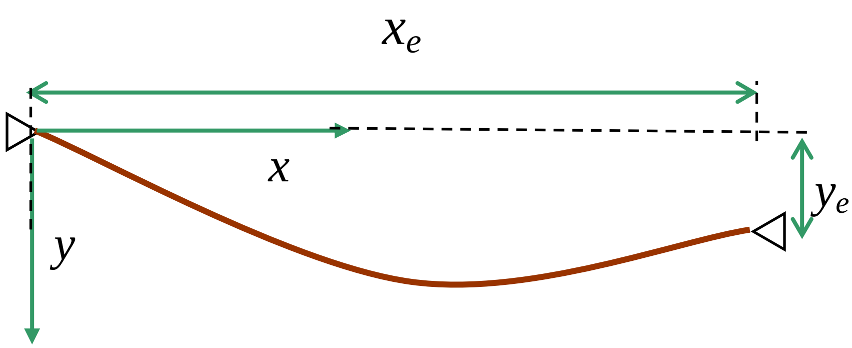
ENGN1300: Structural Analysis

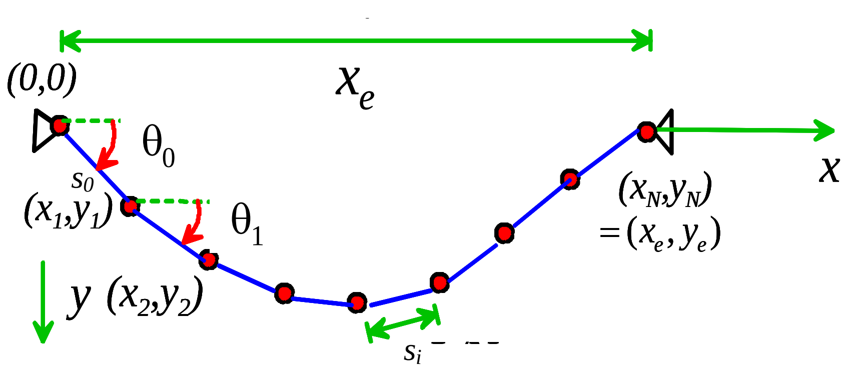
Homework 6
Due Friday, March 19, 2010

Division of Engineering
Brown University|

1. In this problem you will modify your cable spreadsheet from homework 5 to calculate the shape of an elastic cable (modulus of elasticity E and cross sectional area A) with weight per unit length (along the cable) ω, and distributed load $w(x)=w_0$ (across the span).



As described in class, the cable will be approximated as a series of N short segments. Each segment has original length $s = L_O/N$ and weight ωs. When deformed, the length of each segment will be s_i with $s_i > s$. The shape of the cable is determined by the angles $\theta_0, \theta_1, \theta_2 .... \theta_{N-1}$ and lengths $s_0, s_1, s_2 .... s_{N-1}$ of each segment.



The coordinates of successive points on the curve follow as

$$x_1 = s_0 \cos\theta_0, \quad x_2 = x_1 + s_1 \cos\theta_1 \;\; .... \;\; x_i = x_{i-1} + s_{i-1}\cos\theta_{i-1}$$
$$y_1 = s_0 \sin\theta_0, \quad y_2 = y_1 + s_1 \sin\theta_1 \;\; .... \;\; y_i = y_{i-1} + s_{i-1}\sin\theta_{i-1}$$
$$x_N = x_e \quad y_N = y_e$$

Use the solver on Excel to find the values of the angles θ_0, θ_1, θ_2, … θ_{N-1} and $s_0, s_1, s_2 .... s_{N-1}$ that minimize the potential energy V for the cable, subject to the constraint $(x_N, y_N) = (x_e, y_e)$. Note that the total energy for the cable will include the gravitational potential energy of the cable's self weight (as before), the energy associated with the load $w(x)=w_0$ distributed evenly across the span, and the elastic energy in each segment of the cable:

$$V_0^g = -\frac{1}{2}\omega s y_1 \qquad V_1^g = -\frac{1}{2}\omega s(y_1 + y_2) \;\; ..... \;\; V_i^g = -\frac{1}{2}\omega s(y_i + y_{i+1})$$

$$V_0^L = -\frac{1}{2}w_0 x_1 y_1 \qquad V_1^L = -\frac{1}{2}w_0(x_2 - x_1)(y_2 + y_1) \;\; ..... \;\; V_i^L = -\frac{1}{2}w_0(x_{i+1} - x_i)(y_i + y_{i+1})$$

$$V_0^{elast} = \frac{1}{2}\frac{EA}{s}(s_0 - s)^2 \qquad V_1^{elast} = \frac{1}{2}\frac{EA}{s}(s_1 - s)^2 \;\; ..... \;\; V_i^{elast} = \frac{1}{2}\frac{EA}{s}(s_i - s)^2$$

Calculate the tension in each segment. You will have to modify the expression for the tension in the cable near the left endpoint to account for the distributed load w_0:

$$T_0 = \frac{\omega L_0 + w_0 x_e}{\sin\theta_0 - \cos\theta_0 \tan\theta_e}$$

The tension in successive segments is again found from the fact that the horizontal component of force in the cable is constant: $T_i = T_0\cos\theta_0/\cos\theta_i$. Plot the cable tension as a function of position in the cable. Note that you are actually finding the tension at the midpoint of each segment.

Show your results for the following cases:

a. L_0=2 m with x_e=1 m, y_e=0 and EA=10 Newtons, ω=1 Newton per meter, w_0=0. Show plots which compare the cable shape and tension in the elastic and inelastic cables (see hw 5 for the inelastic case)

b. L_0=2 m with x_e=1 m, y_e=0 and EA=10 Newtons, ω=1 Newton per meter, w_0=0.5 Newtons per meter.

2. The George Washington Bridge is supported by 4 main cables, each with a cross sectional area of 800 in^2. The cables are cold-drawn steel, with weight density 0.283 lbs/in^3 and Young's Modulus 30,000 ksi. The yield stress of the steel is 184 ksi. The towers are separated by a linear distance of 3500 feet. The bridge was initially designed to carry 8 lanes of traffic and 4 lanes of rail. It was later modified to carry 14 lanes of traffic. In what follows, the initial design loads are considered. Note on units: kips=kilopounds=10^4 pounds. ksi=kips/in^2, ksi=kips/$foot^2$, kpf=kips/ft, etc.

The cables carry the weight of the suspenders, deck and traffic, which may be calculated as follows:

i. Suspenders (average) 0.6kpf
ii. Deck steel structure: 11kpf
iii. Concrete slabs: 6kpf
iv. Nonstructural elements (lights, rails, etc) 10kpf

The total dead load is therefore 27.6kpf and is evenly distributed across the span

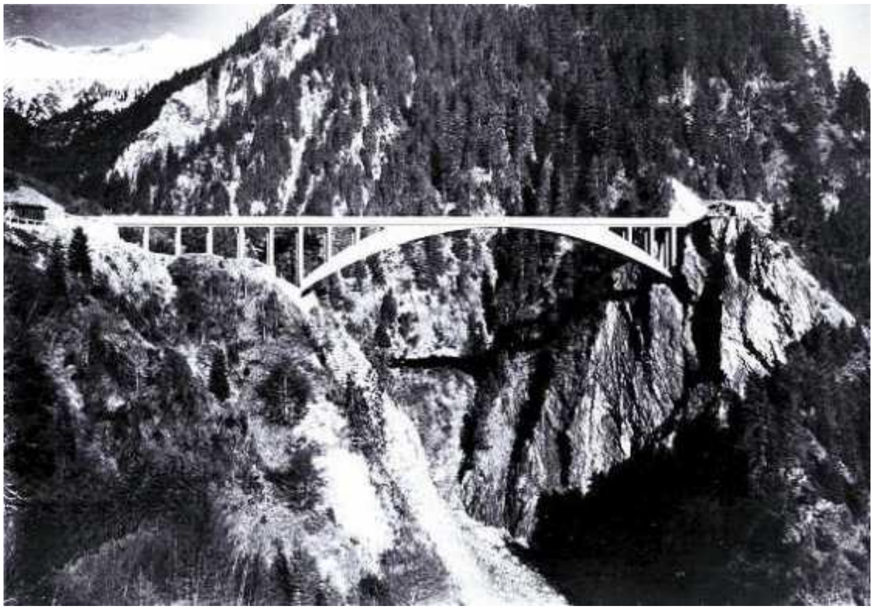
The **worst case** live load, the sidewalks, rail, and roads are simultaneously filled to capacity, bumper-to-bumper and side-to-side, with the heaviest trucks, trains, and pediestrians. This resulting load is estimated as

v. 100 psf for each of two 10 ft-wide sidewalks: 2 kpf
vi. 250 psf for each of the 8 lanes of roadway, 10 ft wide: 20 kpf
(The 250 psf comes from the weight of a 25-ton truck distributed over it's footprint area of 8'×25'=200 ft^2.)
vii. 6 kpf for each of the railway tracks: 46 kpf

This worst-case live load comes to 46 kpf. The design live load was actually much less; two reduction factors were applied. One is a factor C_l =0.25, which takes into account a more realistic spacing between trucks, and the low probability of having any lane filled to capacity across its length. The second factor comes from the low probability of having all lanes simultaneously fully loaded with the heaviest vehicles. This factor is $C_n = 0.5 + \frac{2}{n+3}$, where n is the number of lanes. For the 8-lane bridge, the reduction factor is C=0.682. **The design live load is therefore 46 kpf × 0.25×0.682=7.8kpf, evenly distributed across the span.**

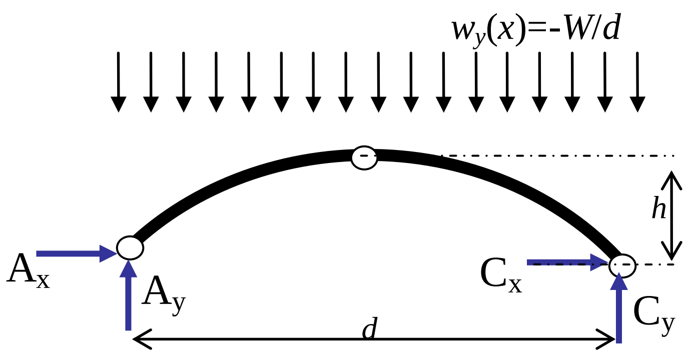
a. The desired cable sag is 316 feet for each cable carrying its self weight and ¼ of the total deck+design live load. Use your spreadsheet to calculate the original (unstretched) length L_0 of cable needed, and the maximum cable tension. Compare the maximum stress in the cables with the yield stress. Compute a safety factor equal to the yield stress/actual stress.
b. For the L_0 found above, load the bridge with the **unreduced** live load. Find the sag and maximum cable tension in this case and compare the cable stress with the yield stress. Compute a safety factor equal to the yield stress/actual stress.

3. The Salginotabel Bridge in the Swiss Alps is considered by many to be a highlight of 20th century bridge architecture. The bridge was designed by a visionary engineer, Robert Maillart and was completed in 1930. It has been named one of 30 "world Monuments" by the American Society of Civil Engineers. (The list of other World Monuments includes the Eiffel Tower and the Panama Canal.)



The bridge is a three-hinged arch with a span of d=90 meters and a rise of h=13 meters. The total weight of the bridge is estimated at W=7500kN.

a. As a first estimate, assume that the weight is evenly distributed across the span, so that $w_y(x)=-W/d$. Find the reactions due to the bridge weight and plot the internal bending moment, shear, and axial force. What is the magnitude and location of the maximum bending moment?



b. Now consider a more refined estimate of the dead loads. The load at the center of the arch is taken to be 57kN/m and extends linearly to the supports, where it has the value of 109kN/m. Determine the reactions due to the bridge weight and plot the internal bending moment, shear, and axial force. Compare the two bending moment distributions. Use symmetry!