Division of Engineering
Brown University

EN130: Structural Analysis and Design

Sample Midterm Examination

General Instructions

- You may refer to your own class, handwritten notes, homework, but no other material.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- Use results from class wherever possible.
- If an answer looks wrong, or you think you're going astray in your methods, say so! Write the reasons for your suspicions.

Please initial the statement below to show that you have read it

By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University.

____

Surgeon General Warning:
These problems have not been adjusted in order to be do-able in 50 minutes. These are simply representative problems intended to give you something to practice on. Solutions will not be posted.

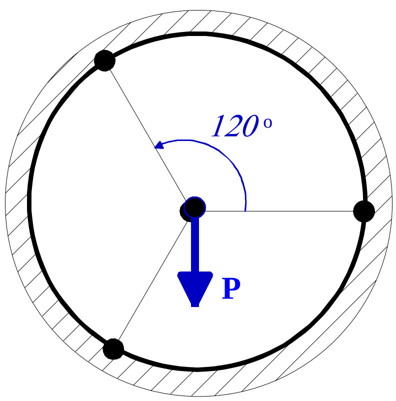
1. Bicycle wheels come with different numbers of spokes.

a) A bike is modeled as a truss. Assume first that the wheel rim is effectively rigid compared with the spokes and the hub is loaded with a single downward force P. In this case, the only nonzero displacement component is the vertical displacement of the hub, u. All spokes have Young's Modulus E and length L_0, with L_0 equal to the wheel radius. Find deflection of:

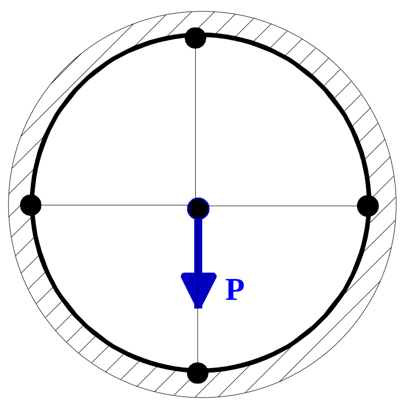
i) A 3-spoked wheel; all spokes have cross sectional area A_3.

ii) A 4-spoked wheel; all spokes have cross sectional area A_4.

iii) If the 3- and 4-spoked wheels are to deflect identical amounts under the load P, determine the required ratio A_3 / A_4. Which of the two wheels is heavier? (The rims in each case have the same weight.)



n=3

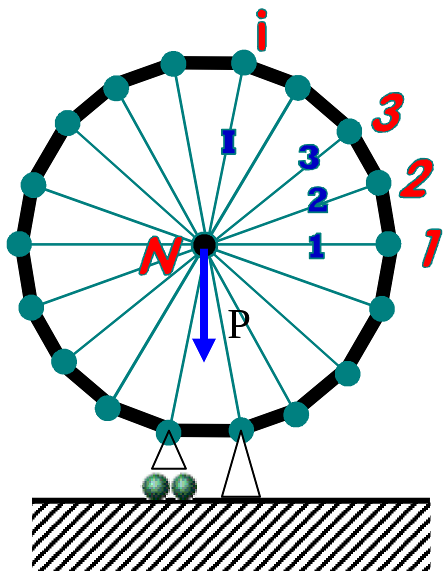


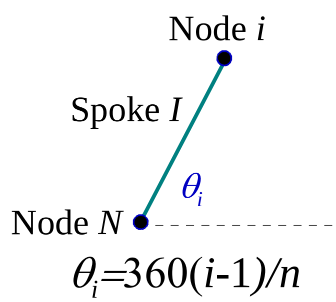
n=4

b) Now consider a general n-spoked wheel truss model that takes into account the rim flexibility. The truss has $N=n+1$ nodes, and $2n$ members: n spokes (shown in green) and n rim elements (black), all of which are elastic. The center node has node number N, and the node at the wheel-end of spoke I has node number i. ($i=1,2,..n$). All spokes are made of identical materials, and have length L_0 and cross-sectional area A.

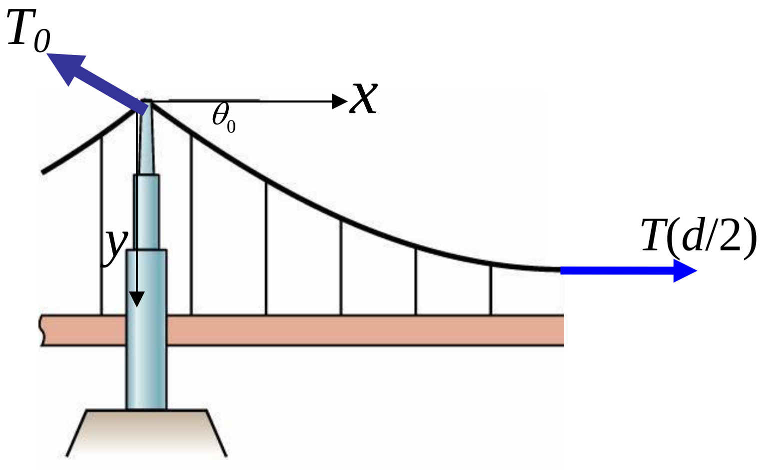
i) How many degrees of freedom are there for this wheel?

ii) Recall that the potential energy stored in spoke I can be written in the form $V^I = \frac{1}{2}\underline{u}^I \cdot \left[k^I\right]\underline{u}^I - \underline{r}^I \cdot \underline{u}^I$, with $\left[k^I\right]$ and $\underline{r}^I$ being the element stiffness matrix and residual vector, and $\underline{u}^I = \left[u_1^N, u_2^N, u_1^i, u_2^i\right]$. Give an expression for $\left[k^I\right]$ in terms of the spoke number I. Give an expression for the element residual vector $\underline{r}^I$ (Suggestion: use class results!)





2. Consider a cable suspending a pipeline as shown below. The structure is symmetric about its midline. The cable is hung across the span *d* with a prescribed sag *h*. You can assume that the weight of the cable is negligible and that the pipeline weight is distributed evenly over the cable span, $w(x)=w_0$.



Determine expressions for the following, as functions of w_0, *d*, and *h*.

a) The tension T_0

b) The tension $T(d/2)$

c) The angle θ_0.

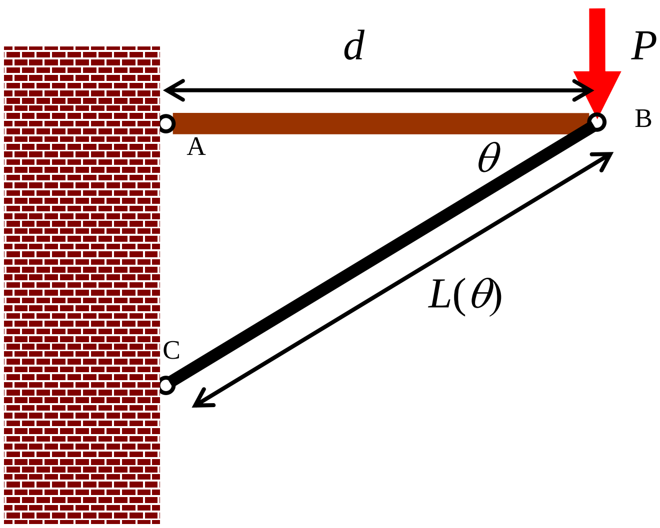
d) The span has length d=70 meters, the sag is 10 meters, and the pipe has diameter of one meter. The pipe, when empty, weighs 1700 Newtons per meter. The cable is to be made from steel, which has a yield stress of 200 MPa. Determine the minimum cross sectional diameter of the cable in order that the tensile stress does not exceed the yield stress **when the pipe is full of water.**

e) Steel has a weight density of about 76MN per meter^3. Calculate the weight of the cable (per unit length). Give an estimate of the tension in the cable at the left endpoint due to the combined effects of the cable self weight and the weight of the full pipe. Explain how you have made your estimate.

Note: there are many ways to make this estimate!

3. A shelf with length d holds a load P at its endpoint as shown in the figure. A strut BC supports the shelf. It is a round steel bar (radius r). Young's modulus is E=200GPa and moment of inertia is $I=\pi r^4/4$.

Your goal is to decide upon the angle θ which will enable the structure to hold the largest possible load without failure due to buckling.



a) Calculate the force in the strut BC as a function of θ and the other parameters of the problem.
b) Find the buckling load P_{CR} for the strut as a function of θ and the other parameters of the problem.
c) Show that load the applied load P must satisfy

$$P \le P_{max} \equiv C \sin\theta \cos^2\theta$$

in order to avoid failure by buckling, where C is a constant. Identify C in terms of the parameters d, E, and I

e) Find the angle θ that maximize the load capacity P_{max} of the shelf

f) The balcony length is d=1 meter. The steel has radius r=1cm. Determine the optimal strut length and the maximum load the optimally designed shelf can support without buckling.

g) You designed the structure against failure due to buckling, but the strut may also fail due to yield if its stress exceeds the yield stress σ_Y=300MPa. If the structure above is loaded with its maximum design load P_{max}, will it fail by yield?

h) For $P=P_{max}$, and r=1cm, calculate the vertical deflection of the structure at point B, assuming rod AB is rigid.