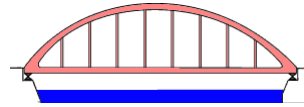


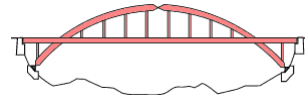
Arches II: Deflections



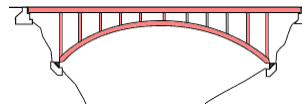
Tied Arch



Three hinged

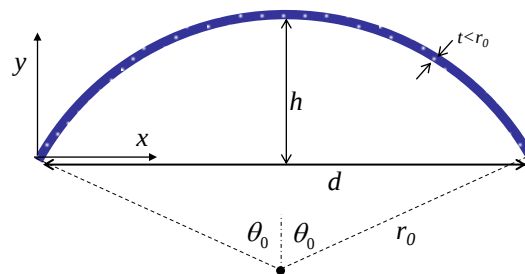


Two Hinged



Circular Arches: Span d , rise h

- Assumes arch thickness is small compared with its initial radius of curvature



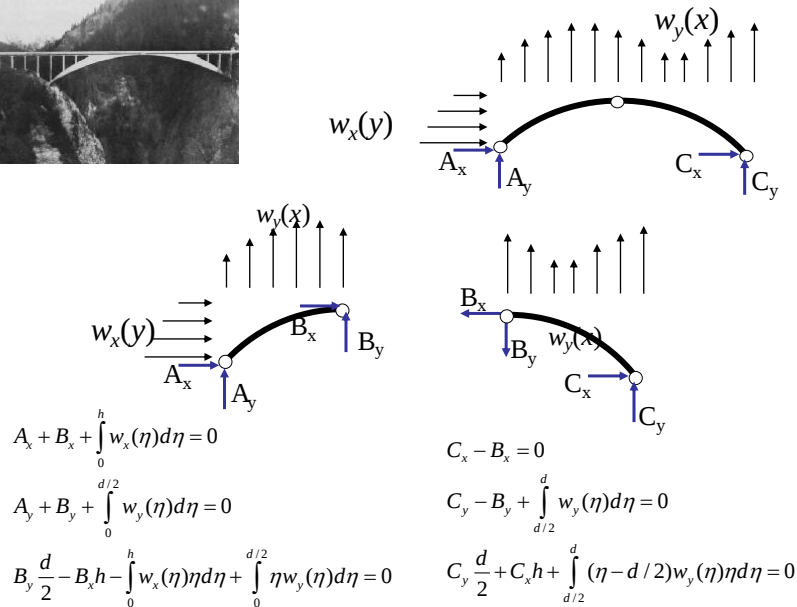
$$r_0 = \frac{1}{2} \left(\frac{d^2}{4h} + h \right), \quad y(x) = \sqrt{r_0^2 - (x - d/2)^2} - r_0 + h$$

$$|\theta| \leq \theta_0 = \tan^{-1} \frac{d}{2(r_0 - h)}$$

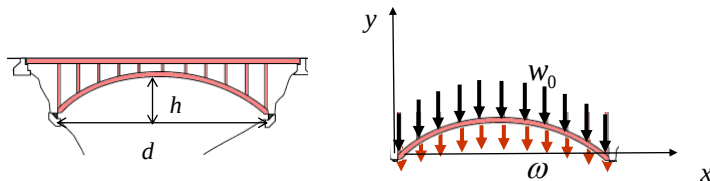
r_0 is the initial radius of curvature of the arch



3-Hinged: Statically determinate



Tied or 2-hinged Arch with dead and self-load: Approximate Analysis

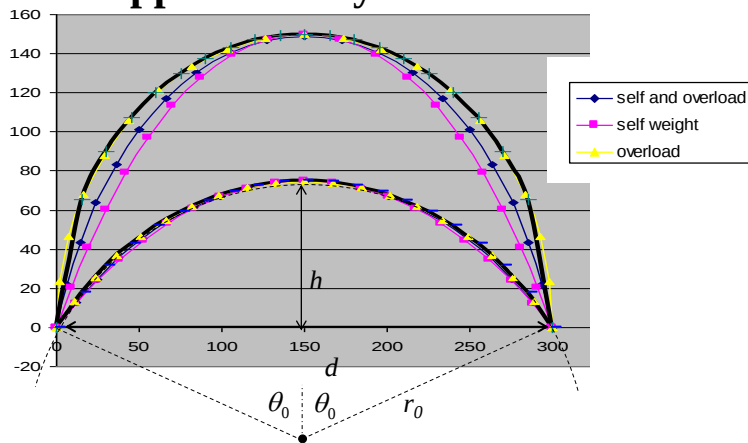


Arch shape for an arch with no internal shear and moment:

$$\frac{d^2 y}{dx^2} = -\frac{w}{R_x} = \frac{w_0}{R_x} - \frac{\omega}{R_x} \sqrt{1 + \left(\frac{dy}{dx} \right)^2}$$

$$\text{BC: } y(0) = y(x_e) = 0, y(x_e/2) = h$$

Approximately Circular

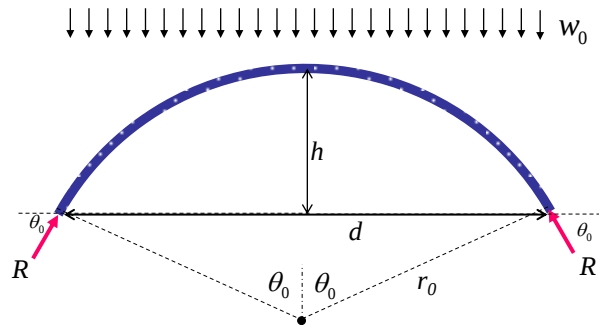


$$r_0 = \frac{1}{2} \left(\frac{d^2}{4h} + h \right), \quad y(x) = \sqrt{r_0^2 - (x - d/2)^2} - r_0 + h$$

$$|\theta| \leq \theta_0 = \tan^{-1} \frac{d}{2(r_0 - h)}$$

Approximate Reactions

Assumes internal shear is small near the supports

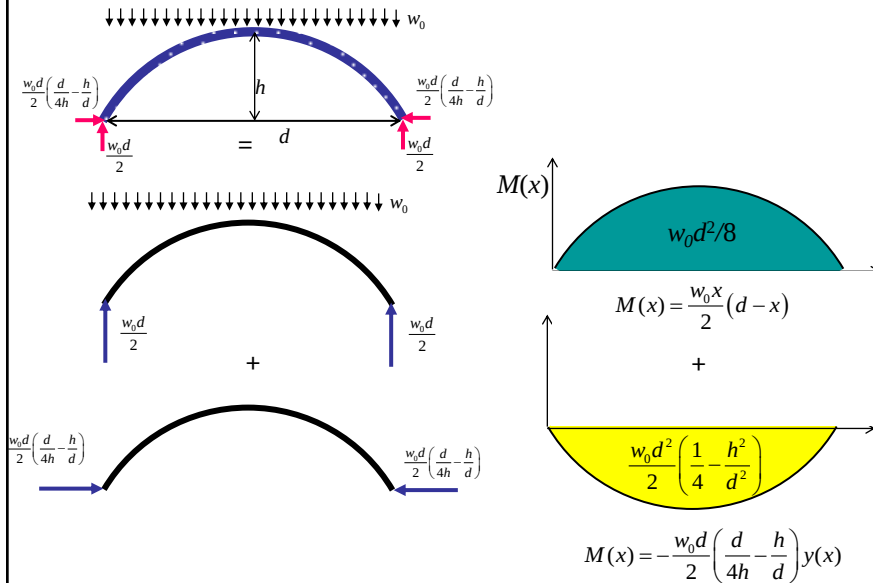


$$r_0 = \frac{1}{2} \left(\frac{d^2}{4h} + h \right) \quad \theta_0 = \tan^{-1} \frac{d}{2(r_0 - h)}$$

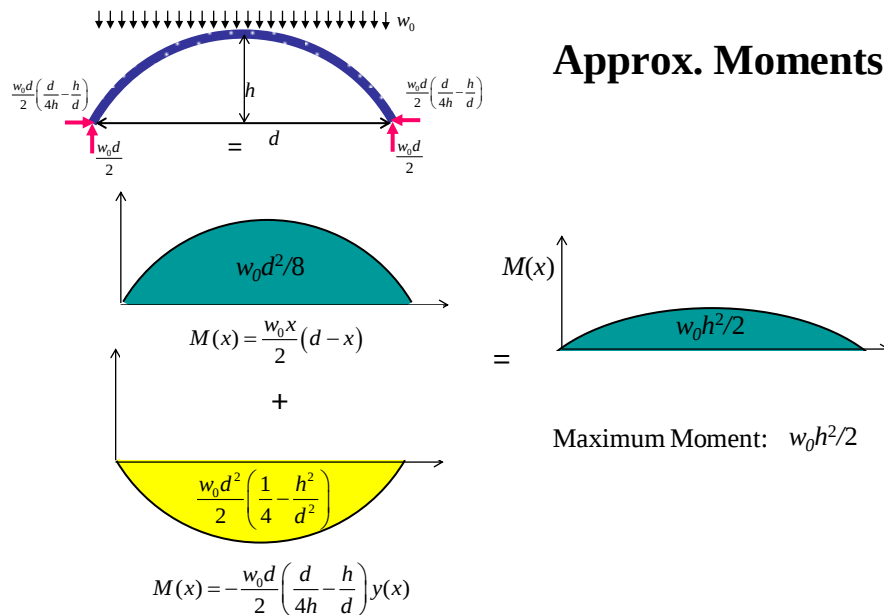
$$R_y = R \sin \theta_0 = \frac{w_0 d}{2}$$

$$R_x = R \cos \theta_0 = \frac{w_0 d}{2 \tan \theta_0} = w_0 (r_0 - h) = \frac{w_0 d}{2} \left(\frac{d}{4h} - \frac{h}{d} \right)$$

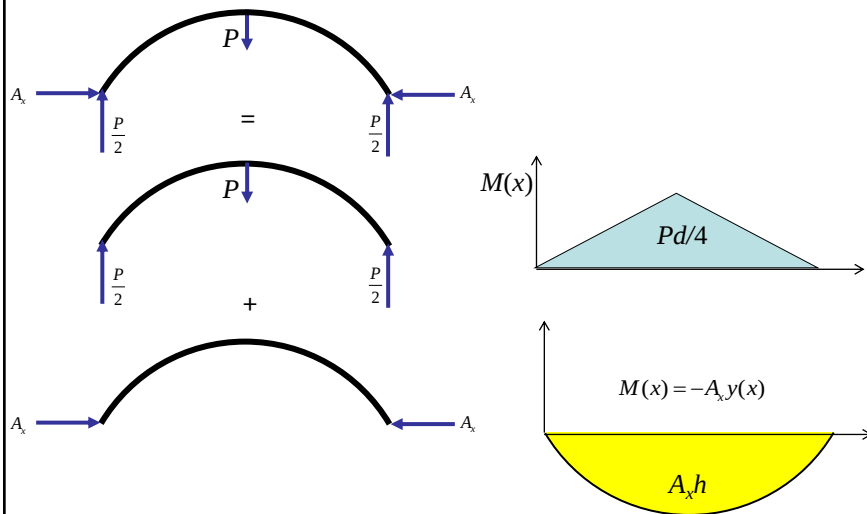
Approximate Moments



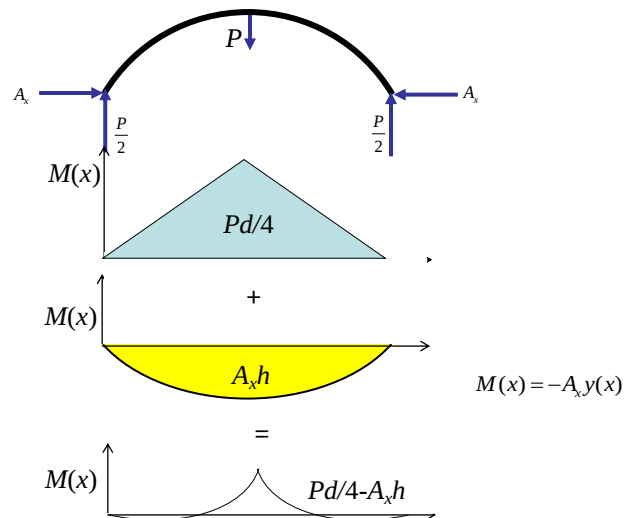
Approx. Moments



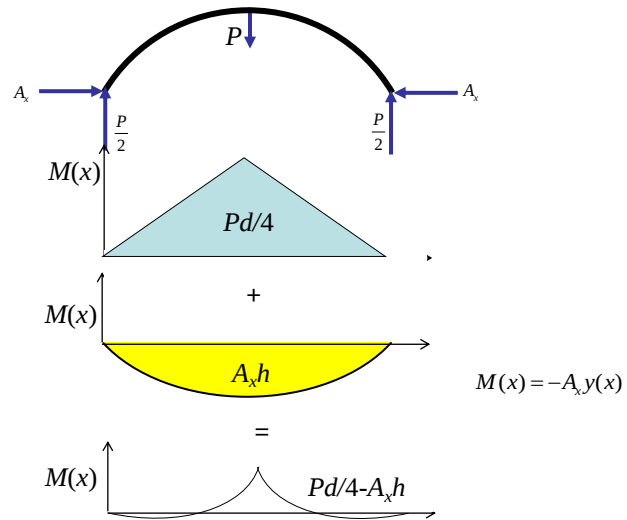
Approximate Analysis: Point Load



Moments



Moments



Typically: $0 \leq A_x \leq \frac{P d}{4 h}$