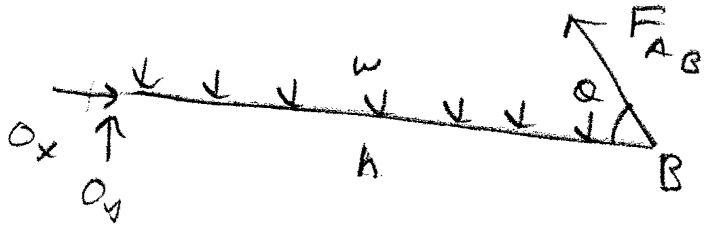
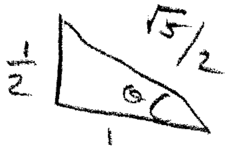


$\tan\theta = \frac{1}{2}$



$\cos\theta = \frac{2}{\sqrt{5}}$

$\sin\theta = \frac{1}{\sqrt{5}}$

$O_y + F_{AB}\sin\theta = wh$

$O_x + F_{AB}\cos\theta = 0$

$\Sigma M_0 = 0 \qquad \frac{wh^2}{2} = F_{AB}\sin\theta\, h \qquad F_{AB} = \frac{wh}{2\sin\theta} = \frac{wh\sqrt{5}}{2}$

$O_x = +F_{AB}\,2/\sqrt{5} = wh \qquad O_y = -F_{AB}\sin\theta + wh$

$= \frac{wh}{2}$

deflection @ B – only vertical if OB does not deflect axially

$\delta_{AB} = \underline{n}^{BA}\cdot(\cancel{\underline{u}^A} - \underline{u}^B) = -\frac{1}{\sqrt{5}}(2\underline{i} - \underline{j})\cdot(-u^B_y\underline{j}) = -\frac{u^B_y}{\sqrt{5}}$

$= \frac{F_{AB}}{EA}L_{AB} = \frac{wh}{EA}\frac{\sqrt{5}}{2}\,h\frac{\sqrt{5}}{2}$

$= \frac{wh^2}{EA}\frac{5}{4}$

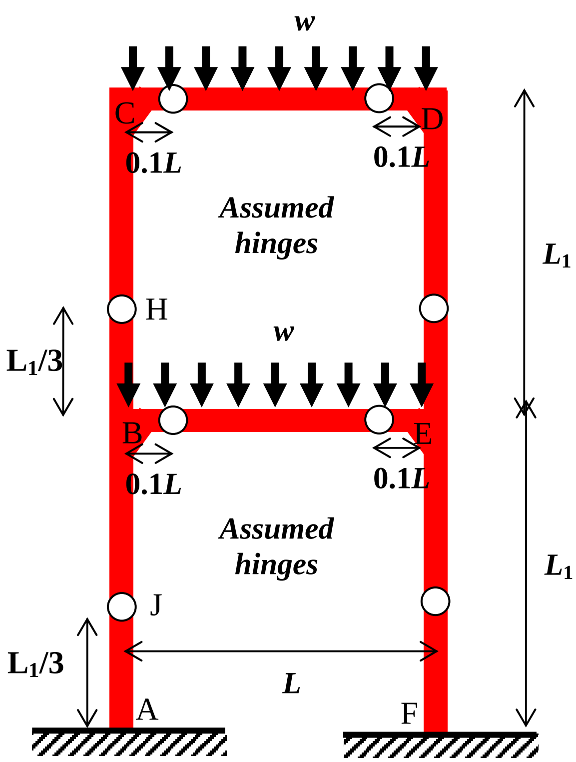
$$\boxed{u^B_y = -\frac{wh^2}{EA}\frac{5\sqrt{5}}{4}}$$

deflection along OB:

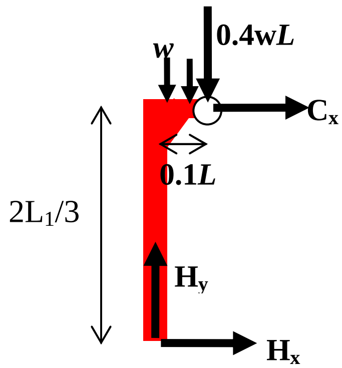
$u^B_y(x) = +u^B_y\frac{x}{h} - \frac{wx}{24EI}(h^3 - 2hx^2 + x^3)$

$= -\frac{wh^2}{EA}\frac{5\sqrt{5}}{4}\frac{x}{h} - \frac{wh^4}{24EI}\frac{x}{h}\left(1 - 2\frac{x^2}{h^2} + \frac{x^3}{h^3}\right)$

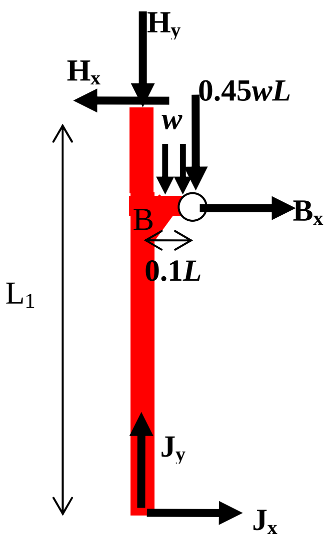
2. See the two story building with uniformly distributed loads shown below. For an approximate analysis, assume there are hinges as shown. Based on this assumption, estimate the reactions at the base *A*.



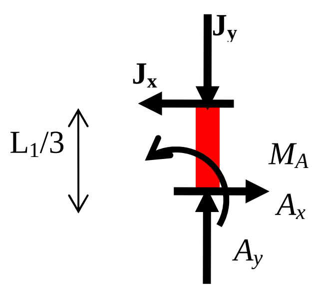
Solution: FBD for the pieces of the column



$$\sum F_y = 0 \Rightarrow H_y = wL/2$$
$$\sum M_H = 0 \Rightarrow -C_x 2L_1/3 - 0.4wL(0.1L) - 0.1wL(0.05L) = 0$$
$$C_x = -(3/2)0.045wL^2/L_1 = -0.0675wL^2/L_1 = -H_x$$
$$H_x = 0.0675wL^2/L_1$$

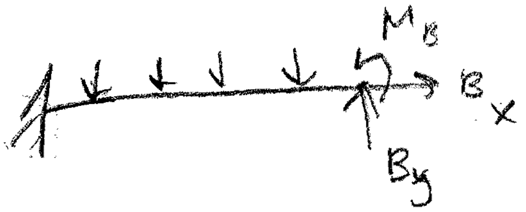


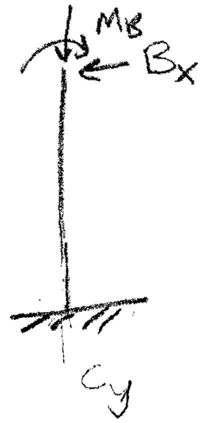
$$\sum F_y = 0 \Rightarrow H_y + wL/2 = J_y = wL$$
$$\sum M_J = 0 \Rightarrow -B_x 2L_1/3 + H_x L_1 - 0.045wL^2 = 0$$
$$B_x = -0.0675wL^2/L_1 + 3H_x/2 = 0.03375wL^2/L_1$$
$$J_x = H_x - B_x = 0.03375wL^2/L_1$$



$$\sum F_y = 0 \Rightarrow A_y = J_y = wL$$
$$\sum F_x = 0 \Rightarrow A_x = J_x = 0.03375wL^2/L_1$$
$$\sum M_O = 0 \Rightarrow M_A = -J_x L_1/3 = -0.01125wL^2$$

3





$$U_x^B = \frac{M_B L^2}{2EI} - \frac{B_x L^3}{3EI} = 0$$

$$\underline{M_B = \frac{B_x L\, 2}{3}}$$

$$\theta_B = -\frac{B_x L^2}{2EI} + \frac{M_B L}{EI}$$

$$= \frac{-B_x L^2}{2EI} + \frac{B_x L^2}{EI}\frac{2}{3}$$

$$= \frac{B_x L^2}{6EI}$$

Top beam $\quad U_y^B = 0 = \frac{-wL^4}{8EI} + \frac{B_y L^3}{3EI} + \frac{M_B L^2}{2EI} = 0$

$$\theta^B = +\frac{wL^3}{6EI} - \frac{M_B L}{EI} - \frac{B_y L^2}{2EI} = 0$$

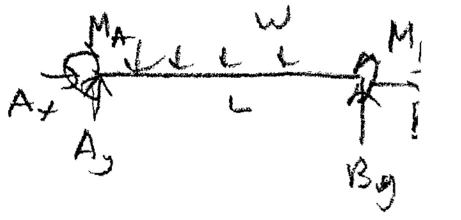
Solve in maple for B_x, B_y, M_B

$$B_x = -\frac{wL}{16} \qquad B_y = \frac{7wL}{16} \qquad M_B = \frac{2B_x L}{3}$$

reactions

$\underline{C_x = B_x \quad C_y = B_y}$

$\underline{M_C = M_B - B_x L}$

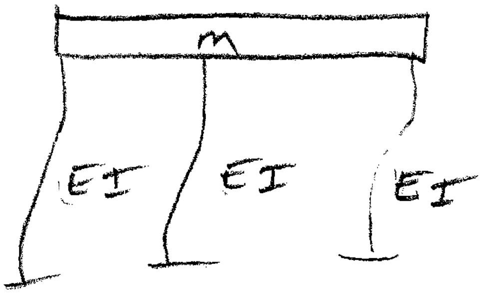


$A_y = wL - B_y$

$A_x = -B_x$

$M_A = -M_B - wL^2/2 + B_y L$

4



side

equiv stiffness

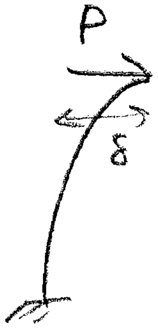
is $K = \frac{3}{2}\,\frac{24EI}{h^3}$ ← 2 column stiffness

$$k = \frac{36EI}{h^3}$$

$$\omega_x^2 = \frac{k}{m} = 36\frac{EI}{mh^3} \qquad \omega = 6\sqrt{\frac{EI}{mh^3}}$$

front

Stiffness



$$\delta = \frac{Ph^3}{3EI}$$

$$K = \frac{P}{\delta} = \frac{3EI}{h^3}$$

one colum

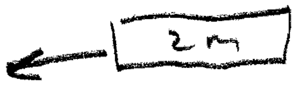
$$K = \frac{9EI}{h^3} \quad \text{3 colums}$$

$$\omega_y^2 = \frac{k}{m} = \frac{9EI}{mh^3} \qquad \omega = 3\sqrt{\frac{EI}{mh^3}}$$

ω_y → lower freq by 1/2

5

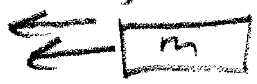
$$\begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$$2m\ddot{u}_2 + k(u_2 - u_1) = 0$$

$k(u_2 - u_1)$

$k(u_1 - u_2)$

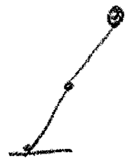


$$m\ddot{u}_1 + 2ku_1 - ku_2 = 0$$

ku_1

Maple for the rest.

$$\omega_1^2 = \sqrt{.22\, k/m} \qquad \underline{A}_1 = \begin{bmatrix} 1 \\ 1.8 \end{bmatrix}$$



$$\omega_2^2 = \sqrt{2.3\, \frac{k}{m}} \qquad \underline{A}_2 = \begin{bmatrix} 1 \\ -.3 \end{bmatrix}$$