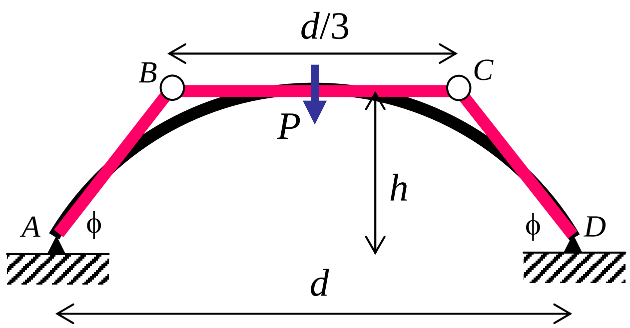
ENGN1300: Structural Analysis

Homework 9
Due Wednesday April 28, 2010

Division of Engineering
Brown University|

1. In class, we talked about a way to approximate the moments and deflection of a two-hinged arch subjected to a point load at its center. For the purposes of the calculation, the arch is represented as the frame shown, with hinges at points B and C.



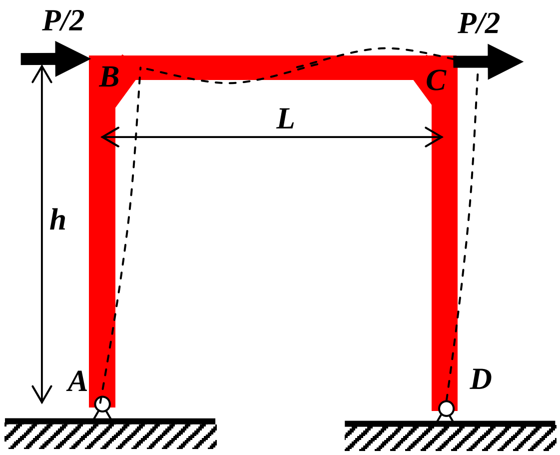
Assume all members of the approximating truss have identical properties *E*, *I*, and *A*, and calculate the deflection under the load.

To do this, note that members AB and CD are two-force members, and the axial forces in those members can be calculated from static equilibrium. This in turn allows you to calculate the axial force in member BC. This member is not a two force member---it deflects due to bending as well as due to the axial load that it carries. Bending does not induce any axial elongation, but the axial load does. If the axial load in that member is F_{BC}, then $F_{BC} = EA(u_x^C - u_x^B)/L_{BC}$. Note that symmetry requires that $u_x^C = -u_x^B$, so $F_{BC} = -2EAu_x^B / L_{BC}$. This means that you can solve for u_x^B in terms of the given parameters of the problem.

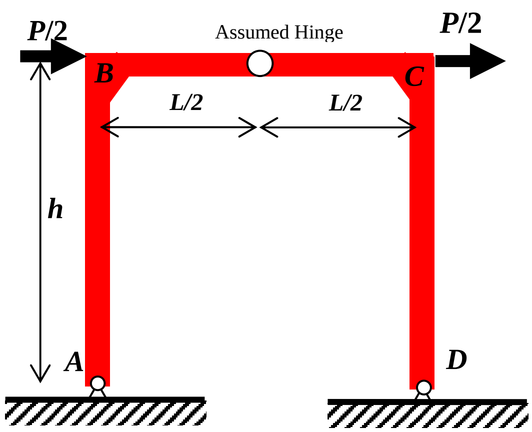
Knowing u_x^B and the forces in the two-force members AB and CD, you can now calculate $u_y^B = u_y^C$ in terms of the parameters of the problem. And once you know these, you can find the vertical deflection under the load. This comes from the deflection of the cross beam (due to bending) and the deflections of the endpoits $u_y^B = u_y^C$.

Evaluate your answer for the case For E=4x10^6 ksf, P=1kip, d=300ft, h=75ft, I=0.0833ft$_4$, A=1ft^2. Run the Excel solver for the two hinged arch and compare.

2. The pinned portal frame shown below is subjected to a lateral load *P* due to wind. An estimate of the deformed shape is shown in dashed lines. The deformed shape appears to have an inflection point near the midpoint of the cross beam. At such a point, the internal moment must be zero. (why?)

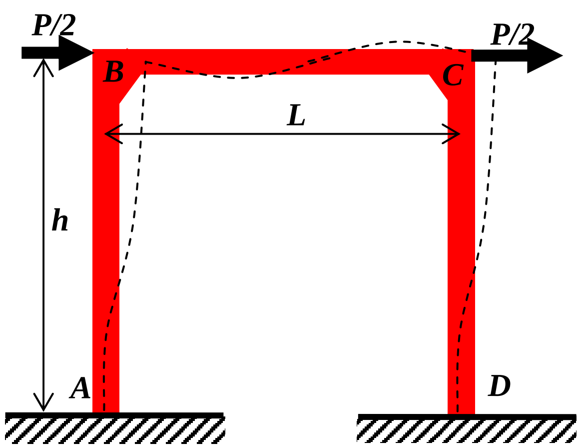


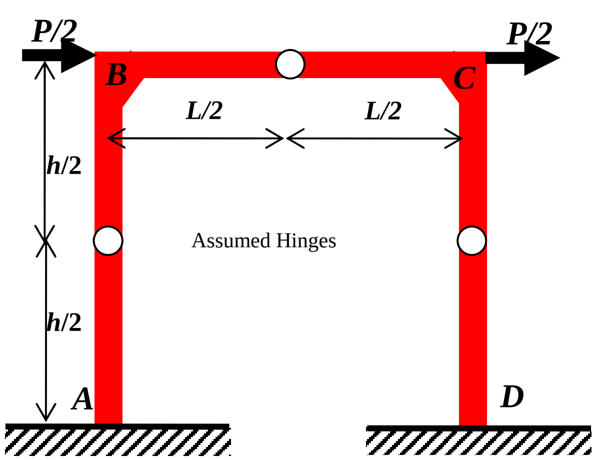
   a. To approximately analyze a pinned frame subject to a horizontal load, a hinge is often assumed at the crossbeam midpoint. Find the base reactions as well as the forces and moments transmitted through the joints B, and C, and the forces transmitted through the central hinge. Draw the moment diagram for the members.



   b. Assume full moment connections for this frame, and calculate the deflections and rotations at each joint. Each member has equal properties *EI*. Use the symmetry of the problem, that tells you that the vertical deflection at the hinge must be zero.

c. If the frame is fixed at the base, rather than hinged, the deflection might looks something like the dotted line shown below. In addition to an inflection point near the crossbeam midpoint, there are also inflection points in the columns. Assuming these to lie at the midpoint of each member, find the reactions and draw the internal moment distributions in the members of the fixed-base frame. Which frame has higher moments?





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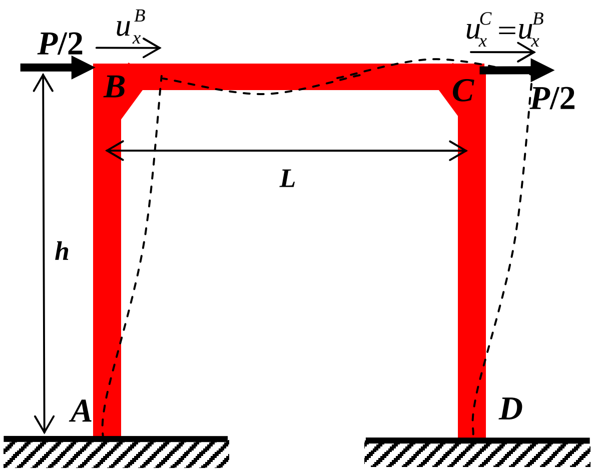
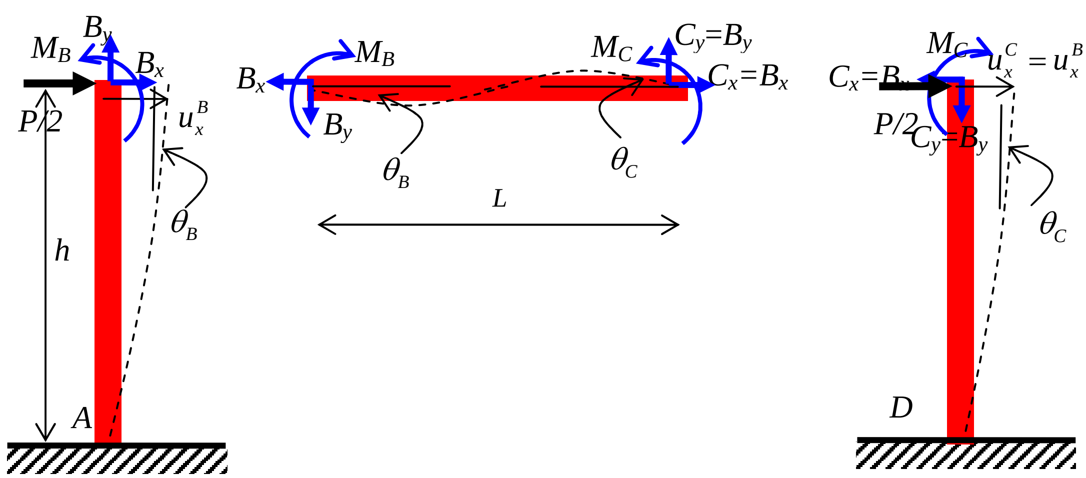
3. In this problem you will find the actual deflections and moments in the frame that is approximated in the previous problem. Assume that the properties *EI* are the same in the columns and the crossbeam. Neglect axial deformation throughout, so that $u_x^B = u_x^C$ and $u_y^B = u_y^C = 0$ and assume full moment connections at B and C.



In the figure below, the forces and moments transmitted through joints B and C are shown, along with rotations and displacements of the joints. Note that $\sum \mathbf{F} = \mathbf{0}$ for the cross beam implies $B_x = C_x$ and $B_y = C_y$.



a. Look at the left hand column and relate the forces P and B_x and moment M_B to u_x^B and θ_B. You can do this by looking up or deriving the displacement and slope of a cantilever beam subjected to an end load ($P+B_x$) and an end moment M_B. **Be very careful about signs!**

b. Derive similar relations between the force $C_x = B_x$ and moment M_C and $u_x^B = u_x^C$ and θ_C by looking at the right hand column.

c. Now consider the cross beam. Relate the moments M_B and M_C to the rotations θ_B and θ_C. You can do this by looking up or deriving the slope of a simply supported beam subjected to end moments M_B and M_C.

d. Finally, use these equations to solve for M_B, B_x, M_c, the rotations θ_B and θ_C, and the lateral displacement $u_x^B = u_x^C$.

e. Look at the crossbeam. Use the equations of static equilibrium to solve for $B_y = C_y$ in terms of M_B.

f. Compare the corner moments M_B obtained here with the approximate result from the previous problem set. Plot $M_B/(Ph/4)$ as a function of L/h. For what range of L/h is the approximate solution valid?