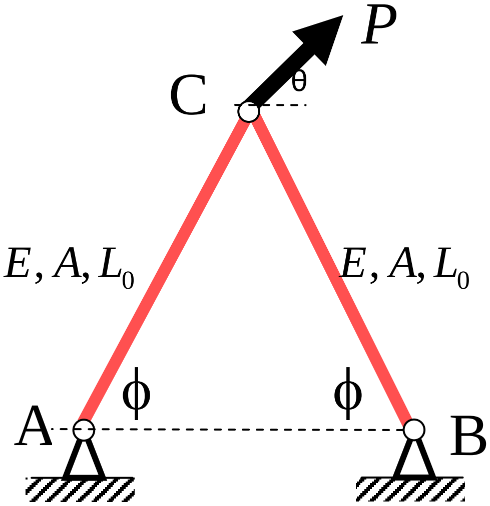
Homework 2
Due Wednesday February 10, 2010

Division of Engineering
Brown University

1. The structure shown below has members of equal length and equal material properties.



Use the stiffness method to determine the member forces and the deflection components of joint C as a function of the angle ϕ.

$$\mathbf{n}_{AC} = \cos\phi\mathbf{i} + \sin\phi\mathbf{j},\ \mathbf{n}_{CB} = \cos\phi\mathbf{i} - \sin\phi\mathbf{j}$$
$$\delta_{AC} = \mathbf{n}_{AC}\bullet\mathbf{u}^{C} = u_x\cos\phi + u_y\sin\phi,$$
$$\delta_{CB} = \mathbf{n}_{CB}\bullet(-\mathbf{u}^{C}) = -u_x\cos\phi + u_y\sin\phi,$$

$$F_{AC} = \frac{EA}{L_0}\delta_{AC} = \frac{EA}{L_0}\left(u_x\cos\phi + u_y\sin\phi\right),$$
$$F_{CB} = \frac{EA}{L_0}\delta_{AC} = \frac{EA}{L_0}\left(-u_x\cos\phi + u_y\sin\phi\right),$$

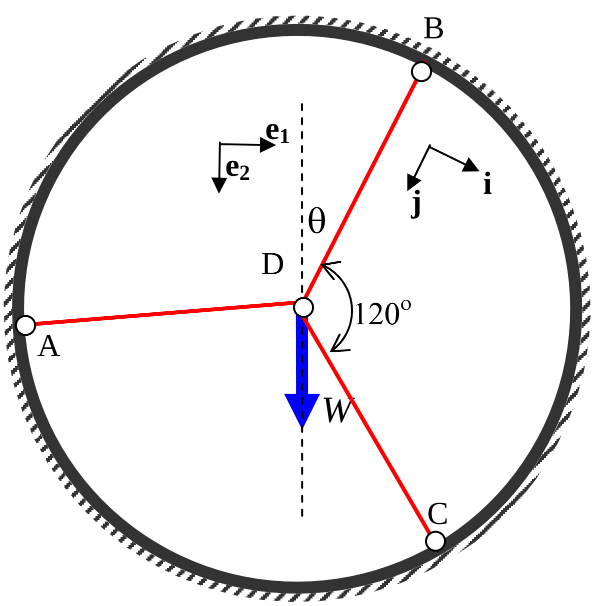
Force balance at joint C:

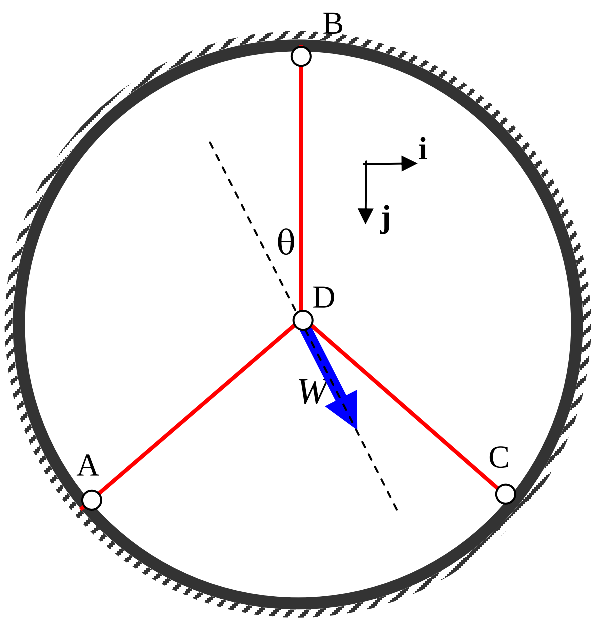
$$\sum F_x = 0 = P\cos\theta + (F_{BC} - F_{AC})\cos\varphi = 0$$
$$\sum F_y = 0 = P\sin\theta - (F_{AC} + F_{BC})\sin\varphi = 0$$
$$P\cos\theta = \left(\frac{2EA}{L_0}\right)u_x(\cos\varphi)^2 \rightarrow u_x = \frac{PL_0\cos\theta}{2EA\,(\cos\varphi)^2}$$
$$P\sin\theta = \left(\frac{2EA}{L_0}\right)u_y(\sin\varphi)^2 \rightarrow u_y = \frac{PL_0\sin\theta}{2EA\,(\sin\varphi)^2}$$

$$F_{AC} = \frac{EA}{L_0}\delta_{AC} = \frac{P\cos\theta}{2\sin\phi} + \frac{P\sin\theta}{2\cos\phi},$$

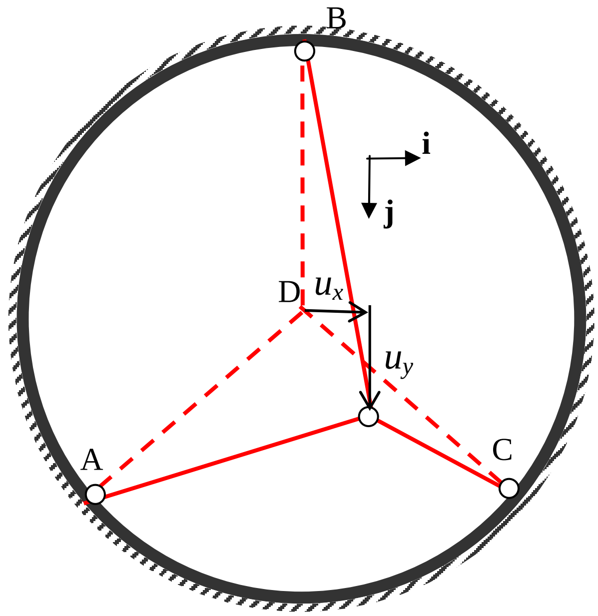
$$F_{CB} = \frac{EA}{L_0}\delta_{AC} = \frac{P\cos\theta}{2\sin\phi} - \frac{P\sin\theta}{2\cos\phi}.$$

2. The figure shows a simple model for a front bicycle wheel with 3 spokes. The rim is assumed to be rigid, and the spokes are pinned at each end. The central hub is subjected to a force W as shown. For simplicity, assume all spokes have uniform cross section *A*, length *L*, and Young's Modulus *E*. In this problem you will examine the forces in the spokes as a function of the orientation θ of the wheel. The second picture is simply rotated to align the axes with the horizontal and vertical directions.





Due to the load on the hub, the hub displaces as shown below, with displacement components u_x, u_y.



a. Use the stiffness method to determine the member forces and the displacement components u_x, u_y as a function of *W*, θ, L_0 and *EA*.

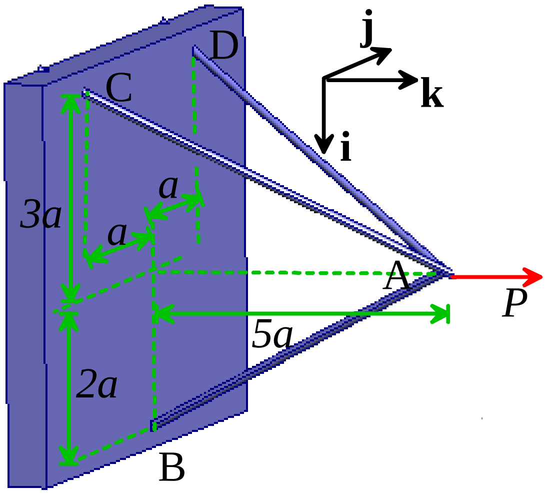
See Maple for the solution to this problem

b. Find the internal forces in members DA, DB and DC in terms of *W*and θ. Make a single graph showing T_{DA}, T_{DB}, and T_{DA} (normalized by *W*) versus the wheel orientation θ.

c. For a given value of the load *W,* which orientation has the highest spoke forces?

d. Find the components of displacement u_1 and u_2 in the $\mathbf{e_1}$, $\mathbf{e_2}$ basis in terms of W, θ, L_0 and *EA*.

3. Consider the structure shown below and discussed in a lecture. The loading at point A is slightly different here.



Find the deflection of the joint A and forces in the members under the load P. Use the stiffness method.

Unit vectors:

$$\mathbf{n}_{AB} = (2a\mathbf{i} - 5a\mathbf{k})/\sqrt{29a^2} = (2\mathbf{i} - 5\mathbf{k})/\sqrt{29} \qquad \mathbf{n}_{AC} = (-3\mathbf{i} - \mathbf{j} - 5\mathbf{k})/\sqrt{35}$$

$$\mathbf{n}_{AD} = (-3\mathbf{i} + \mathbf{j} - 5\mathbf{k})/\sqrt{35}$$

By symmetry, the only nonzero components of displacement at joint A are u_x and u_z.

Elongations are:

$$\delta_{AB} = \mathbf{n}_{AB} \bullet (-\mathbf{u}^C) = (-2u_x + 5u_z)/\sqrt{29}$$

$$\delta_{AC} = \mathbf{n}_{AC} \bullet (-\mathbf{u}^C) = (3u_x + 5u_z)/\sqrt{35}$$

$$\delta_{AD} = \mathbf{n}_{AB} \bullet (-\mathbf{u}^C) = (3u_x + 5u_z)/\sqrt{35}$$

Forces:

$$T_{AB} = \frac{EA}{a\sqrt{29}}\delta_{AB} = k\frac{(-2u_x + 5u_z)}{29}$$

$$T_{AC} = \frac{EA}{a\sqrt{35}}\delta_{AC} = k\frac{(3u_x + 5u_z)}{35} = T_{AD}$$

$$k = \frac{EA}{a}$$

Force Balance:

$$T_{AB}\mathbf{n}_{AC} + T_{AC}\mathbf{n}_{AC} + T_{AD}\mathbf{n}_{AD} + P\mathbf{k} = 0$$

$$T_{AB}(2\mathbf{i} - 5\mathbf{k})/\sqrt{29} + T_{AC}(-3\mathbf{i} - \mathbf{j} - 5\mathbf{k})/\sqrt{35} + T_{AD}(-3\mathbf{i} + \mathbf{j} - 5\mathbf{k})/\sqrt{35} + W\mathbf{i} = \mathbf{0}$$

Maple Time!