

**ENGN1300: Structural Analysis** 

Homework 3 Due Wednesday, March 3, 2010

Division of Engineering Brown University

1. For the statically indeterminate structure shown below, all members have identical values of *EA*. Members 12 and 23 have length *L*.



- a. Write the total potential energy V of the structure as a function of the two displacement components  $(u_x, u_y)$  of joint 2. Assume these are small compared with L.
- b. Apply the minimum potential energy principle, so that  $\frac{dV}{du_x} = \frac{dV}{du_y} = 0$ . Hence,

derive a pair of equations for  $(u_x, u_y)$ . Put the equations into the matrix form **Ku=p** and identify the stiffness matrix for the structure.

- c. Solve for  $(u_x, u_y)$  and the member forces in terms of the problem parameters.
- Using energy minimization, find the member forces and nodal displacements in the following three-dimensional truss. To simplify the problem, you can assume members CD, BC, and BD are very stiff compared with the vertical members, so that joints C, D, and B are essentially pinned. The other members have equal properties EA. Assume small displacements.
  - a. Find unit vectors parallel to each member.
  - b. Find the extension  $\delta$  for each member in terms of the vertical displacement  $u=-u_z$ . of joint A. Note that symmetry tells you that  $u_x=u_y=0$ .
  - c. Write the total potential energy as a function of *u*.
  - d. Minimize this expression and solve for *u*.

- e. Calculate the force in each member using this *u*.
- f. The stand is made from aluminum tubes with outer diameter 1 inch and inner diameter 0.8 inches. Calculate the critical buckling load for each leg of the structure.
- g. What value of the applied load *P* (in pounds) will cause the legs to buckle?



- 3. Study the <u>Maple Truss Analysis program</u>. Download a copy of program and the tutorial truss input file. Use the program to check your answers to problem 1. Let EA=1 and run the program once for each of the cases in which  $(P_1, P_2)=(1,0)$  and  $(P_1, P_2)=(0,1)$
- 4. Use the program to analyze the tandem frame considered in Problem Set 1, but subjected to the following loads:



Assume that all the members are made from steel, and have a hollow circular cross-section with OD=2cm and ID=1.8cm. Take L=0.6m and W=700N.

Compressive members of the frame are subject to failure by buckling. Find the buckling load for each compressive member. You might want to use Excel or some such program to find the member lengths and buckling load.

Report a safety factor for each compressive member (the safety factor is defined as the internal force that will cause the member to fail, divided by the actual internal force in the member). Calculate the overall factor of safety for the design, which is the smallest factor of safety in any member. Which member is the weak link?

5. The Maple truss program is coded to solve two-dimensional problems only. For this assignment you are asked to modify the code to solve three-dimensional problems. To simplify your assignment, you need not worry about the on-screen graphics. Download and modify the non-graphical version of the 2-D truss code

http://www.engin.brown.edu/courses/en130/maple\_fem/texttruss.mws

This way the graphics commands won't be in the way, and you can concentrate on the rest of the code. For anyone who wants big league extra credit, you may add some or all of the graphics to your 3-D code.

Turn in your code and the input/output files for the following two problems:

- a. Run problem 2 above. Put EA=P=1.
- b. Model the stand from problem 2 to include the effects of members CD, BC, and BD. Assume that they have the same stiffness as the other members.(*EA*=1). Assume that joint C is pinned, joint D cannot move in the *y* or *z* direction, and B cannot move in the vertical direction.