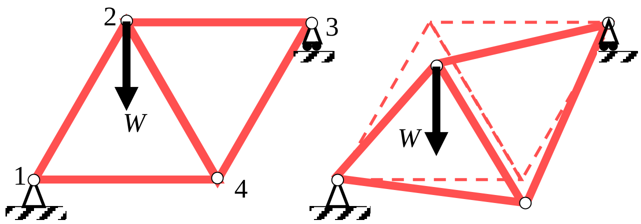
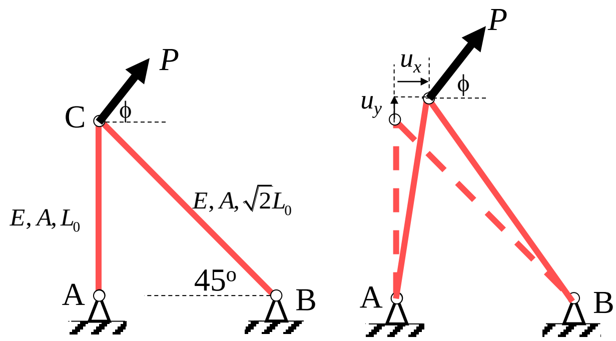
Stiffness Formulation of Truss analysis: forces and deflections



$$\underline{u} = \begin{bmatrix} u_1^{(1)} \\ u_2^{(1)} \\ u_1^{(2)} \\ u_2^{(2)} \\ u_1^{(3)} \\ u_2^{(3)} \\ \vdots \end{bmatrix}$$

Fundamental Unknowns
are Nodal (joint) Displacements

Example



Method of Joints:

$$\sum F_{Cx} = F_{BC} / \sqrt{2} + P\cos\phi = 0$$

$$\sum F_{Cy} = -F_{AC} - F_{BC} / \sqrt{2} + P\sin\phi = 0$$

$$F_{AC} = P(\sin\phi + \cos\phi),\ F_{BC} = -P\sqrt{2}\cos\phi$$

Stiffness Matrix

- Deflection in AC $\delta_{AC} = u_y$
- Force in AC $F_{AC} = \frac{EA}{L_0} u_y$
- Deflection in BC $\delta_{BC} = \frac{1}{\sqrt{2}}(u_y - u_x)$
- Force in BC: $F_{BC} = \frac{EA}{2L_0}(u_y - u_x)$
- Matrix Equation:

$$\frac{EA}{2\sqrt{2}L_0}\begin{bmatrix} 1 & -1 \\ -1 & 1+2\sqrt{2} \end{bmatrix}\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} P\cos\phi \\ P\sin\phi \end{bmatrix}$$

Maple 11 - [stiffness ex1.mws - [Server 1]]

```
> restart;with(linalg):
> K:=(EA/L/2/sqrt(2))*matrix([[1,-1],[-1,2*sqrt(2)+1]]);
```

$$K := \frac{1}{4}\,\frac{EA\sqrt{2}\begin{bmatrix} 1 & -1 \\ -1 & 2\sqrt{2}+1 \end{bmatrix}}{L}$$

```
> Pv:=vector([P*cos(phi),P*sin(phi)]);
```

$$Pv := [P\cos(\phi), P\sin(\phi)]$$

```
> u:=linsolve(K,Pv);
```

$$u := \left[\frac{P\,L\,(\sin(\phi)+\cos(\phi)+2\sqrt{2}\cos(\phi))}{EA}, \frac{P\,L\,(\sin(\phi)+\cos(\phi))}{EA}\right]$$

```
> M:=(EA/L)*matrix([[0,1],[-1/2,1/2]]);
```

$$M := \frac{EA\begin{bmatrix} 0 & 1 \\ \frac{-1}{2} & \frac{1}{2} \end{bmatrix}}{L}$$

```
> forces:=multiply(M,u);
```

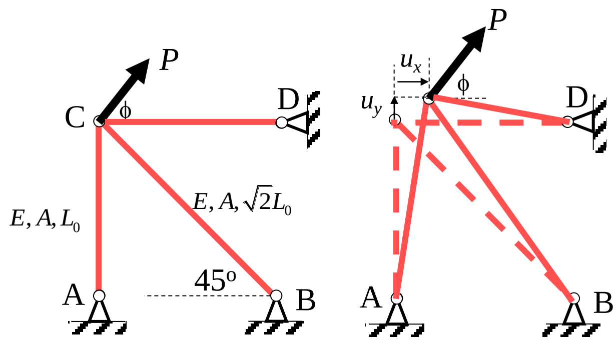
$$forces := \left[P\,(\sin(\phi)+\cos(\phi)), -\frac{1}{2}P\,(\sin(\phi)+\cos(\phi)+2\sqrt{2}\cos(\phi))+\frac{1}{2}P\,(\sin(\phi)+\cos(\phi))\right]$$

```
> simplify(forces);
```

$$[P\,(\sin(\phi)+\cos(\phi)), -P\sqrt{2}\cos(\phi)]$$

```
>
```

Works for Statically indeterminate too.



Maple 11 - [stiffness ex1.mws - [Server 1]]

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```
> K2:=(EA/L/2/sqrt(2))*matrix([[2*sqrt(2)+1,-1],[-1,2*sqrt(2)+1]]);
```

$$K2 := \frac{1}{4}\,\frac{EA\sqrt{2}\begin{bmatrix} 2\sqrt{2}+1 & -1 \\ -1 & 2\sqrt{2}+1 \end{bmatrix}}{L}$$

```
> u2:=linsolve(K2,Pv);
```

$$u2 := \left[\frac{1}{2}\,\frac{P\,L\,(\sin(\phi)+\cos(\phi)+2\sqrt{2}\cos(\phi))}{EA\,(\sqrt{2}+1)},\ \frac{1}{4}\,\frac{\sqrt{2}\,P\,L\,(4\sin(\phi)+\sqrt{2}\cos(\phi)+\sqrt{2}\sin(\phi))}{EA\,(\sqrt{2}+1)}\right]$$

```
> M2:=(EA/L)*matrix([[0,1],[-1/2,1/2],[-1,0]]);
```

$$M2 := \frac{EA\begin{bmatrix} 0 & 1 \\ \frac{-1}{2} & \frac{1}{2} \\ -1 & 0 \end{bmatrix}}{L}$$

```
> forces:=multiply(M2,u2);
```

$$forces := \left[\frac{1}{4}\,\frac{\sqrt{2}\,P\,(4\sin(\phi)+\sqrt{2}\cos(\phi)+\sqrt{2}\sin(\phi))}{\sqrt{2}+1},\ -\frac{1}{4}\,\frac{P\,(\sin(\phi)+\cos(\phi)+2\sqrt{2}\cos(\phi))}{\sqrt{2}+1}+\frac{1}{8}\,\frac{\sqrt{2}\,P\,(4\sin(\phi)+\sqrt{2}\cos(\phi)+\sqrt{2}\sin(\phi))}{\sqrt{2}+1},\ -\frac{1}{2}\,\frac{P\,(\sin(\phi)+\cos(\phi)+2\sqrt{2}\cos(\phi))}{\sqrt{2}+1}\right]$$

```
> simplify(forces);
```

$$\left[\frac{1}{4}\,\frac{\sqrt{2}\,P\,(4\sin(\phi)+\sqrt{2}\cos(\phi)+\sqrt{2}\sin(\phi))}{\sqrt{2}+1},\ \frac{1}{2}\,\frac{P\sqrt{2}\,(-\sin(\phi)+\cos(\phi))}{\sqrt{2}+1},\ -\frac{1}{2}\,\frac{P\,(\sin(\phi)+\cos(\phi)+2\sqrt{2}\cos(\phi))}{\sqrt{2}+1}\right]$$