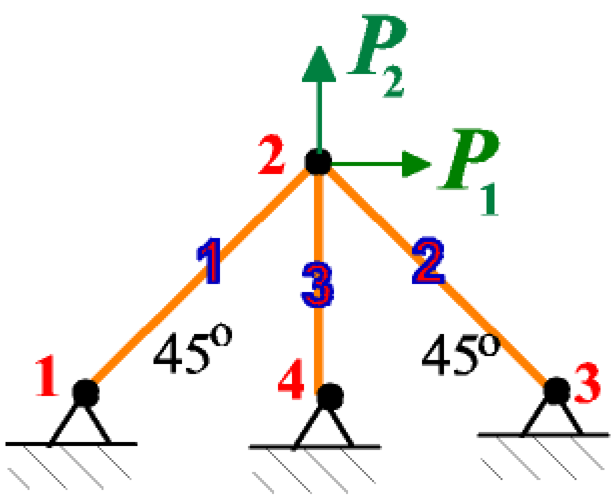
ENGN1300: Structural Analysis

Homework 4
Due Wednesday, March 3, 2010

Division of Engineering
Brown University

1. For the statically indeterminate structure shown below, all members have identical values of *EA*. Members 12 and 23 have length *L*.



See Maple. Unit vectors are $\mathbf{n}^{12} = (\mathbf{i}+\mathbf{j})/\sqrt{2}, \mathbf{n}^{42} = \mathbf{j}, \mathbf{n}^{23} = (\mathbf{i}-\mathbf{j})/\sqrt{2}$

   a. Write the total potential energy V of the structure as a function of the two displacement components (u_x,u_y) of joint 2. Assume these are small compared with *L*.

   b. Apply the minimum potential energy principle, so that $\frac{dV}{du_x} = \frac{dV}{du_y} = 0$. Hence, derive a pair of equations for (u_x,u_y). Put the equations into the matrix form **Ku=p** and identify the stiffness matrix for the structure.

   c. Solve for (u_x,u_y) and the member forces in terms of the problem parameters.

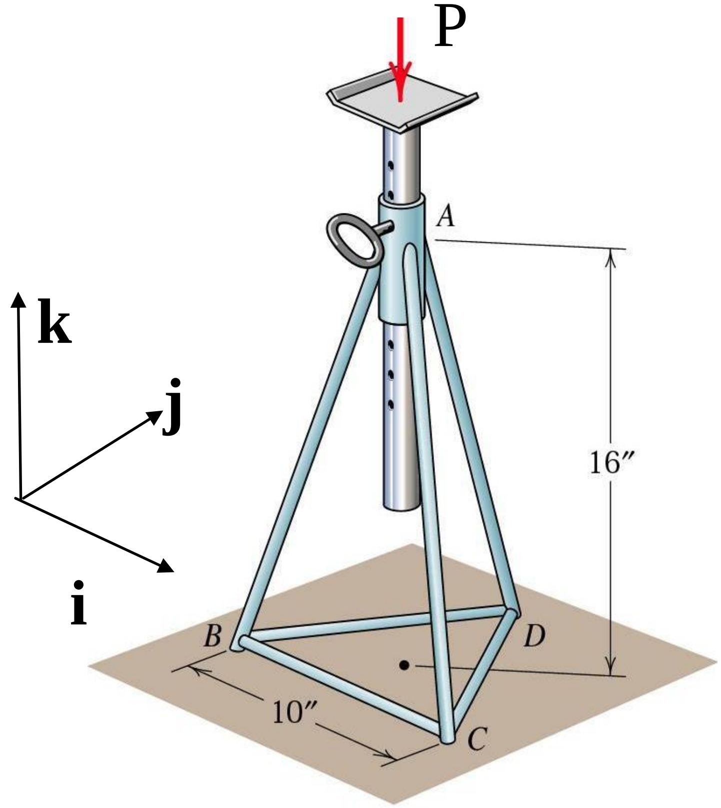
2. Using energy minimization, find the member forces and nodal displacements in the following three-dimensional truss. To simplify the problem, you can assume members CD, BC, and BD are very stiff compared with the vertical members, so that joints C, D, and B are essentially pinned. The other members have equal properties EA. Follow the steps.

   a. Find unit vectors parallel to each member.

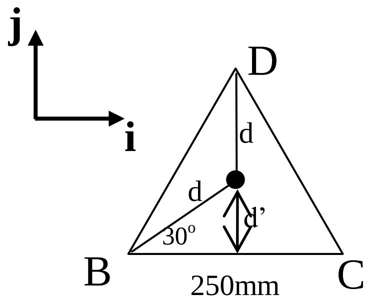
   b. Find the extension δ for each member in terms of the vertical displacement $u=-u_z$. of joint A. Note that symmetry tells you that $u_x=u_y=0$.

   c. Write the total potential energy as a function of *u*.

d. Minimize this expression and solve for *u*.
e. Calculate the force in each member using this *u*.
f. The stand is made from aluminum tubes with outer diameter 1 inch and inner diameter 0.8 inches. Calculate the critical buckling load for each legs of the structure.
g. What value of the applied load *P* (in pounds) will cause the legs to buckle?



Base Geometry shown below.



cos30=125/d d=5.8 in
tan30=d'/5 d'=2.9 in

$\mathbf{n}^{AB}$=(-5**i**-2.9**j**-16**k**)/sqrt(52+2.92+162)= -.29**i**-.17**j**-.94**k**
$\mathbf{n}^{AC}$=.29**i**-.17**j**-.94**k**
$\mathbf{n}^{AD}$=(5.8**j**-16**k**)/sqrt(5.82+162)=0.34**j**-.94**k**
Stretch in each member is the same since $\mathbf{u}^A=u_z\mathbf{k}$, and

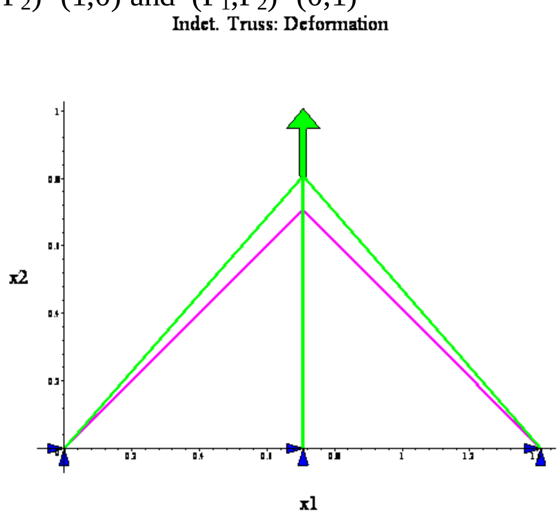
$$\delta^{AB}=-\mathbf{u}^A\cdot\mathbf{n}^{AB}=0.94u_z$$

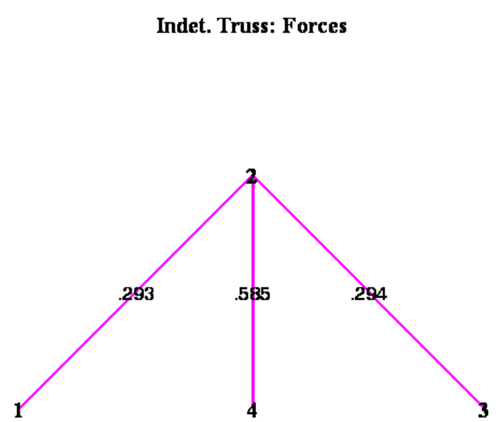
$$\delta^{AD}=-\mathbf{u}^A\cdot\mathbf{n}^{AD}=0.94u_z$$

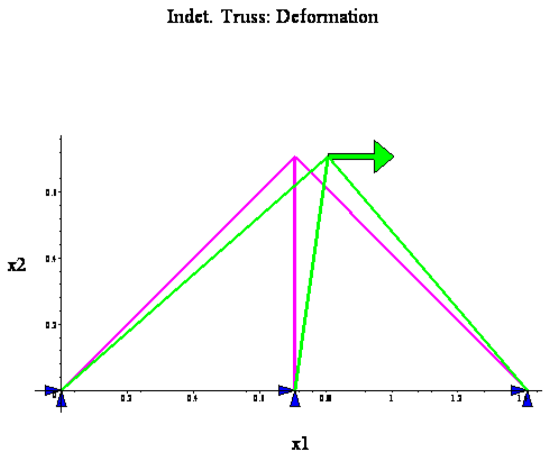
$$\delta^{AC}=-\mathbf{u}^A\cdot\mathbf{n}^{AB}=0.94u_z$$

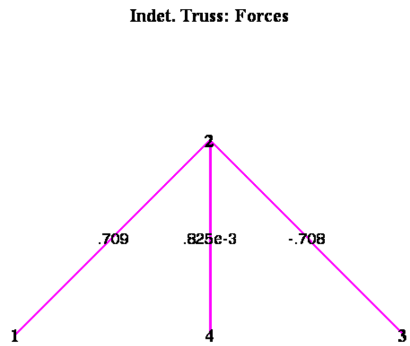
$$V=3\frac{1}{2}K\left(\delta^{AB}\right)^2-(-P\mathbf{k}\cdot\mathbf{u}^A)=\frac{3}{2}K\left(0.94u_z\right)^2+Pu_z$$ K=EA/L. See Maple

3. Study the Maple Truss Analysis program. Download a copy of the truss analysis program and the tutorial truss input file. Use the program to check your answers to problem 1. Let EA=1 and run the program once for each of the cases in which (P_1, P_2)=(1,0) and (P_1,P_2)=(0,1)

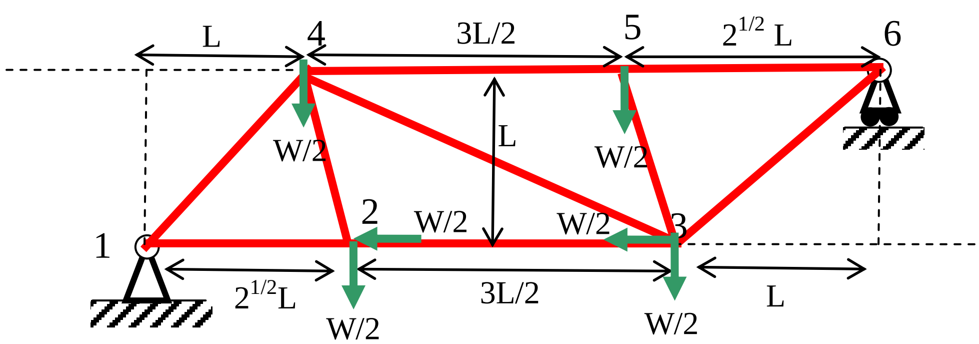








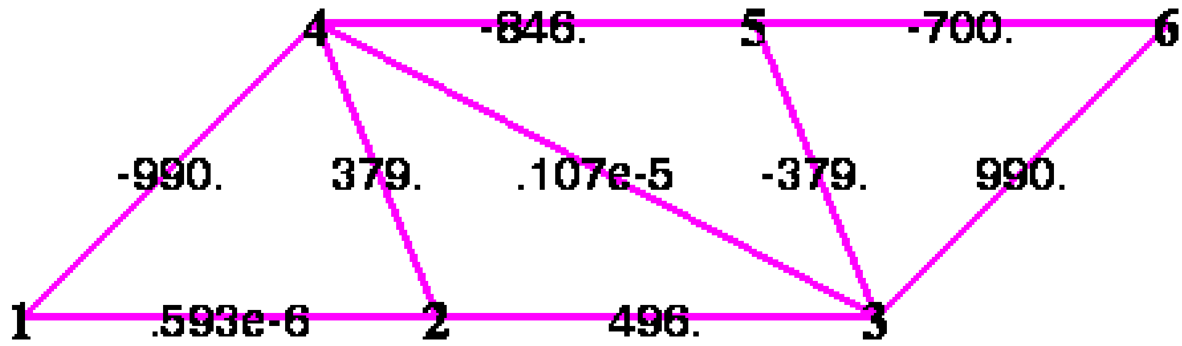
4. Use the program to analyze the tandem frame considered in Problem Set 1, but subjected to the following loads:



Assume that all the members are made from steel, and have a hollow circular cross-section with OD=2cm and ID=1.8cm. Take $L=0.6$m and $W=700$N.
Compressive members of the frame are subject to failure by buckling. Find the buckling load for each compressive member. You might want to use Excel or some such program to find the member lengths and buckling load.

Report a safety factor for each compressive member (the safety factor is defined as the internal force that will cause the member to fail, divided by the actual internal force in the member). Calculate the overall factor of safety for the design, which is the smallest factor of safety in any member. Which member is the weak link?

Output forces shown below (Newtons)



Compressive members of the frame are subject to failure by buckling. Find the buckling load for each compressive member. You might want to use Excel or some such program to find the member lengths and buckling load

Excell calculation:

OD	0.02	m
ID	0.018	m
I	2.70E-09	m^4
E	2.10E+11	N/m^2

Member	**L (m)**	**Stiffness N/m**	**Buckling Load N**	**Force N**	**Safety**
12	0.848528	1.50E+07	-7775.15	0	
23	0.9	1.40E+07	-6911.24	495.8	
36	0.848528	1.50E+07	-7775.15	989.9	
14	0.848528	1.50E+07	-7775.15	-989.9	7.85448
24	0.649435	1.90E+07	-13273	379.1	
34	1.295907	9.67E+06	-3333.96	0	
35	0.649435	1.90E+07	-13273	-379.1	35.01187
56	0.848528	1.50E+07	-7775.15	-700	11.10736
45	0.9	1.40E+07	-6911.24	-846	8.169314

Report a safety factor for each compressive member (the safety factor is defined as the internal force that will cause the member to fail, divided by the actual internal force in the member). Calculate the overall factor of safety for the design, which is the smallest factor of safety in any member. Which member is the weak link?

Safety factor is 7.85; weak member is 14.