In General

1. Express *M* member elongations δ_{ij} in terms of $\underline{u}$ (M eqns)
2. Enforce the boundary constraints on the displacements (R eqns)
3. Express *M* member forces F^{ij} in terms of δ_{ij} (M eqns)
4. Enforce static equilibrium at each node (2J or 3J eqns)

Total number of equations: 2M+R+DJ

Unknowns

Nodal displacements $\underline{u}$: DJ

Member elongations $\delta_{ij:}$ M

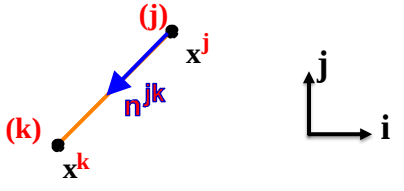
Member forces F^{ij} : M

Reaction forces $R^{k:}$ *R*

Total number of unknowns: 2M+R+DJ

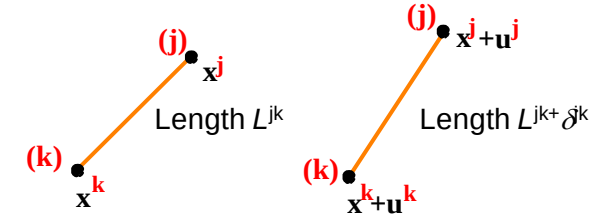
Member Forces and elongations in terms of $\underline{u}$:

For a member connecting joints (j) and (k):



$$L^{jk} = \left|(\mathbf{x}^k - \mathbf{x}^j)\right|, \quad \mathbf{n}^{jk} = (\mathbf{x}^k - \mathbf{x}^j) / L^{jk}$$

Member Elongations



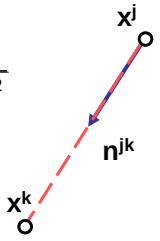
$$L^{jk}+\delta^{jk} \equiv \left|\mathbf{x}^k+\mathbf{u}^k-\mathbf{x}^j-\mathbf{u}^j\right| =$$

$$\sqrt{(\mathbf{x}^k+\mathbf{u}^k-\mathbf{x}^j-\mathbf{u}^j)\bullet(\mathbf{x}^k+\mathbf{u}^k-\mathbf{x}^j-\mathbf{u}^j)}$$

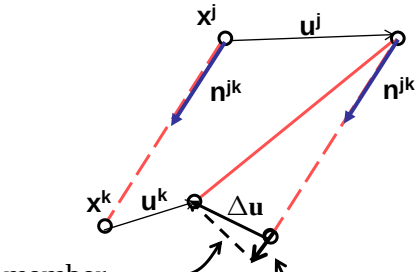
$$= \sqrt{\left|\mathbf{x}^k-\mathbf{x}^j\right|^2+2(\mathbf{x}^k-\mathbf{x}^j)\bullet(\mathbf{u}^k-\mathbf{u}^j)+\left|\mathbf{u}^k-\mathbf{u}^j\right|^2}$$

$$\approx L^{jk}\left(1+(\mathbf{u}^k-\mathbf{u}^j)\bullet(\mathbf{x}^k-\mathbf{x}^j)/L^{jk}\right)$$

$$= L^{jk}+(\mathbf{u}^k-\mathbf{u}^j)\bullet\mathbf{n}^{jk}$$

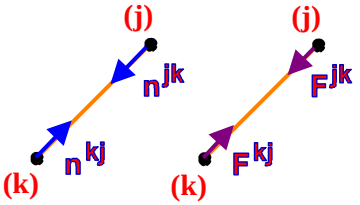


Finally $\quad \delta^{jk} \equiv \delta^{kj} = (\mathbf{u}^k-\mathbf{u}^j)\cdot\mathbf{n}^{jk}$



$$\mathbf{n}^{jk} = (\mathbf{x}^k-\mathbf{x}^j)/L^{jk}, \quad L^{jk} = \left|\mathbf{x}^k-\mathbf{x}^j\right|$$

Axial Force in the member:



$$F^{jk} = \frac{E^{jk} A^{jk}}{L^{jk}} \quad \delta^{jk} \equiv F^{kj} = \left(\frac{EA}{L} \right)^{jk} (\mathbf{u}^{k} - \mathbf{u}^{j}) \cdot \mathbf{n}^{jk}$$

Vectorial force on joint j through the member jk:

$$\mathbf{F}^{jk} = F^{jk} \mathbf{n}^{jk}$$