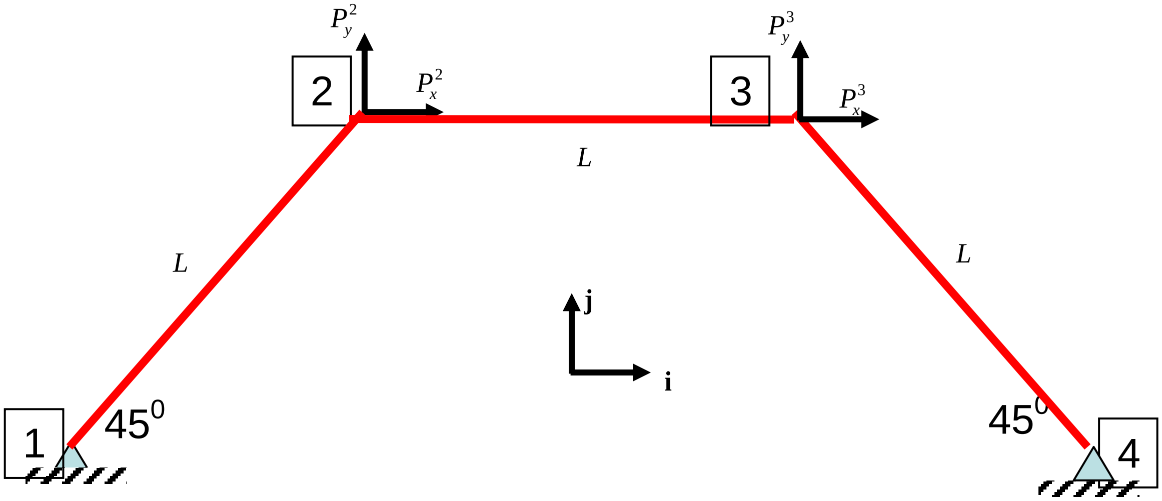
ENGN1300: Structural Analysis

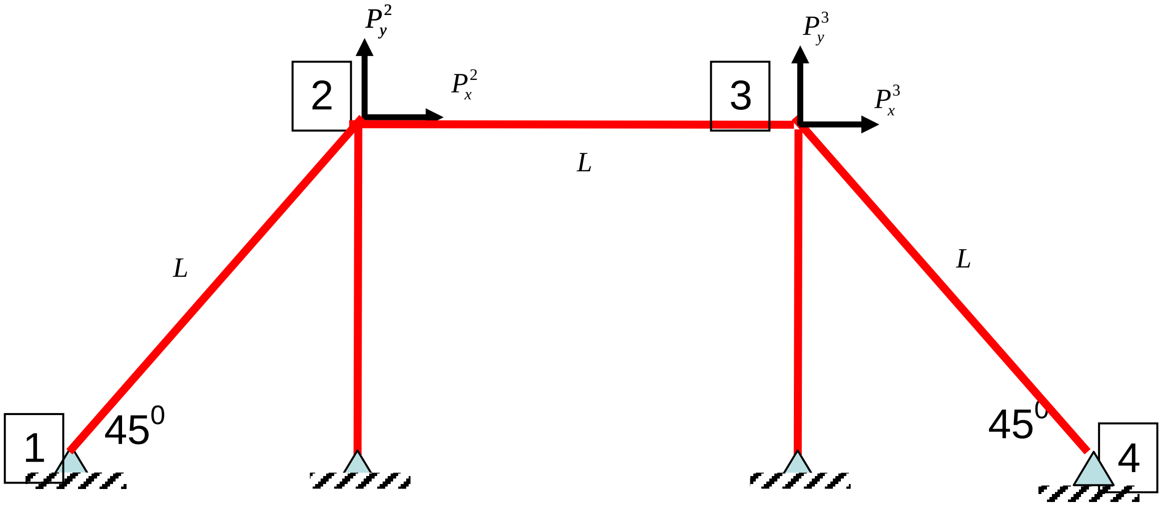
Homework 3
Due Wednesday February 17, 2010

Division of Engineering
Brown University

1. For the structure shown, all members have equal properties *EA* and *L*. Determine the unit vectors for the members, use these to determine the stiffness matrix for the structure. Find the eigenvalues and null vectors and explain the mechanism modes for the structure.



2. Modify the structure above by adding two members as shown below. (Same *EA*) Determine the stiffness matrix for the modified structure.

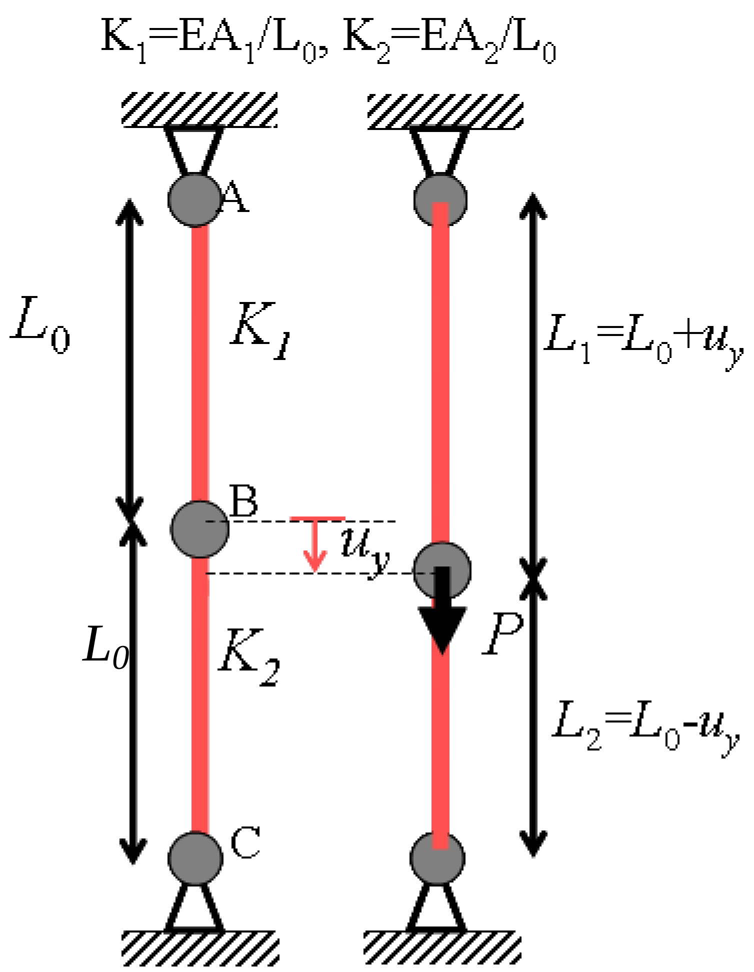


Determine the joint displacements and member forces for the cases in which:

a) $P_x^2 = P_x^3 = 0,\ P_y^2 = P_y^3 = P_y.$

b) $P_x^2 = P_x^3 = P_x,\ P_y^2 = P_y^3 = 0.$

3. The 1D truss below is statically indeterminate. The rods have equal *E* and *L*. The initial cross sections for the two rods are A_1 (rod AB) and A_2 (rod BC). A force P is applied at joint B, and that joint undergoes a displacement u_y as shown. The goal of this problem is to use the principal of minimum potential energy to find the displacement of joint B and the forces in the members.



a. Write an expression for the potential energy $V(u_y)$ of the structure in terms of u_y, P, K_1, and K_2. Include the potential energy due to stretching of each member and the potential energy due to the load applied at joint B.

$$V_{stretching}(u_y) = \frac{1}{2}K_1u_y^2 + \frac{1}{2}K_2u_y^2 \text{ and } V_{load}(u_y) = -Pu_y$$

$$\Rightarrow V(u_y) = -Pu_y + \frac{1}{2}(K_1+K_2)u_y^2$$

b. Find the value of u_y that minimizes the total potential energy of the structure. for given values of P, u_y, K_1, and K_2.

$$V(u_y) = -Pu_y + \frac{1}{2}(K_1+K_2)u_y^2$$

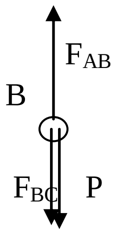
$$V'(u_y) = -P + (K_1+K_2)u_y = 0$$

$$\Rightarrow u_y = P/(K_1+K_2)$$

c. Using the minimizing value of the displacement found in part b, compute the force in each member. be sure to state whether the members are in tension or compression.

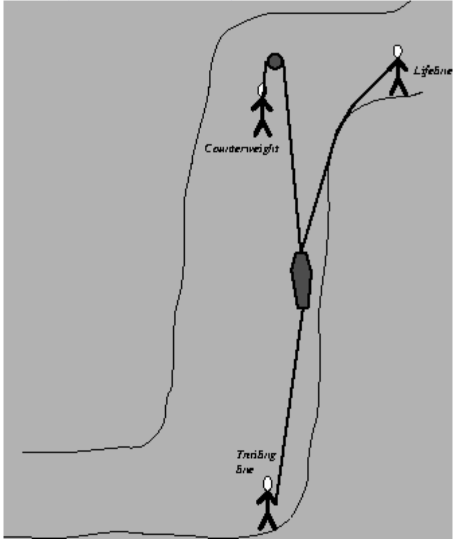
$$F_{AB} = K_1(L_1\text{-}L_0) = K_1\ u_y = PK_1/(K_1+K_2)\ \text{(tension)}$$
$$F_{BC} = K_2(L_2\text{-}L_0) = \text{-}K_2\ u_y = \text{-}PK_2/(K_1+K_2)\ \text{(compression)}$$

d. Draw a free body diagram for joint B. Using the forces found in part c, verify that the there is no net force at joint B. **(3 point)**

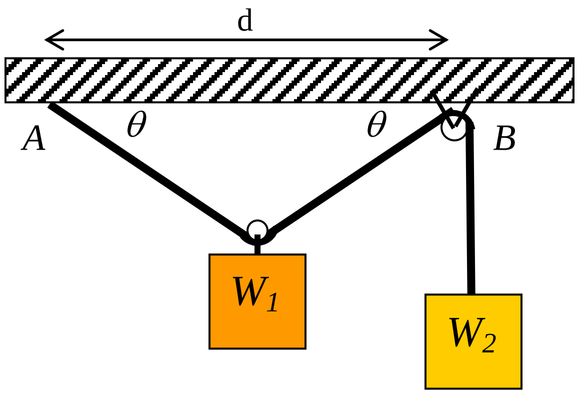


$$F_{AB} - F_{BC} - P = P\frac{K_1 + K_2}{K_1 + K_2} - P = 0$$

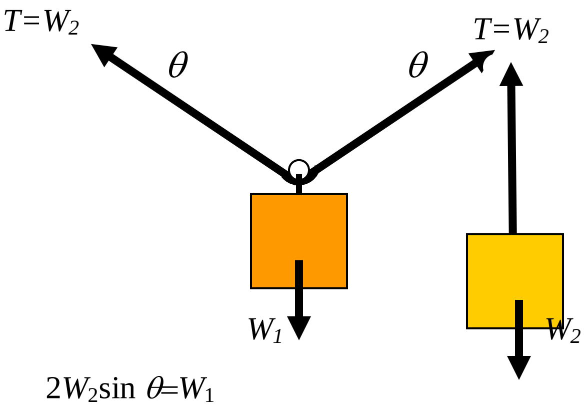
4. A common rescue technique in mountaineering or caving accidents is called the counterweight method. A rescuer or a weighted backpack hangs from a pulley, serving as a counterweight, while the injured person is suspended at center rope. See the figure below. http://www.mcs.le.ac.uk/~glowe/Caving/rescue/rescue.html



An idealization is shown below. A weight W_1 is suspended by a cable, which is attached to a counter weight W_2 as shown. The pulley at B is frictionless. The goal of this problem is to determine the angle θ between the cable and the ceiling when the configuration is in equilibrium. The problem is used here to demonstrate the power of the minimum potential energy principal for mechanical systems.



a. Draw a free body diagrams for weights W_1 and W_2 and hence derive an expression for the angle θ as a function of the ratio W_1/W_2.



$$2W_2\sin\theta=W_1$$

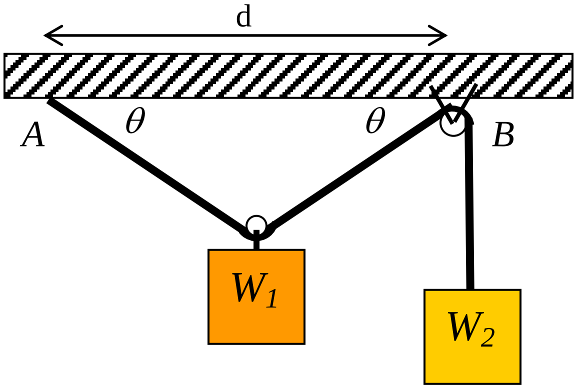
$$\theta = \sin^{-1}[W_1/(2W_2)]$$

b. Assume the cable is very long, compared with the dimension d. Using your answer to part b, show that the arrangement of weights does not have an equilibrium solution unless $W_1<2W_2$. What do you think would happen if you tried to hang weights with $W_1>2W_2$?

Follows since $\sin\theta = W_1/(2W_2)<1$

c. In practice, the rope has a finite length L, and the counterweight W_2 will hit pulley B if W_1 is too large. Show that the weight W_2 hits the pulley if

$W_1 \geq 2W_2\sqrt{1-d^2/L^2}$.



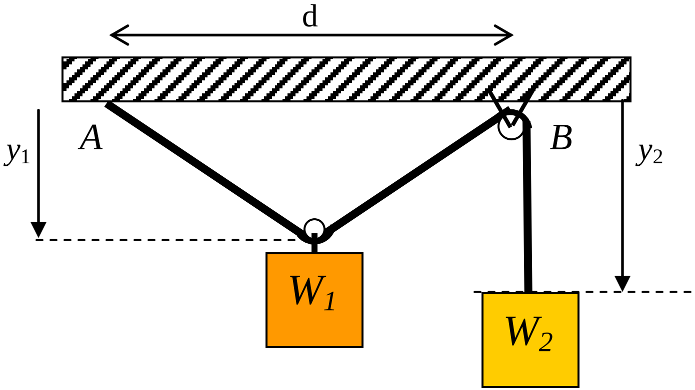
$s = d/(2\cos\theta) = d/\sqrt{1-\sin^2\theta} = d/\sqrt{1-(W_1/2W_2)^2}$

When W_2 hits, $L = 2s$.

$(L/d)^2 = 1-(W_1/2W_2)^2$, so $W_1 = 2W_2\sqrt{1-d^2/L^2}$.

d. In this part of the problem, you will find the equilibrium position of the system using minimum energy principles.

Let y_1 and y_2 denote the distance below the supports that each weight hangs. Express y_1 and y_2 in terms of the angle θ, and hence write the total potential energy of the system as a function of θ.



$$y_1 = \left(\frac{d}{2}\right)tan\theta, y_2 = L - \frac{d}{cos\theta}$$

$$V(\theta) = -W_1\left(\frac{d}{2}tan\theta\right) - W_2\left(L - \frac{d}{cos\theta}\right)$$

$$V'(\theta) = -\frac{W_1 d}{2cos\theta} + \frac{W_2 dsin\theta}{\cos^2\theta}$$

$$\frac{d}{\cos^2\theta}\left(W_2 sin\theta - \frac{W_1}{2}\right) = 0 \rightarrow W_2 sin\theta = \frac{W_1}{2} \rightarrow \theta = \sin^{-1}(\frac{W_1}{2W_2})$$

e. Find the value of θ that minimizes the total potential energy of the system. Compare the answer with the one found in part b above.

f. For a caving rescue, suppose d=2m and L=100m the injured climber (weight W_1=150lbs) is to be brought to a distance $y_1 \leq$ 1m. Determine the minimum counter weight W_2 required.

$$y_1 = \frac{d}{2}\tan\theta = \frac{d}{2}\frac{\sqrt{1-(W_1/2W_2)^2}}{(W_1/2W_2)} = \frac{d}{2}\sqrt{(2W_2/W_1)^2-1}.$$

$$W_2 = \frac{W_1}{2}\sqrt{1+(2y_1/d)^2} = 75\,\text{lbs}\sqrt{1+1} = 106\,\text{lbs}$$