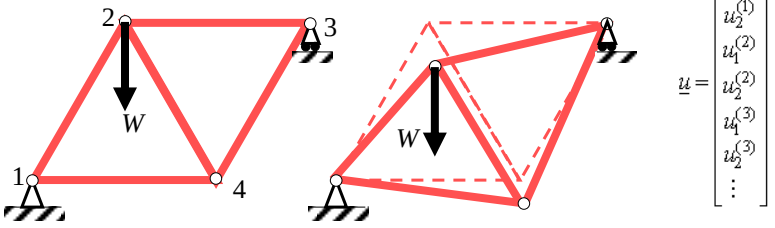
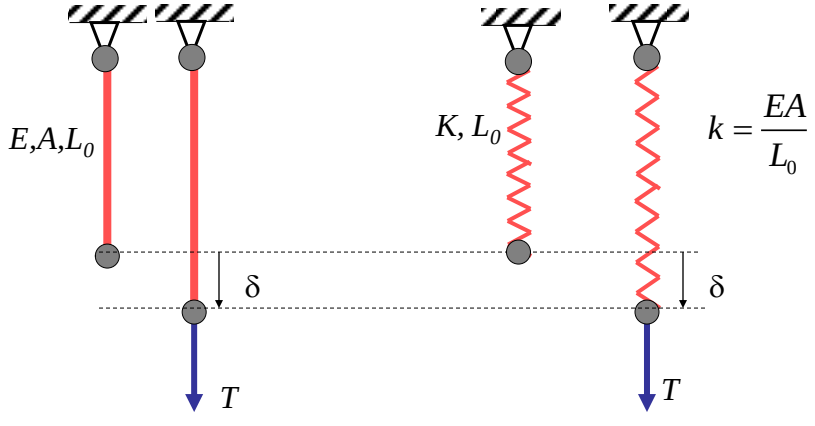
Stiffness Formulation of Truss analysis: forces and deflections



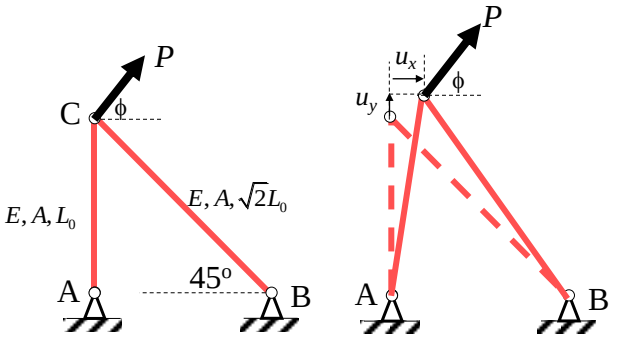
Fundamental Unknowns
are Nodal (joint) Displacements

Elastic rods are linear springs



$$\sigma = \frac{T}{A} = E\frac{\delta}{L_o} \Rightarrow \frac{EA}{L_0}\delta = T$$

Example



Method of Joints:

$$\sum F_{Cx} = F_{BC}/\sqrt{2} + P\cos\phi = 0$$

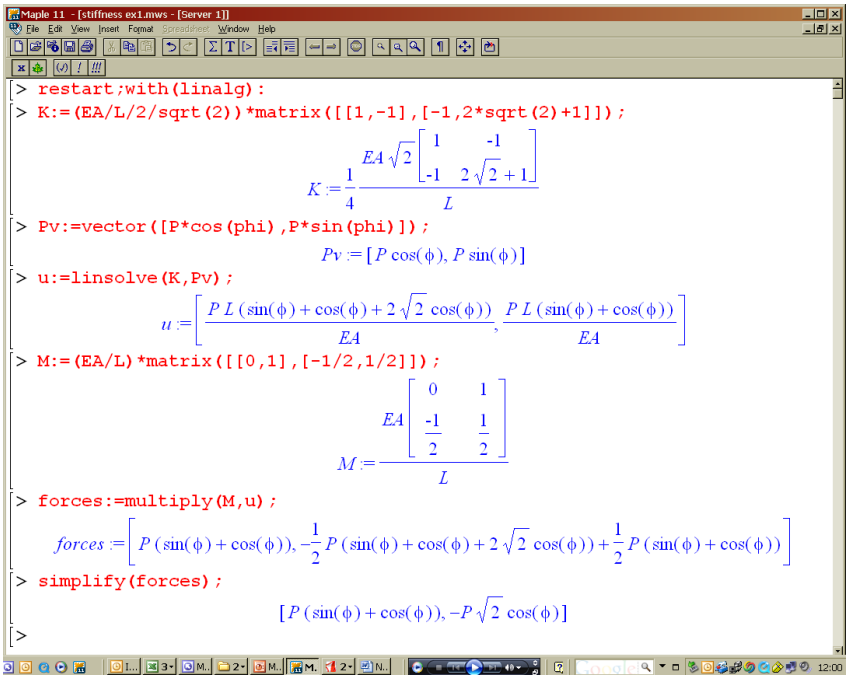
$$\sum F_{Cy} = -F_{AC} - F_{BC}/\sqrt{2} + P\sin\phi = 0$$

$$F_{AC} = P(\sin\phi + \cos\phi),\ F_{BC} = -P\sqrt{2}\cos\phi$$

Stiffness Matrix

- Deflection in AC $\delta_{AC} = u_y$
- Force in AC $F_{AC} = \dfrac{EA}{L_0} u_y$
- Deflection in BC $\delta_{BC} = \dfrac{1}{\sqrt{2}}(u_y - u_x)$
- Force in BC: $F_{BC} = \dfrac{EA}{2L_0}(u_y - u_x)$
- Matrix Equation:

$$\frac{EA}{2\sqrt{2}L_0}\begin{bmatrix} 1 & -1 \\ -1 & 1+2\sqrt{2} \end{bmatrix}\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} P\cos\phi \\ P\sin\phi \end{bmatrix}$$


Maple 11 - [stiffness ex1.mws - [Server 1]]
File Edit View Insert Format Spreadsheet Window Help
> restart;with(linalg):
> K:=(EA/L/2/sqrt(2))*matrix([[1,-1],[-1,2*sqrt(2)+1]]);
$$K := \frac{1}{4}\,\frac{EA\sqrt{2}\begin{bmatrix} 1 & -1 \\ -1 & 2\sqrt{2}+1 \end{bmatrix}}{L}$$
> Pv:=vector([P*cos(phi),P*sin(phi)]);
$$Pv := [P\cos(\phi), P\sin(\phi)]$$
> u:=linsolve(K,Pv);
$$u := \left[\frac{P\,L\,(\sin(\phi)+\cos(\phi)+2\sqrt{2}\,\cos(\phi))}{EA}, \frac{P\,L\,(\sin(\phi)+\cos(\phi))}{EA}\right]$$
> M:=(EA/L)*matrix([[0,1],[-1/2,1/2]]);
$$M := \frac{EA\begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}}{L}$$
> forces:=multiply(M,u);
$$forces := \left[P\,(\sin(\phi)+\cos(\phi)), -\frac{1}{2}P\,(\sin(\phi)+\cos(\phi)+2\sqrt{2}\,\cos(\phi))+\frac{1}{2}P\,(\sin(\phi)+\cos(\phi))\right]$$
> simplify(forces);
$$[P\,(\sin(\phi)+\cos(\phi)), -P\sqrt{2}\,\cos(\phi)]$$
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