

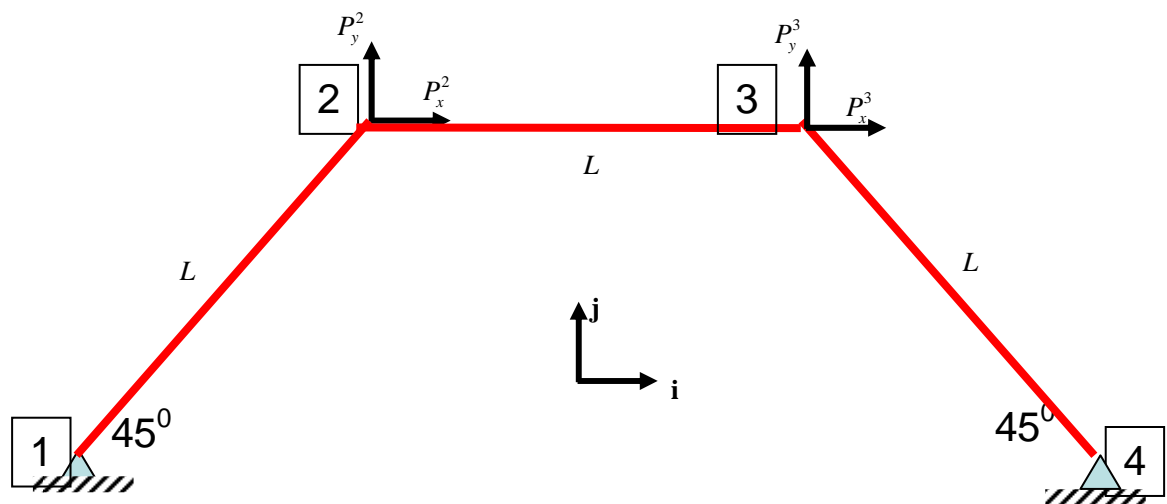


## ENGN1300: Structural Analysis

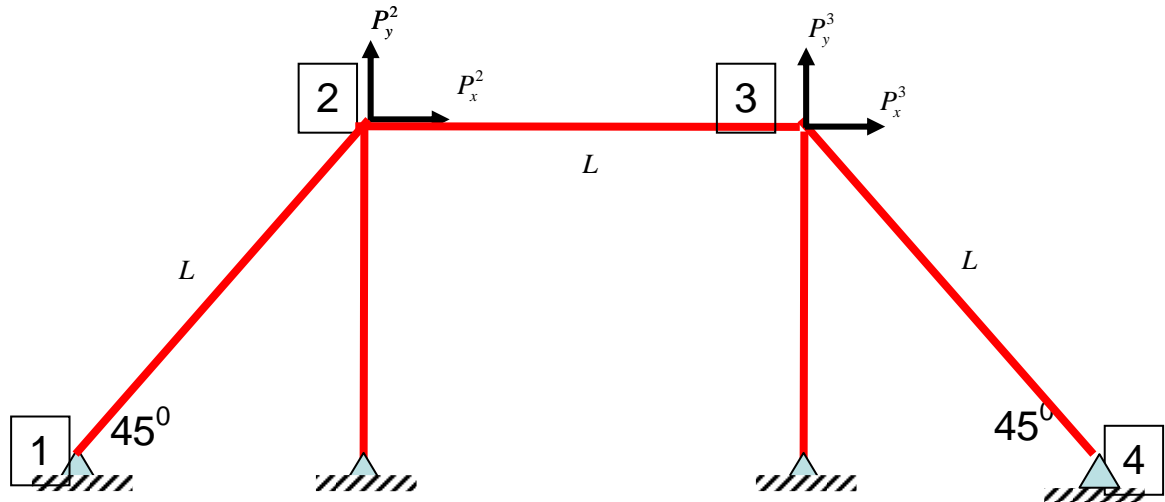
### Homework 3 Due Friday February 19, 2010

Division of Engineering  
Brown University

1. For the structure shown, all members have equal properties  $EA$  and  $L$ . Determine the unit vectors for the members, use these to determine the stiffness matrix for the structure. Find the eigenvalues and null vectors and explain the mechanism modes for the structure.



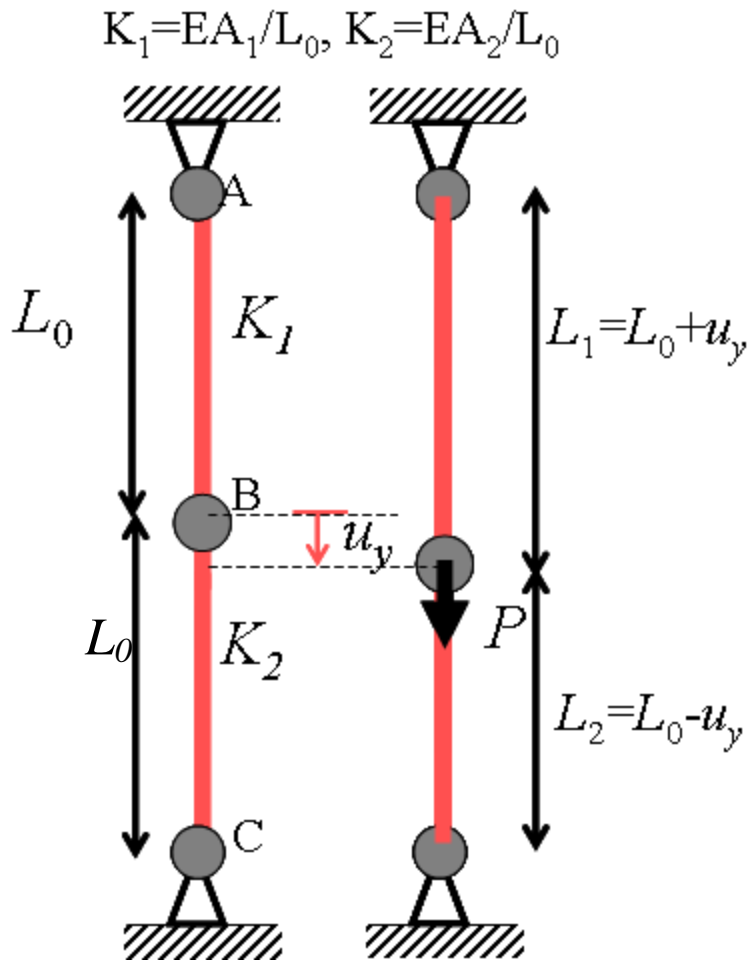
2. Modify the structure above by adding two members as shown below. (Same  $EA$ )  
Determine the stiffness matrix for the modified structure.



Determine the joint displacements and member forces for the cases in which:

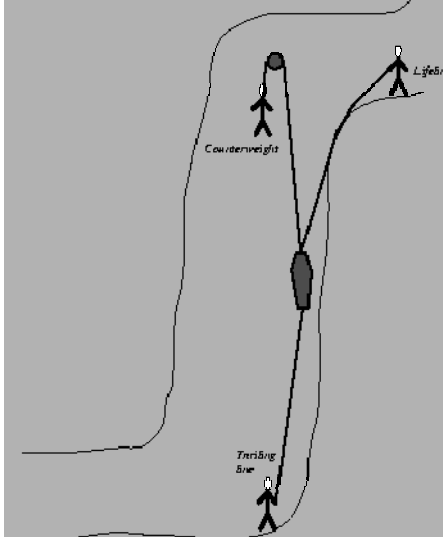
- a)  $P_x^2 = P_x^3 = 0$ ,  $P_y^2 = P_y^3 = P_y$ .  
b)  $P_x^2 = P_x^3 = P_x$ ,  $P_y^2 = P_y^3 = 0$ .

3. The 1D truss below is statically indeterminate. The rods have equal  $E$  and  $L$ . The initial cross sections for the two rods are  $A_1$  (rod AB) and  $A_2$  (rod BC). A force  $P$  is applied at joint B, and that joint undergoes a displacement  $u_y$  as shown. The goal of this problem is to use the principal of minimum potential energy to find the displacement of joint B and the forces in the members.

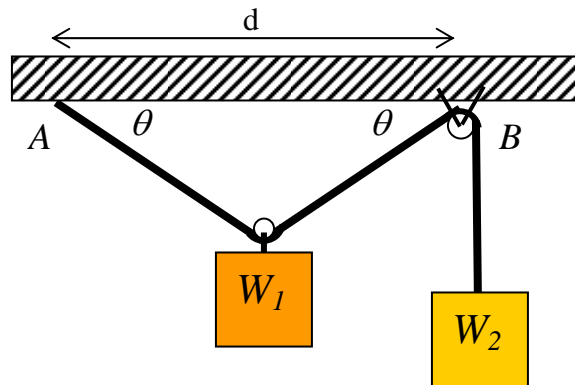


- Write an expression for the potential energy  $V(u_y)$  of the structure in terms of  $u_y$ ,  $P$ ,  $K_1$ , and  $K_2$ . Include the potential energy due to stretching of each member and the potential energy due to the load applied at joint B.
- Find the value of  $u_y$  that minimizes the total potential energy of the structure. for given values of  $P$ ,  $u_y$ ,  $K_1$ , and  $K_2$ .
- Using the minimizing value of the displacement found in part b, compute the force in each member. Be sure to state whether the members are in tension or compression.
- Using the forces found in part c, verify that there is no net force at joint B.

4. A common rescue technique in mountaineering or caving accidents is called the counterweight method. A rescuer or a weighted backpack hangs from a pulley, serving as a counterweight, while the injured person is suspended at center rope. See the figure below. <http://www.mcs.le.ac.uk/~glowe/Caving/rescue/rescue.html>

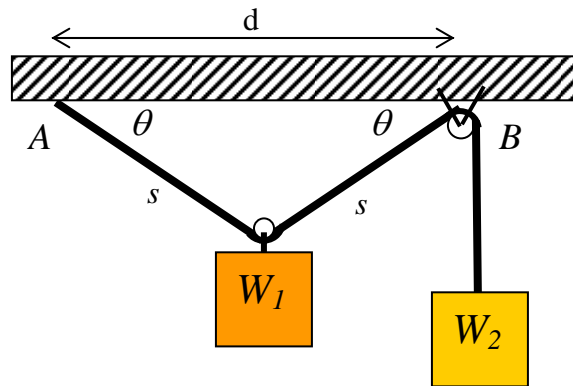


An idealization is shown below. A weight  $W_1$  is suspended by a cable, which is attached to a counter weight  $W_2$  as shown. The pulley at  $B$  is frictionless. The goal of this problem is to determine the angle  $\theta$  between the cable and the ceiling when the configuration is in equilibrium. The problem is used here to demonstrate the power of the minimum potential energy principal for mechanical systems.



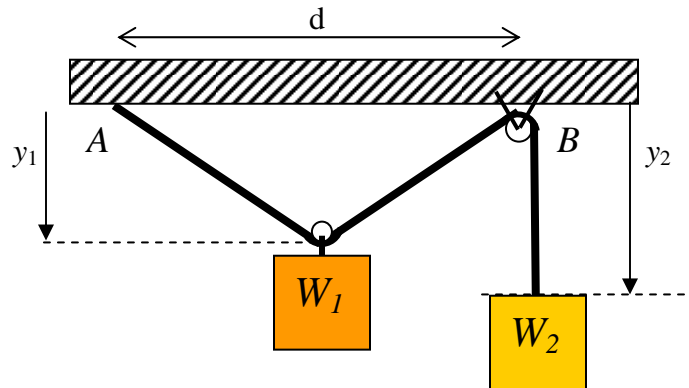
- a. Draw a free body diagrams of weight  $W_1$  and  $W_2$  and derive an expression for the angle  $\theta$  as a function of the ratio  $W_1/W_2$ .

- b. Assume the cable is very long, compared with the dimension  $d$ . Using your answer to part b, show that the arrangement of weights does not have an equilibrium solution unless  $W_1 < 2W_2$ . What do you think would happen if you tried to hang weights with  $W_1 > 2W_2$ ?
- c. In practice, the rope has a finite length  $L$ , and the counterweight  $W_2$  will hit pulley  $B$  if  $W_1$  is too large. Show that the weight  $W_2$  hits the pulley if  $W_1 \geq 2W_2 \sqrt{1 - d^2 / L^2}$ .



- d. In this part of the problem, you will find the equilibrium position of the system using minimum energy principles.

Let  $y_1$  and  $y_2$  denote the distance below the supports that each weight hangs. Express  $y_1$  and  $y_2$  in terms of the angle  $\theta$ , and hence write the total potential energy of the system as a function of  $\theta$ .



- e. Find the value of  $\theta$  that minimizes the total potential energy of the system. Compare the answer with the one found in part b above.
- f. For a caving rescue, suppose  $d=2\text{m}$  and  $L=100\text{m}$  the injured climber (weight  $W_1=150\text{lbs}$ ) is to be brought to a distance  $y_I \leq 1\text{m}$ . Determine the minimum counter weight  $W_2$  required.