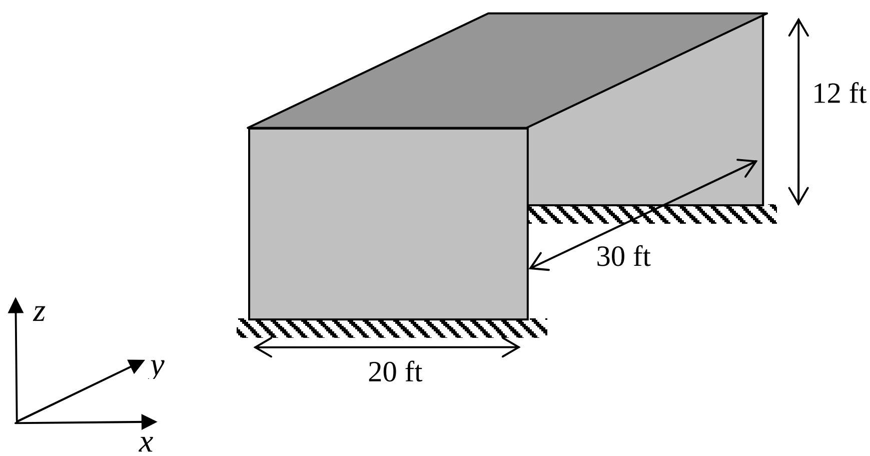
ENGN1300: Structural Analysis

Homework 10
Due Friday May 7, 2010

Division of Engineering
Brown University|

1. A one-story industrial building consists of a stiff roof truss, supported by walls on two sides. For lateral motion of the roof in the *x*-direction, these walls act as shear walls. For lateral motion of the roof in the *y*-direction, the walls act as columns in bending. The roof slab has a weight distribution of 60 psf. the walls are steel, *E*=29,000ksi. The steel walls are 1 inch thick. Calculate the natural frequencies of vibration of the structure (in Hz) for lateral vibrations in the two directions.



$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. Mass of the rooftop is $m = \frac{60\text{lbs/ft}^2 \times 600\text{ft}^2}{386\text{in}/\text{sec}^2} = 93.3\text{lb-sec}^2/\text{in}$

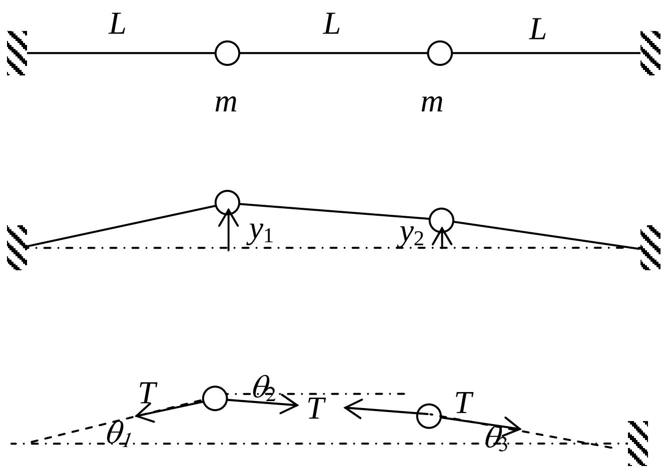
For motion in the *y*-direction: $k = \frac{24EI}{h^3}$. $I = \frac{240'' \times (1'')^3}{12} = 20\text{in}^4$.

$$k = \frac{24 \times 29000 \times 10^3\text{lbs/in}^2 \times 20\text{in}^4}{(144\text{in})^3} = 4.7 \times 10^3\text{lbs/in}. \quad f_n = \frac{1}{2\pi}\sqrt{\frac{4.7 \times 10^3\text{lbs/in}}{93\text{lb}-\text{sec/in}}} = 1.13Hz$$

For motion in the *x*-direction is $k = \frac{2EA}{h}$. $A = 240'' \times 1'' = 240\text{in}^2$.

$$k = \frac{2 \times 29000\text{kips/in}^2 \times 240\text{in}^2}{144\text{in}} = 96.7\text{kips/in}. \quad f_n = \frac{1}{2\pi}\sqrt{\frac{96.7\text{kips/in}}{93\text{lb}-\text{sec/in}}} = 5.1Hz$$

2. Two masses are attached to a string with tension T. For small motion of the masses (relative to L), find the natural frequencies and the mode shapes of the free vibration of the system. Assume that the tension does not change during the motion.



$$m\ddot{y}_1 + T\sin\theta_1 + T\sin\theta_2 = 0$$
$$m\ddot{y}_2 + T\sin\theta_3 - T\sin\theta_2 = 0$$

Small motions $\sin\theta_1=\theta_1=y_1/T$, $\sin\theta_2=\theta_2=(y_1-y_2)/L$, $\sin\theta_3=\theta_3=y_3/L$.

$$m\ddot{y}_1 + \frac{T}{L}(2y_1 - y_2) = 0$$

$$m\ddot{y}_2 + \frac{T}{L}(2y_2 - y_1) = 0$$

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} + \frac{T}{L}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

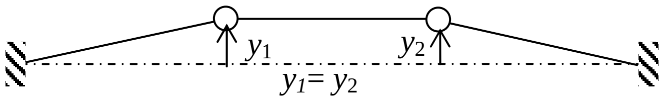
Natural frequencies and mode shapes:

Eigenvalues and Eigenvectors of

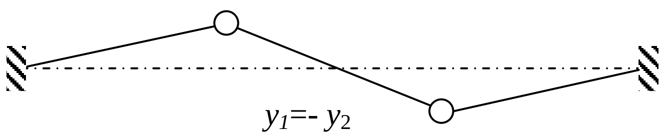
$$[H] = \frac{T}{Lm}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

Maple results: Mode 1 $\omega_n = \sqrt{\frac{T}{Lm}}, \; A_n = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Mode 1:



Mode 2: $\omega_n = \sqrt{\dfrac{3T}{Lm}},\ A_n = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$




```
> restart;
> with(linalg):
> H:=matrix(2,2,[[2,-1],[-1,2]]);#prob2
```

$$H := \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

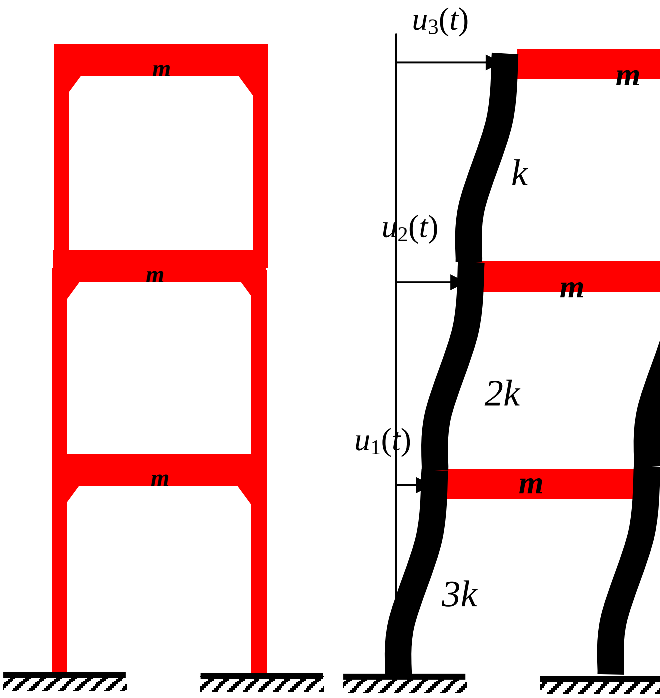
```
> eigenvalues(H);
```

$$3, 1$$

```
> eigenvectors(H);
```

[1, 1, {[1, 1]}], [3, 1, {[-1, 1]}]

Buildings often use lighter columns to support upper floors. A model of a 3-story building is shown below.Find the natural frequencies and mode shapes in terms of *k* and *m*



$$\begin{bmatrix} m & & \\ & m & \\ & & m \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} + \begin{bmatrix} 5k & -2k & 0 \\ -2k & 3k & -k \\ 0 & -k & k \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Eigenvaules of $[H] = \frac{k}{m}\begin{pmatrix} 5 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{pmatrix}$

Maple:

```
> #prob 3
> H:=matrix(3,3,[[5,-2,0],[-2,3,-1],[0,-1,1]]);
```

$$H := \begin{bmatrix} 5 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

```
> eomgs:=evalf(eigenvalues(H)); #tiny imaginary part is from
the numerical solutions.
>
```

$eomgs := 6.289945083 - 0.1\ 10^{-9}\ I,\ 0.4157745575 - 0.8660254040\ 10^{-9}\ I,$
$2.294280360 + 0.8660254040\ 10^{-9}\ I$

```
> omega3:=evalf(sqrt(Re(eomgs[1])))*sqrt(k/m);omega1:=evalf(sqrt(Re(eomgs[2])))*sqrt(k/m);omega2:=evalf(sqrt(Re(eomgs[3])))*sqrt(k/m);
```

$$\omega 3 := 2.507976292\ \sqrt{\frac{k}{m}}$$

$$\omega 1 := 0.6448058293\ \sqrt{\frac{k}{m}}$$

$$\omega 2 := 1.514688206\ \sqrt{\frac{k}{m}}$$

```
> evalf(eigenvectors(H)); #same deal with the imaginary part
```

$[6.289945083 - 0.1\ 10^{-9}\ I,\ 1.,$
$\{[1.,\ -0.6449725414 + 0.\ I,\ 0.12192425 - 0.6289945085\ 10^{-9}\ I]\}],\ [$
$0.4157745575 - 0.8660254040\ 10^{-9}\ I,\ 1.,$
$\{[1.,\ 2.292112721 + 0.4330127020\ 10^{-9}\ I,\ 3.923336013 + 0.3104030287\ 10^{-8}\ I]\}],$
$[2.294280360 + 0.8660254040\ 10^{-9}\ I,\ 1.,$
$\{[1.,\ 1.352859819 - 0.4330127020\ 10^{-9}\ I,\ -1.045260255 - 0.1477196540\ 10^{-8}\ I]\}]$

```
> a1:=vector([1., 2.292112721, 3.923336013]);
```

$$a1 := [1.,\ 2.292112721,\ 3.923336013]$$

```
> a3:=vector([1., -.6449725414, .12192425]);
```

$$a3 := [1.,\ -0.6449725414,\ 0.12192425]$$

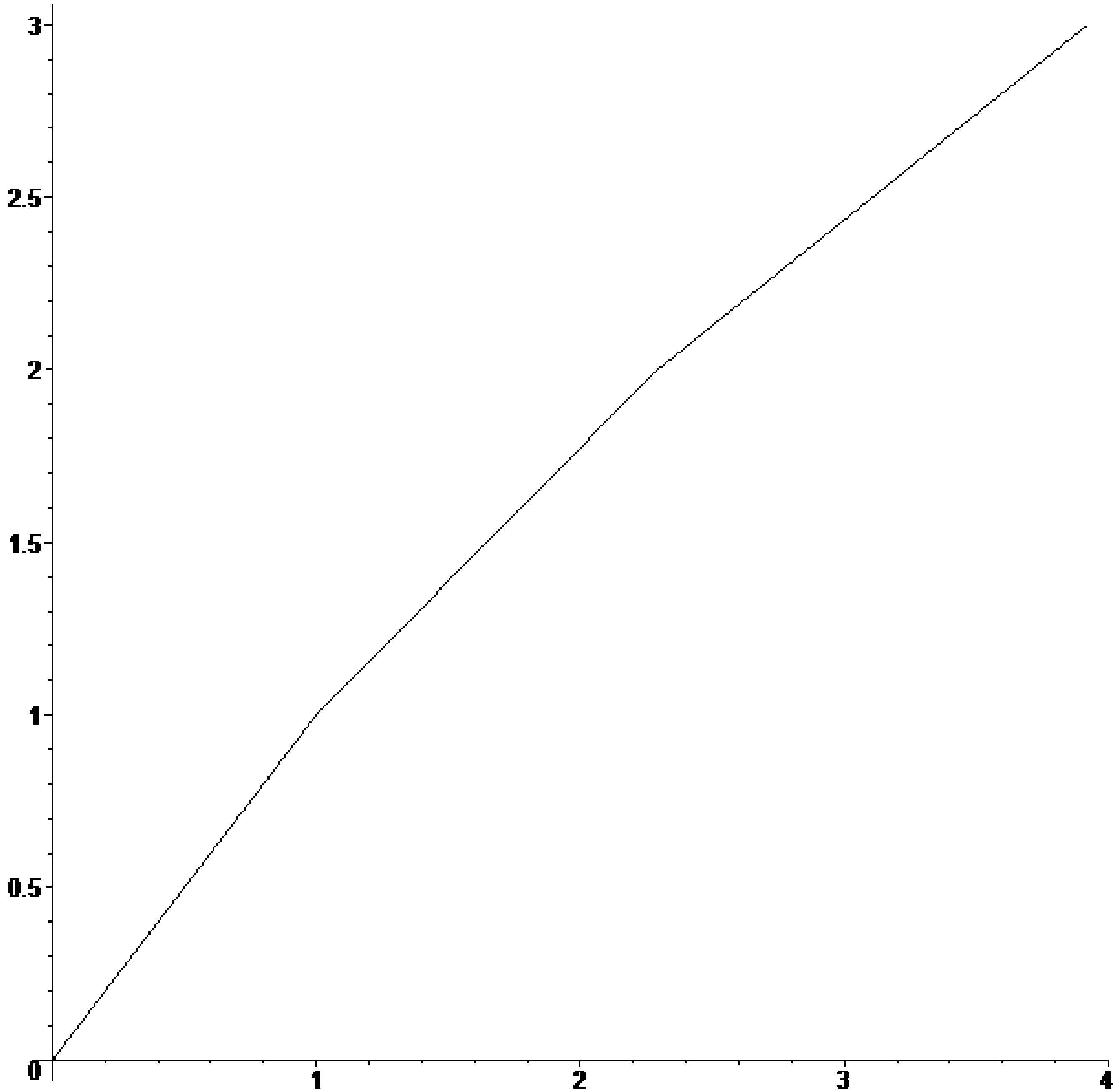
```
> a2:=vector([1., 1.352859819, -1.045260255]);
```

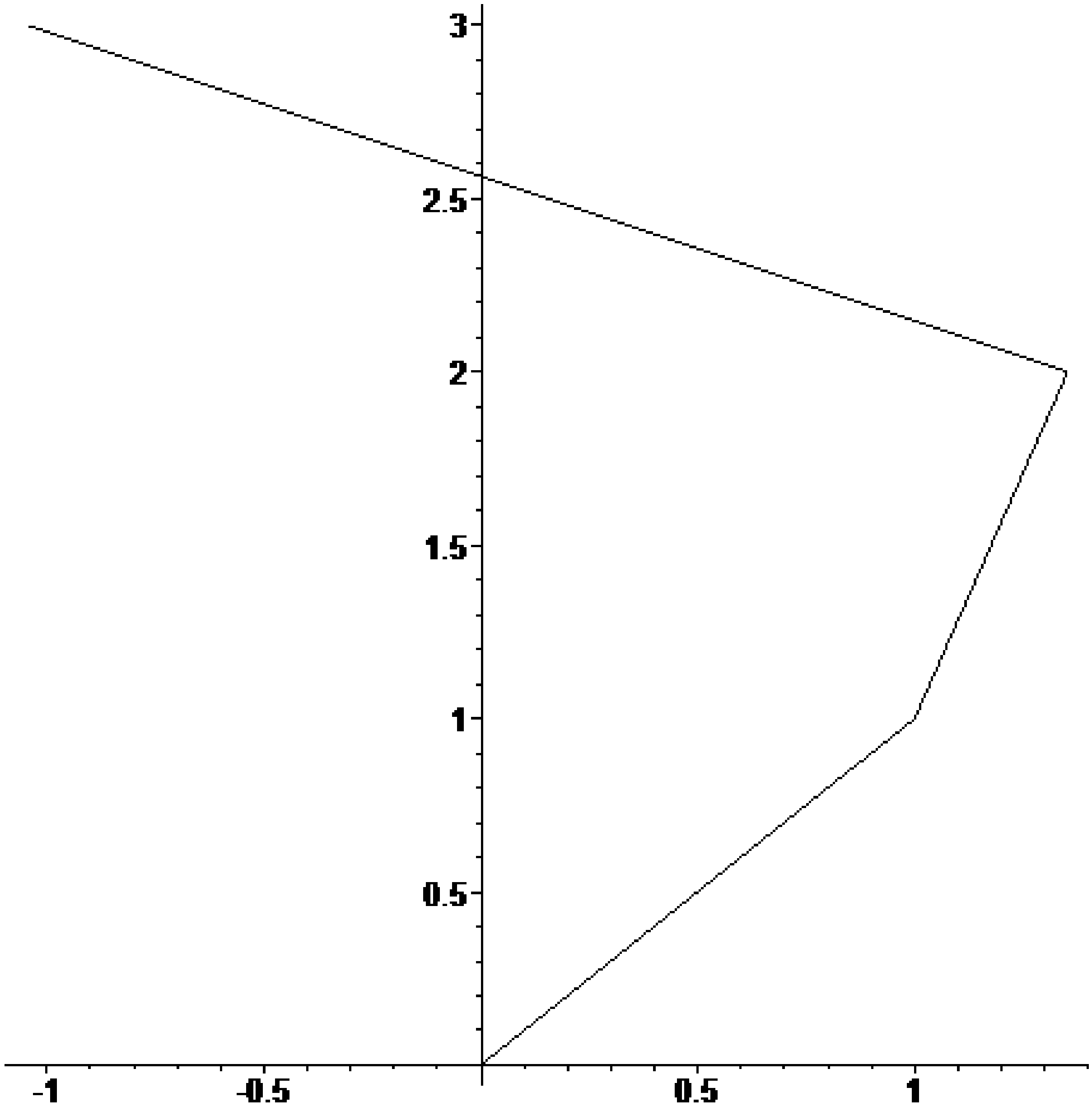
$$a2 := [1.,\ 1.352859819,\ -1.045260255]$$

```
> with(plots);
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *graphplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]

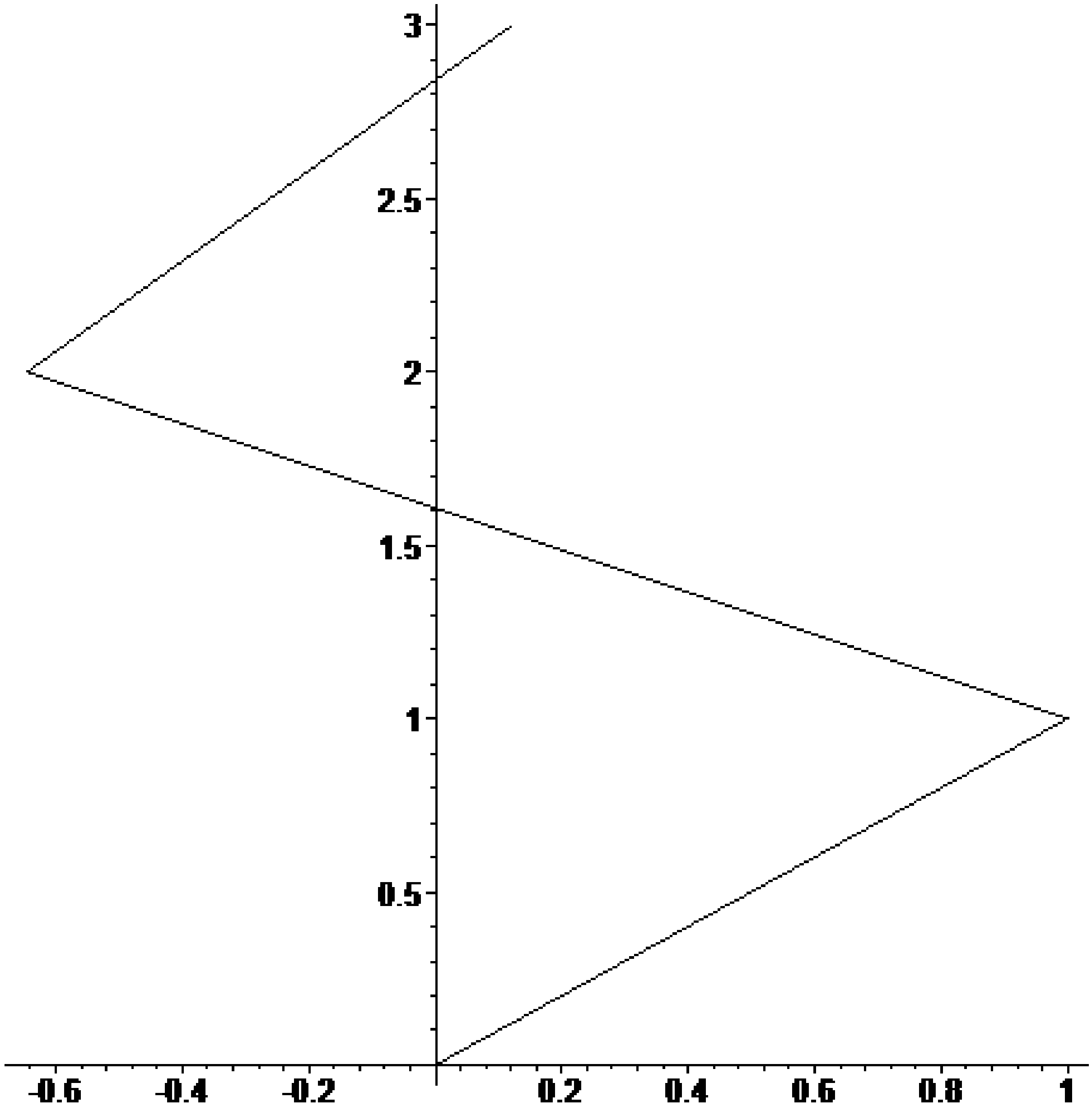
```
> 
pointplot([[0,0],[a1[1],1],[a1[2],2],[a1[3],3]],connect=true); #mode 1
```



>
```
pointplot([[0,0],[a2[1],1],[a2[2],2],[a2[3],3]],connect=true); #mode 2
```


>

```
pointplot([[0,0],[a3[1],1],[a3[2],2],[a3[3],3]],connect=true); #mode 3
```



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