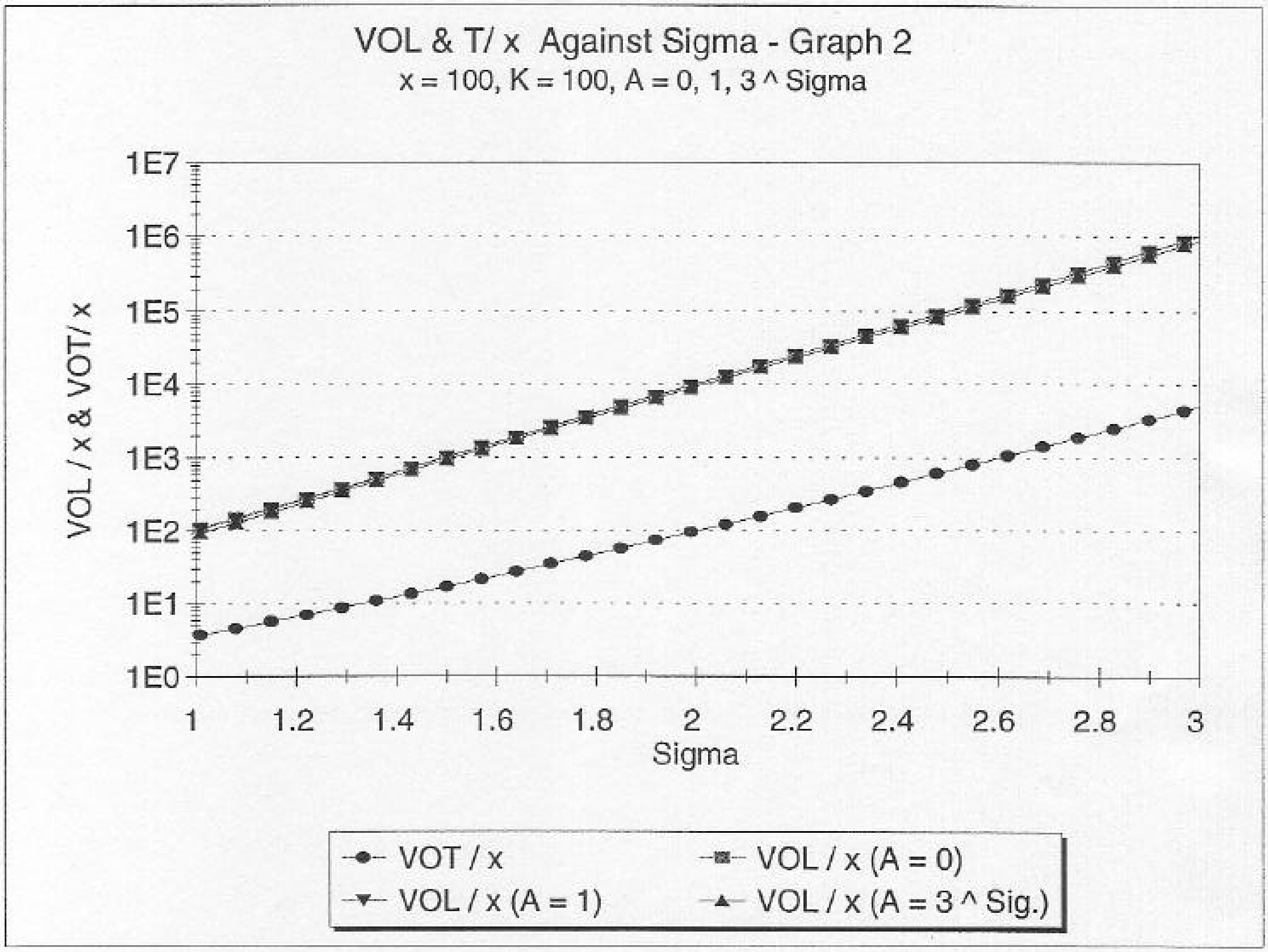


VOL & T/ x Against Sigma - Graph 1
x = 100, K = 100, A = 0, 1, 3 ^ Sigma
VOL / x & VOT/ x
10
8
6
4
2
0
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Sigma
VOT / x
VOL / x (A = 0)
VOL / x (A = 1)
VOL / x (A = 3 ^ Sig.)



VOL & T/ x Against Sigma - Graph 2
x = 100, K = 100, A = 0, 1, 3 ^ Sigma
1E7
1E6
1E5
1E4
1E3
1E2
1E1
1E0
VOL / x & VOT/ x
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
Sigma
VOT / x
VOL / x (A = 0)
VOL / x (A = 1)
VOL / x (A = 3 ^ Sig.)