1. **The simplest possible derivation of the Kelvin State.** Assume that the solution is to be generated from Papkovich-Neuber potentials $\phi, \Psi$ satisfying

$$\Psi_{1, \kappa k}(x) = -b_j \delta(x - \xi) \quad \phi_{1, \kappa k}(x) = -x_j b_j \delta(x - \xi)$$

Here, we have chosen the body force to be a Dirac delta sequence centered at $\xi$. By taking Fourier transforms of the governing equations for $\phi, \Psi$ above, find the Papkovich-Neuber potentials that generate the required solution.

2. **Papkovich-Neuber potentials for the Doublet states.** Let $\psi^{(k)}$, $\phi^{(k)}$ denote the Papkovich-Neuber potentials for the normalized Kelvin state, i.e. a point force of unit magnitude acting in the $e_k$ direction at the origin. Let $S^{(k,l)} = [u^{(k,l)}, e^{(k,l)}, \sigma^{(k,l)}]$ denote the doublet states, i.e.

$$u_i^{(k,l)} = u_i^{(k)}, \quad e_{ij}^{(k,l)} = e_{ij}^{(k)}, \quad \sigma_{ij}^{(k,l)} = \sigma_{ij}^{(k)}$$

Show that $S^{(k,l)}$ may be generated from Papkovich-Neuber potentials

$$\psi^{(k,l)} = \psi_i^{(k)}, \quad \phi^{(k,l)} = \phi_i^{(k)} - \psi_i^{(k)}$$

Hence, verify that the Papkovich Neuber potentials for the doublet states centered at the origin are

$$\psi_i^{(k,l)} = -\frac{1}{4\pi} \frac{\delta_{kl} x_i}{r^3} \quad \phi_i^{(k,l)} = -\frac{1}{4\pi} \frac{\delta_{kl}}{r}$$
3. **Center of Compression.** Using the results of the preceding section, find the displacement, strain and stress fields associated with a center of compression at the origin, i.e., find

\[ \mathbf{S}^{(k,k)} = [u^{(k,k)}, \varepsilon^{(k,k)}, \sigma^{(k,k)}] \]

4. **Center of compression in a sphere.** Using superposition and the results of problem (3), find the displacement fields induced by a center of compression at the center of a sphere of radius \( a \). Assume that the surface of the sphere is free of traction.

5. **Dilatation at the center of a sphere due to arbitrary surface traction.** Using the result of problem (4), show that the dilatation at the center of a sphere of radius \( a \) due to a self-equilibrating distribution of traction \( \mathbf{t} \) acting on its surface is

\[ \varepsilon_{kk} = \frac{3(1-2\nu)}{8\pi(1+\nu)\mu} \frac{1}{a^3} \int_B \mathbf{t} \cdot \mathbf{r} \, dA \]

where \( \mathbf{r} \) is the position vector of a point on the sphere’s surface relative to the origin, and \( B \) denotes the surface of the sphere. Verify the predictions that were made in our proof of Saint-Venants principle. What happens if the tractions act tangent to the surface of the sphere?