Division of Engineering
Brown University

ENGN1300: Structural Analysis

Final Exam
Monday, May 17, 2010

NAME: ____

General Instructions

- Exam is open notes and homework. The course website may be used, but no other web or printed materials are allowed.
- You can use Maple. No other websites or computational aids are allowed.
- All work must be shown in these pages. You can use Maple for the calculations, but write down your final answers on these pages.
- If you do use Maple, please save your maple file and email it to me. Use a single Maple file for each problem. Be aware that I don't plan to grade the Maple---I should be able to follow your work by looking at these pages alone.

Sign your name to the statement below:

I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University.

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1. (12 points) ______________

2. (8 points) ______________

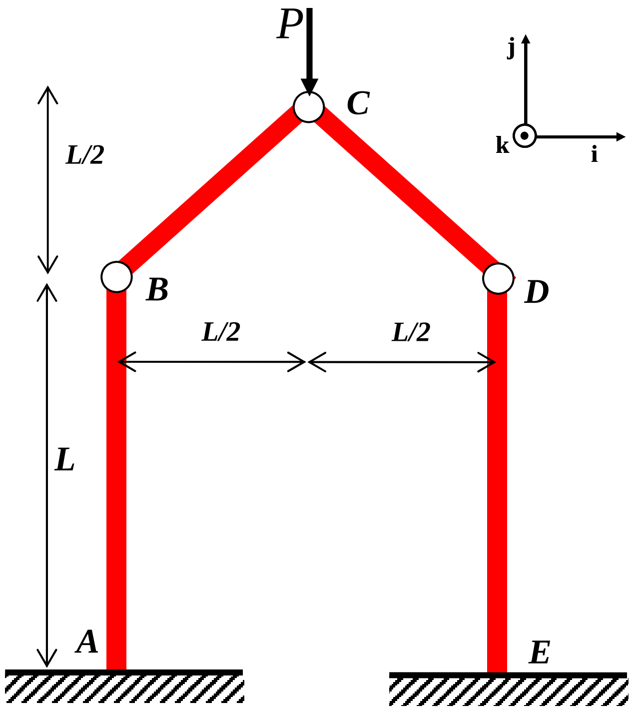
3. (12 points) ______________

4. (10 points) ______________

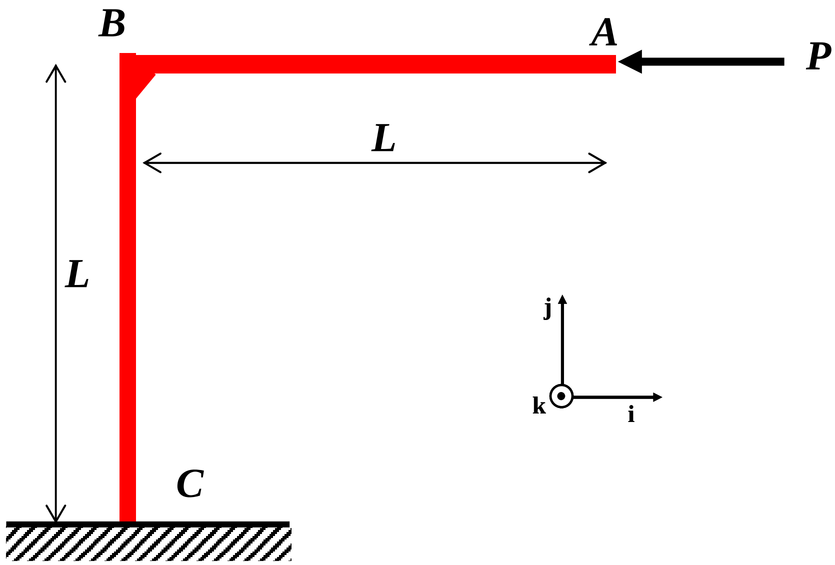
5. (8 points) ______________

TOTAL (50 points) ____________

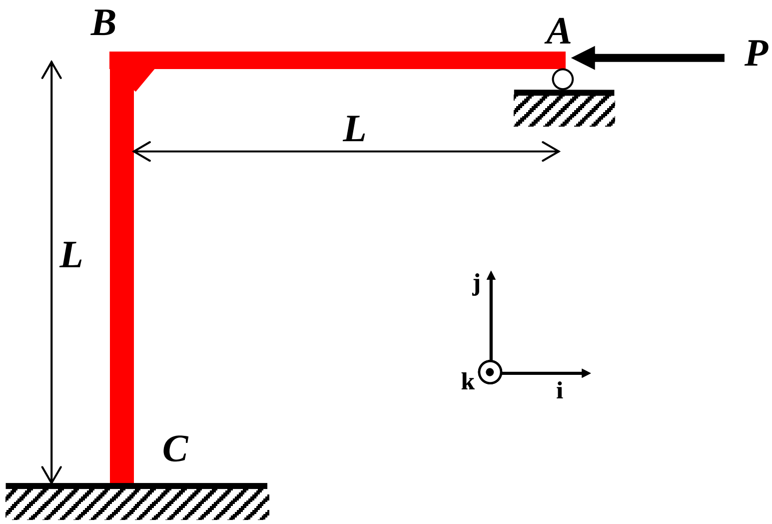
1. The structure shown below is subjected to a load *P*. All members have identical values of E, I, and A. Determine the base reactions and the deflection at point C. Neglect deformation due to axial extension.



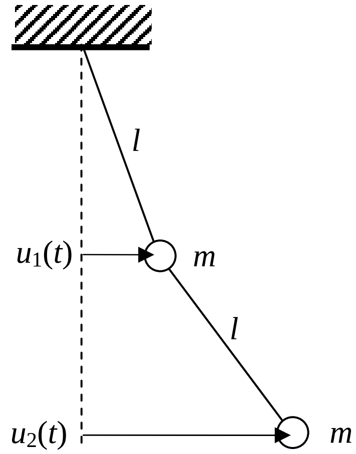
2. The structure is subjected to a lateral load *P* at point A. Determine the vertical and lateral deflection at point *A*. Both members have identical properties *EI*, and joint B is a full moment connection.



3. A roller support is added at point A. Determine the reactions at points A and C.



4. A double pendulum is shown below.



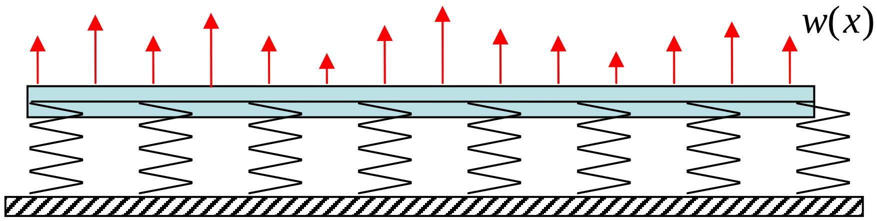
The equations of motion are:

$$2m\ddot{u}_1 + m\ddot{u}_2 + 2m(g/l)u_1 = 0$$
$$m\ddot{u}_1 + m\ddot{u}_2 + m(g/l)u_2 = 0$$

a. Put these equations in matrix form.

b. Determine the natural frequencies and mode shapes. Sketch the mode shapes. A rough hand-drawn sketch is fine.

5. A beam on an elastic foundation (eg a footing or roadway on a soil base) can be represented as a beam resting on a distribution of elastic springs with stiffness *k* per unit length (force/length^2). When the beam deflects with displacement $u(x)$ the springs provide a reaction force distribution $-ku(x)$ (force per unit length).

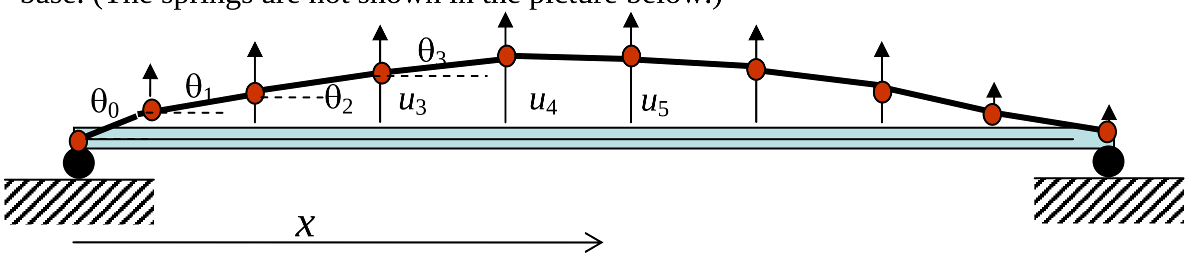


The potential energy of the beam is

$$V = \int_0^{L_0} \left(\frac{1}{2} EI\kappa(x)^2 + \frac{1}{2} ku(x)^2 - w(x)u(x) \right) dx \,,$$

where u is the upwards deflection and κ is the curvature.

If you were to modify the excel beam bending calculator to handle a simply supported beam on an elastic foundation, you would have to include the contribution due to the elastic base. (The springs are not shown in the picture below.)



As before the beam deflections are determined by the angles θ_i:

$u_0 = 0, \;\; u_1 = s\sin\theta_0, \quad u_2 = u_1 + s\sin\theta_1 \;\; .... \;\; u_i = u_{i-1} + s\sin\theta_{i-1}$. (*)

and the curvature at segment i is $\kappa_i = (\theta_i - \theta_{i-1})/s$. ($s = L/N$). The elastic energy due to bending is computed at node i as $V_i = \frac{s}{2} EI\kappa_i^2$. The load at node i is $W_i = w(x_i)s$ ($i = 1,2,\dots N-1$), $W_0 = w(0)s/2$, $W_N = w(L_0)s/2$, and so the associated energy is $W_i u_i$.

The total energy for the beam is the sum of elastic and load energy:

$$V = \frac{1}{2} EIs \sum_{i=1}^{N} \frac{\left(\theta_i - \theta_{i-1}\right)^2}{s^2} - \sum_{i=0}^{N} W_i u_i + \text{terms due to the elastic foundation.}$$

a. Specifically, what are the terms that should be included to take into account the elastic foundation? Explain your answer. Be careful to treat the end joints properly.

b. Suppose you want to calculate the deflection of a beam on an elastic foundation that has **no** additional end supports. The beam rests solely on the elastic foundation. Assuming that you have the proper expression for the potential energy in part a, explain how you would modify equation (*) above and what variables/constraints you would enter into the solver.