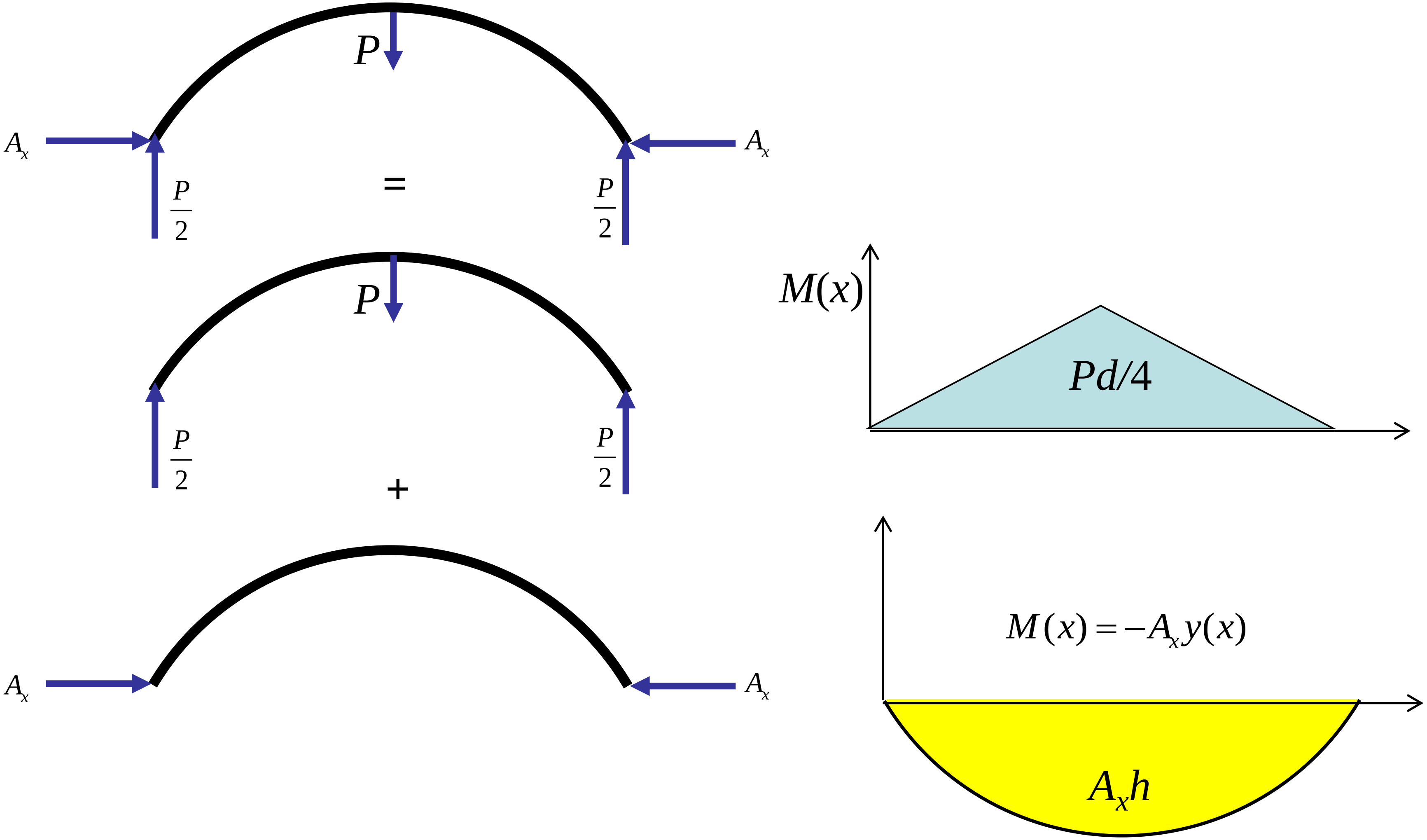
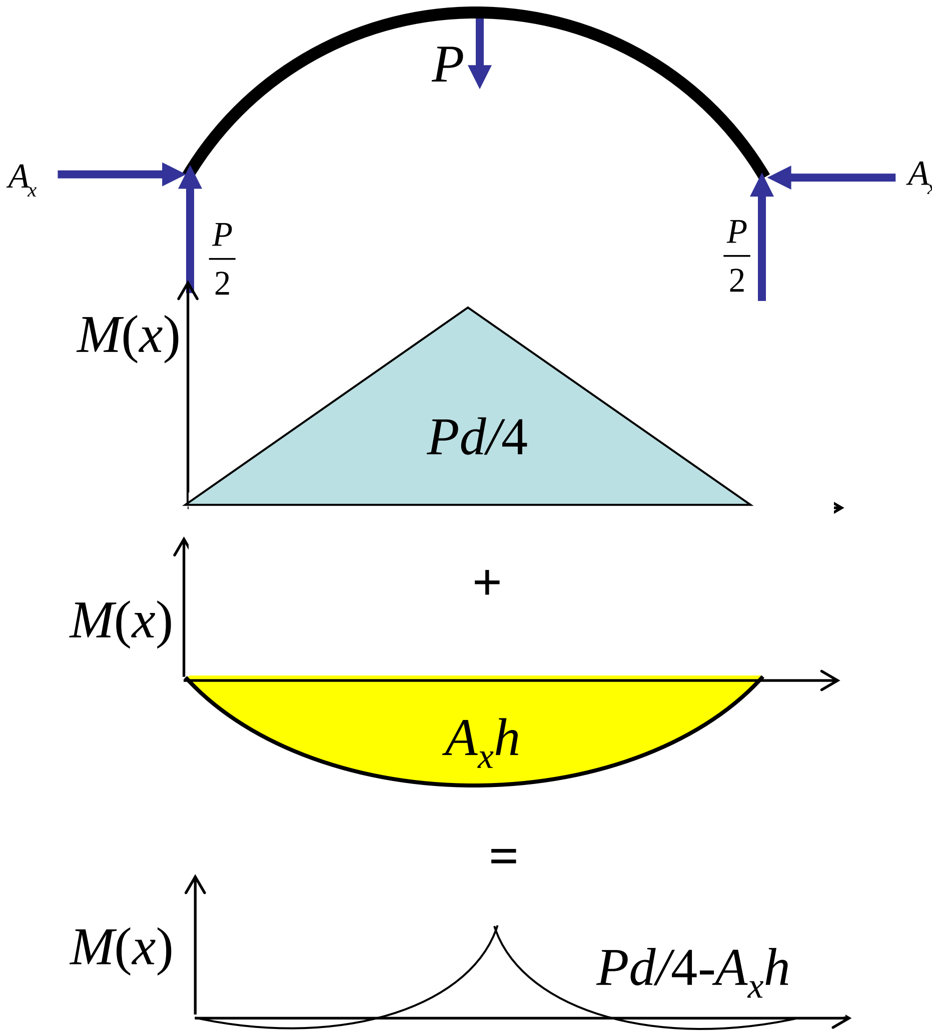
Approximate Analysis: Point Load

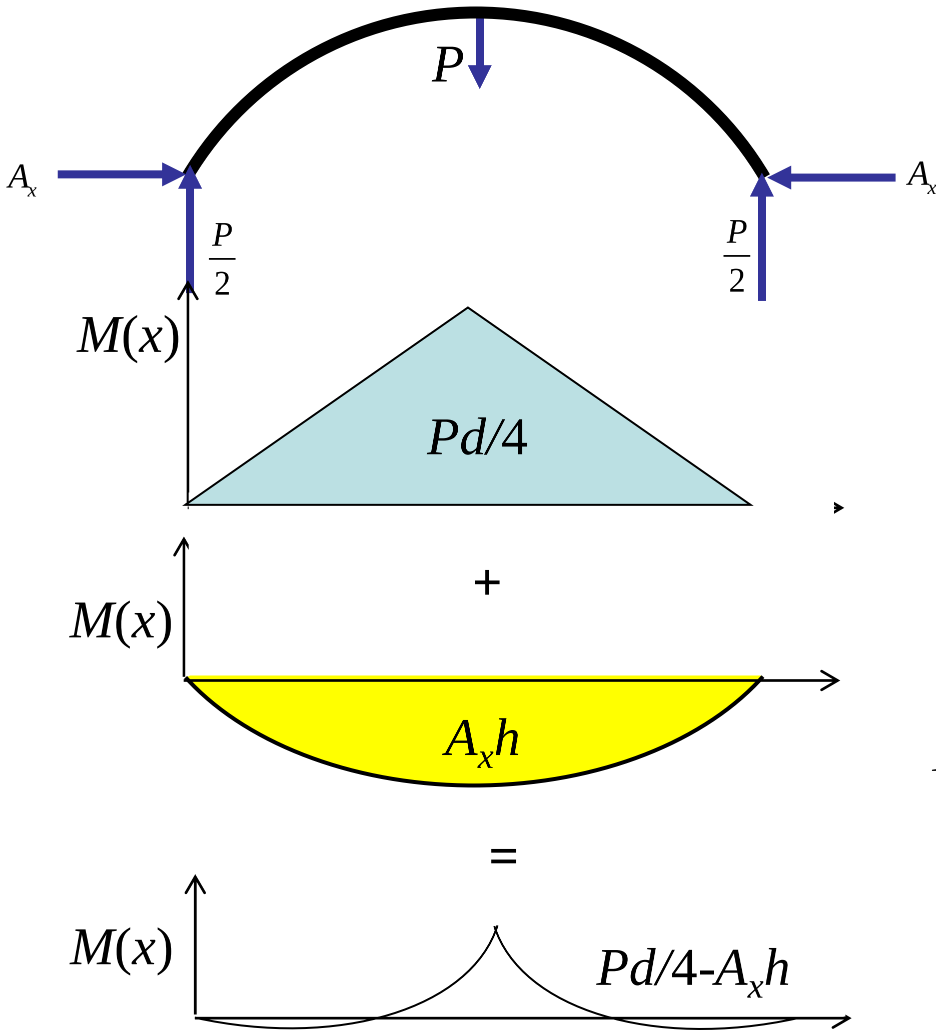


Moments



$$M(x) = -A_x y(x)$$

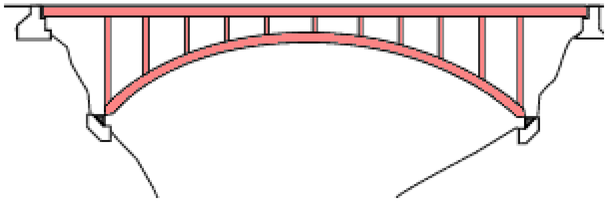
Moments

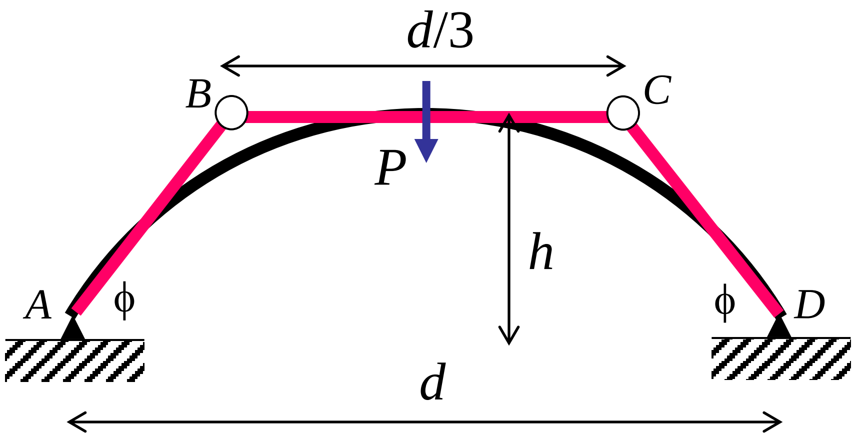


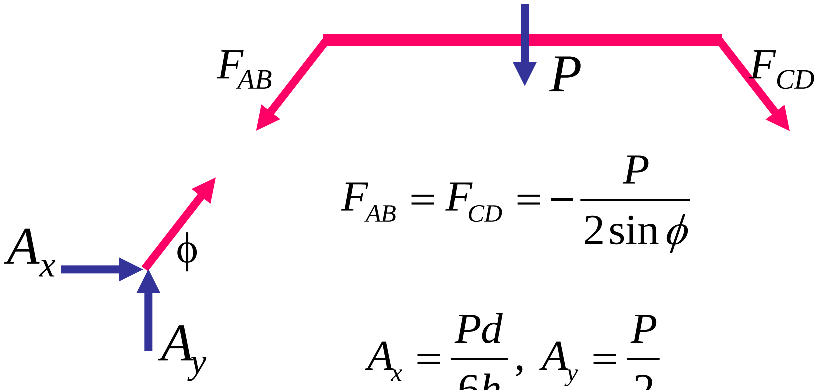
$$M(x) = -A_x y(x)$$

Typically: $0 \le A_x \le \frac{P}{4}\frac{d}{h}$

Approximate Analysis



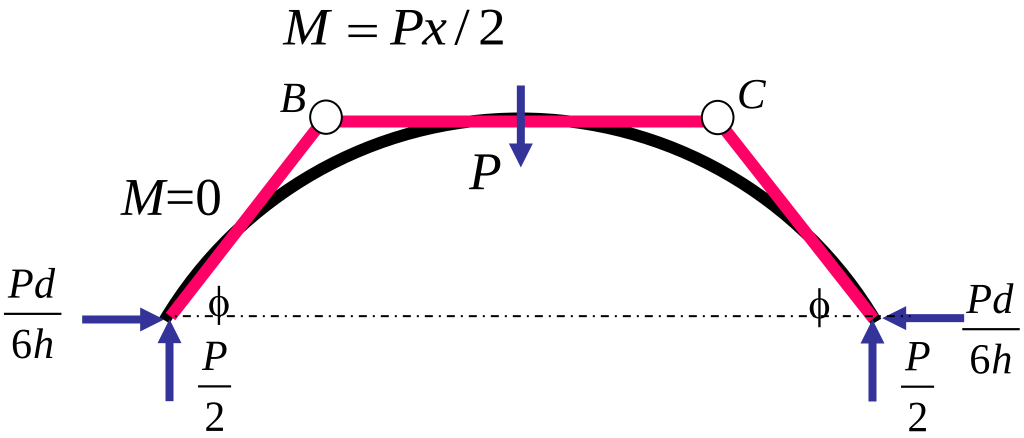


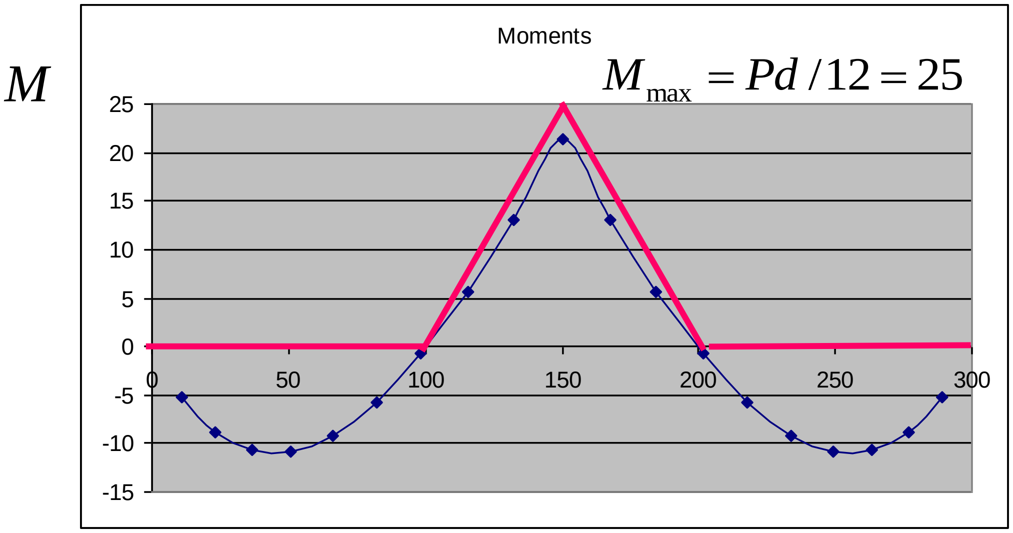


$$F_{AB} = F_{CD} = -\frac{P}{2\sin\phi}$$

$$A_x = \frac{Pd}{6h},\; A_y = \frac{P}{2}$$

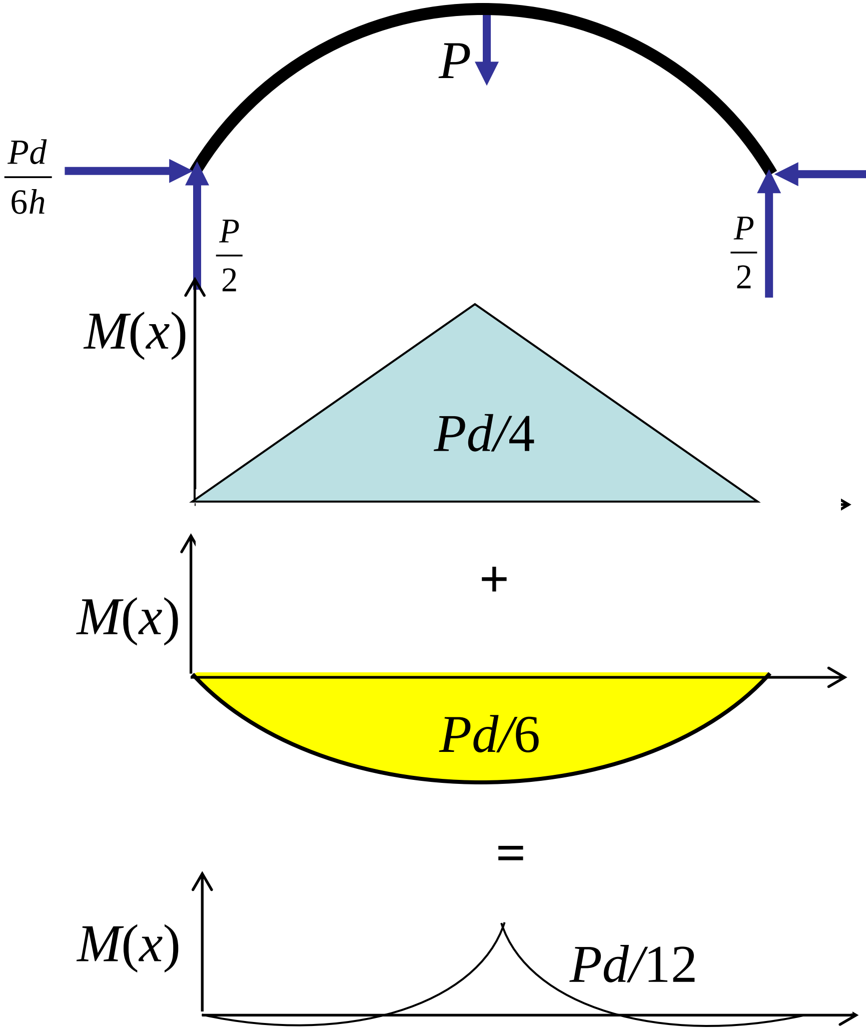
Moments (1)





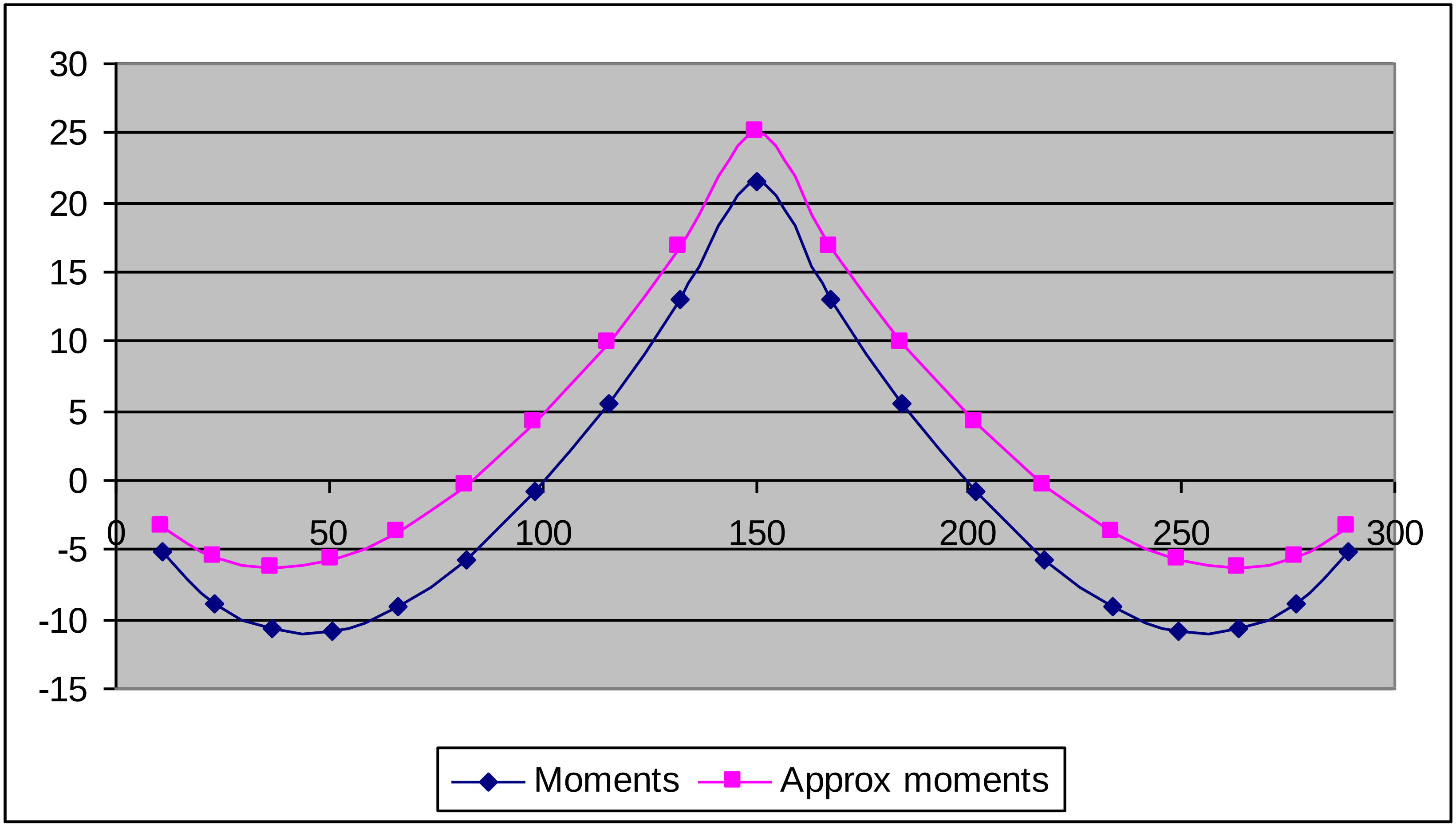
For E=10^6, P=1, d=300ft, h=75ft, I=0.0833ft^4, A=1ft^2

Moments (2)

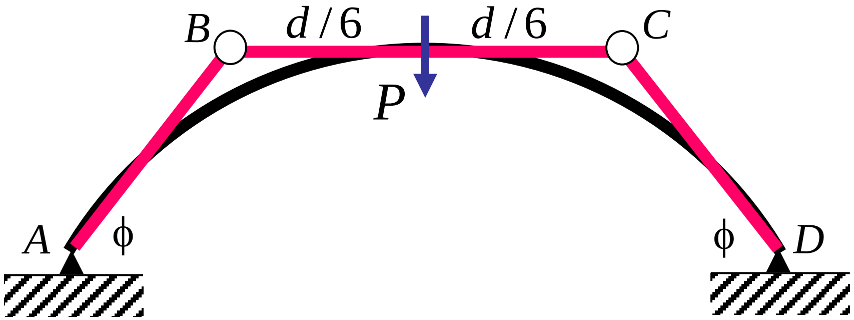


$$M(x) = -\frac{Pd}{6h}\,y(x)$$

P=1, d=300ft, h=75ft



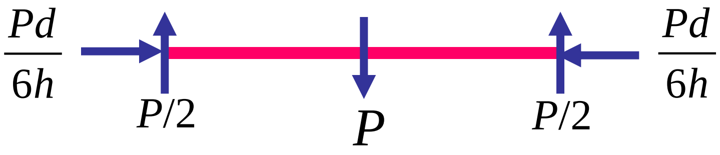
Deflections



$$\tan\phi = \frac{3h}{d},$$

$$L_{AB} = \sqrt{h^2 + d^2/9}$$

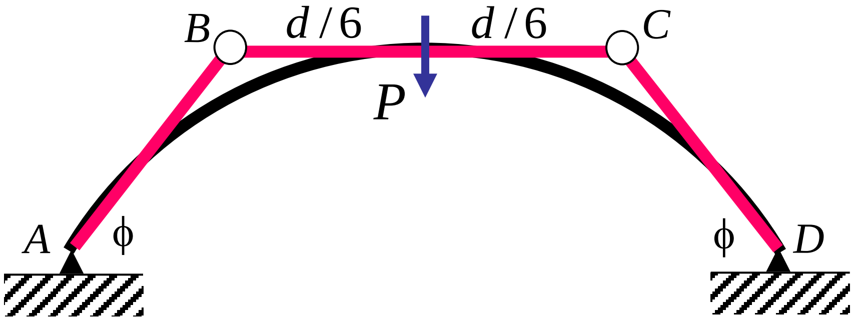
$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA} = \mathbf{u}_B \bullet \mathbf{n}_{AB} = u_x^B \cos\phi + u_y^B \sin\phi = -\frac{PL_{AB}}{2EA\sin\phi}.$$



$$u_x^B = -\frac{Pd}{6h}\frac{d}{6EA} = \frac{Pd^2}{36hEA}$$

$$u_y^P = u_y^B - \frac{Pd/3}{48EI}$$

Deflections



$$\tan\phi = \frac{3h}{d}$$

$$\sin\phi = \frac{h}{L_{AB}}$$

$$L_{AB} = \sqrt{h^2 + d^2/9}$$

$$\frac{Pd^2}{36hEA}\cos\phi + u_y^B \sin\phi = -\frac{PL_{AB}}{2EA\sin\phi}$$

$$\Rightarrow u_y^B = -\frac{PL_{AB}}{2EA\sin^2\phi} - \frac{Pd^2}{36EAh\tan\phi} = -\frac{P(h^2+d^2/9)^{3/2}}{2EAh^2} - \frac{Pd^3}{108EAh^2}$$

$$u_y^P = u_y^B - \frac{P\left(d/3\right)^3}{48EI}$$

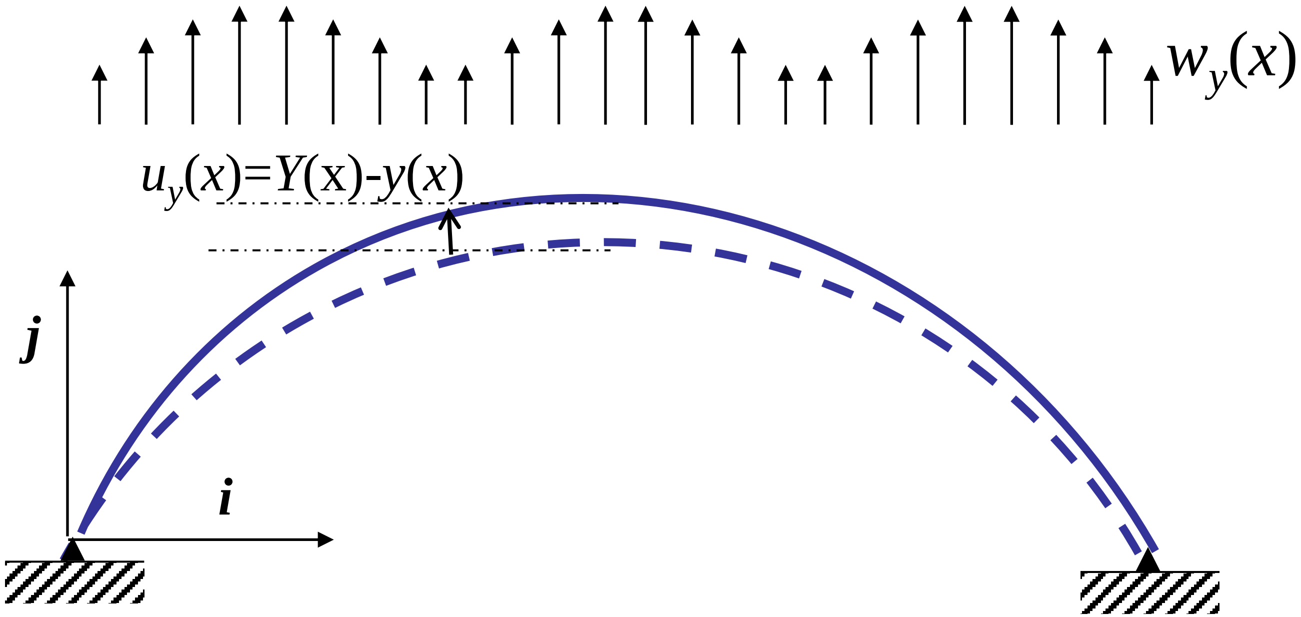
For E=10⁶, P=1, d=300ft, h=75ft, I=0.0833ft⁴, A=1ft²

$$u_y^P = -0.25\text{ft}$$

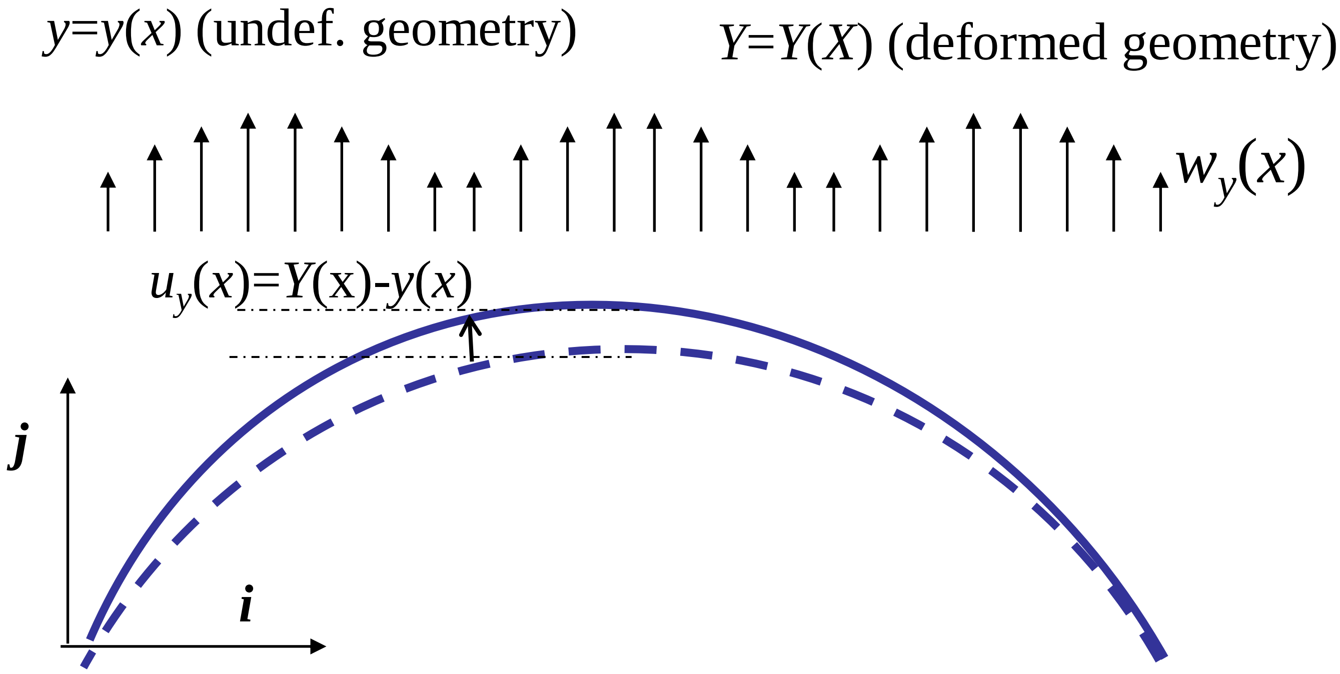
Actual: 0.28 feet

Arch Deflections: Energy Methods

$y=y(x)$ (undef. geometry) $Y=Y(X)$ (deformed geometry)



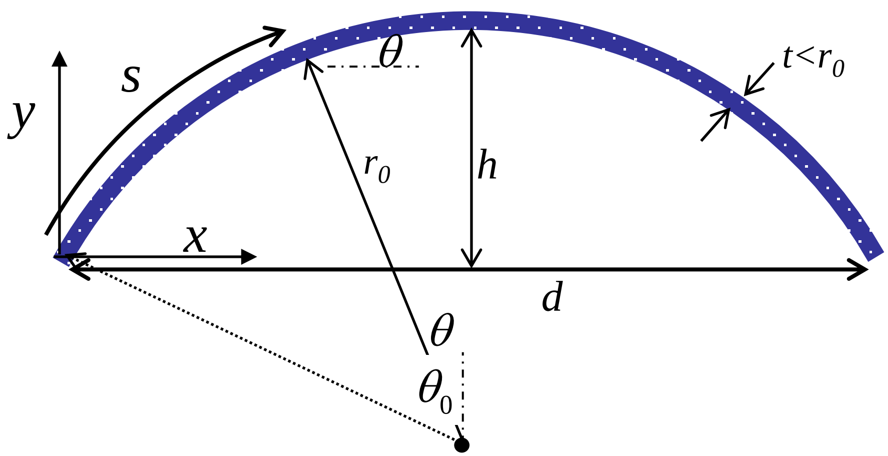
Energy due to the load w_y:



$$V^{\text{load}} = -\int w_y(x) u_y(x) dx$$

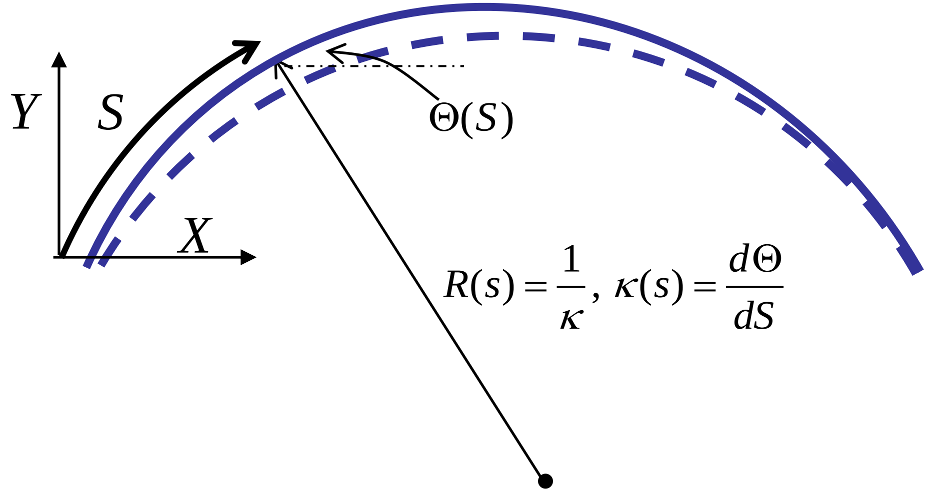
Strain Energy

Undeformed geometry y=y(x): constant curvature κ_0



$$\theta(s) = \theta_0 - \frac{s}{r_0};\ \kappa_0 = \frac{d\theta}{ds} = -r_0$$

Deformed geometry: Y=Y(X) Curvature κ(s)



$$R(s) = \frac{1}{\kappa},\ \kappa(s) = \frac{d\Theta}{dS}$$

Moment is

$$M = EI\left(\kappa - \kappa_0\right)$$

Strain Energy is

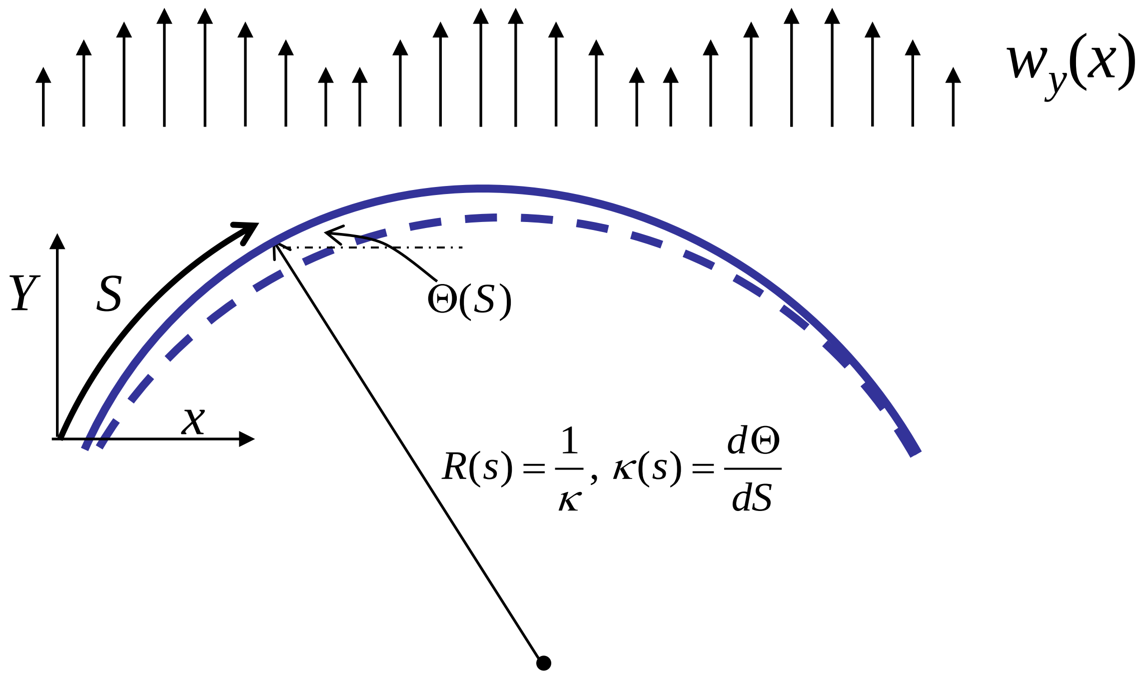
$$V = \int \frac{1}{2} EI\left(\kappa_0 - \kappa\right)^2 ds$$

$$+\int \frac{1}{2} EA\left(\frac{dS}{ds}\right)^2 ds$$

Total Energy to be minimized

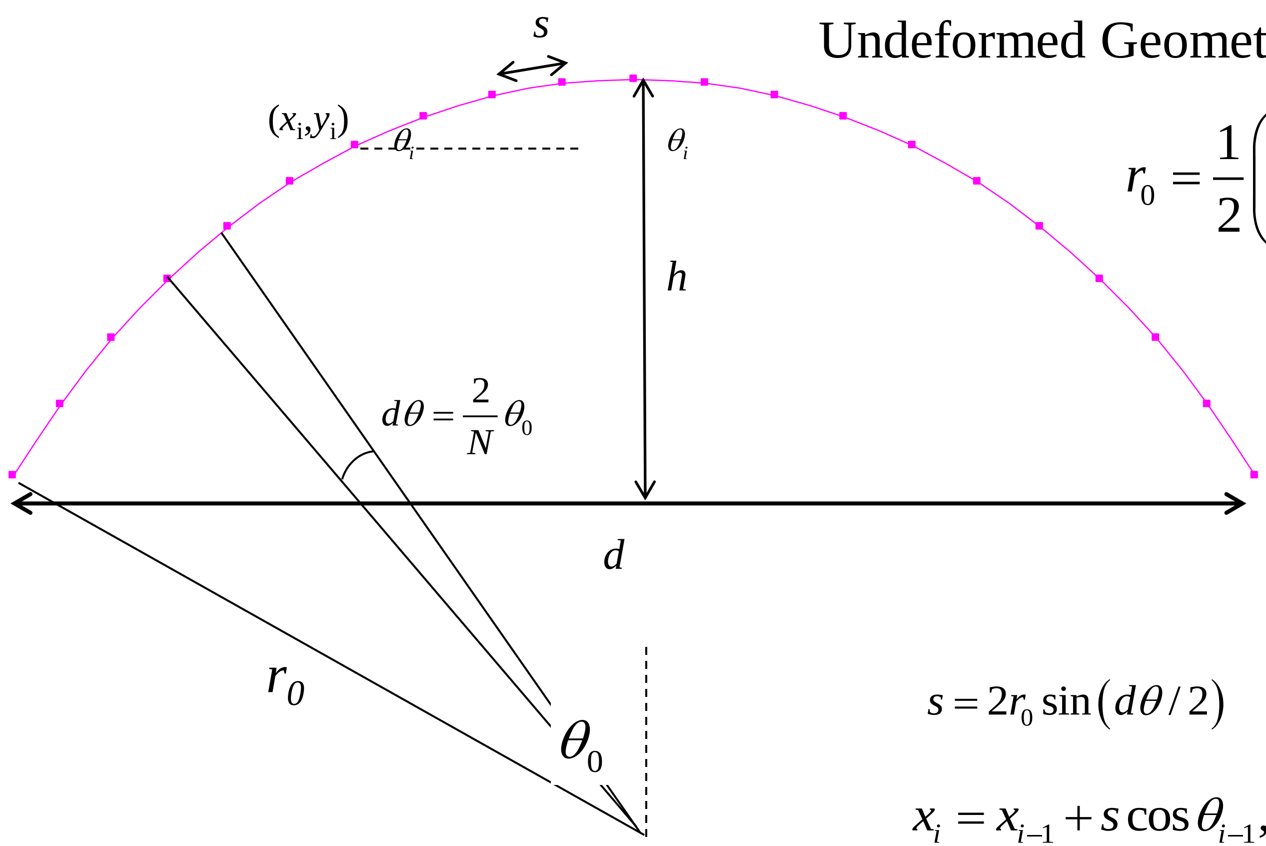
$y=y(x)$ (undef. geometry) $Y=Y(X)$ (deformed geometry)

$u_y(x)=Y(\mathrm{x})-y(x)$



$$V=\int\frac{1}{2}EI\left(\kappa_0-\kappa\right)^2 ds+\int\frac{1}{2}EI\left(\frac{dS}{ds}\right)^2 ds-\int w_y(x)u_y(x)dx$$

Undeformed Geometry: N segments



Undeformed Geometry

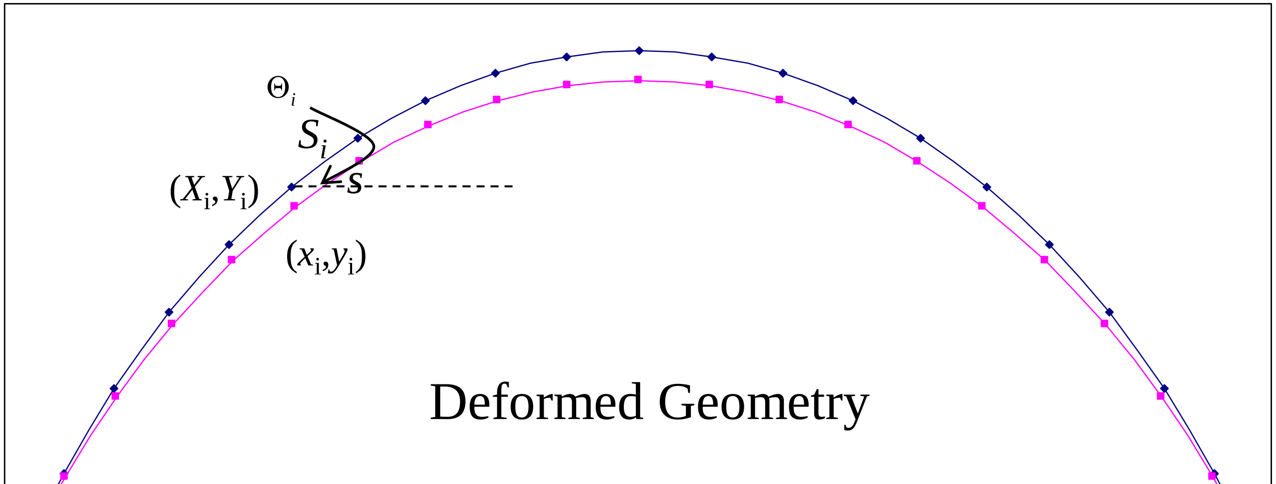
$$r_0 = \frac{1}{2}\left(\frac{d^2}{4h} + h\right)$$

$$\theta_0 = \tan^{-1}\frac{d}{2(r_0 - h)}$$

$$s = 2r_0 \sin\left(d\theta/2\right) \qquad \theta_i = \theta_0 - d\theta(i + 1/N)$$

$$x_i = x_{i-1} + s\cos\theta_{i-1}, \quad y_i = y_{i-1} + s\sin\theta_{i-1}$$

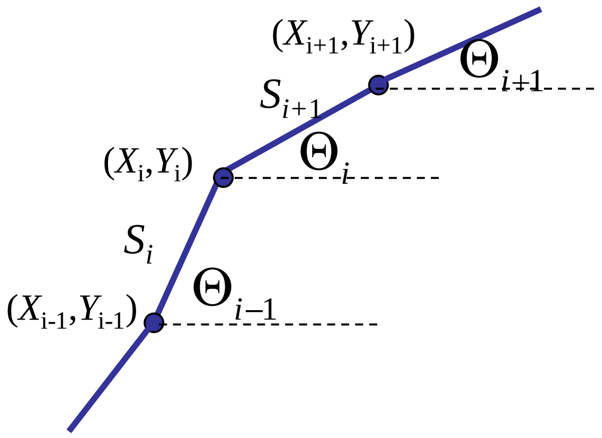
Ugly...



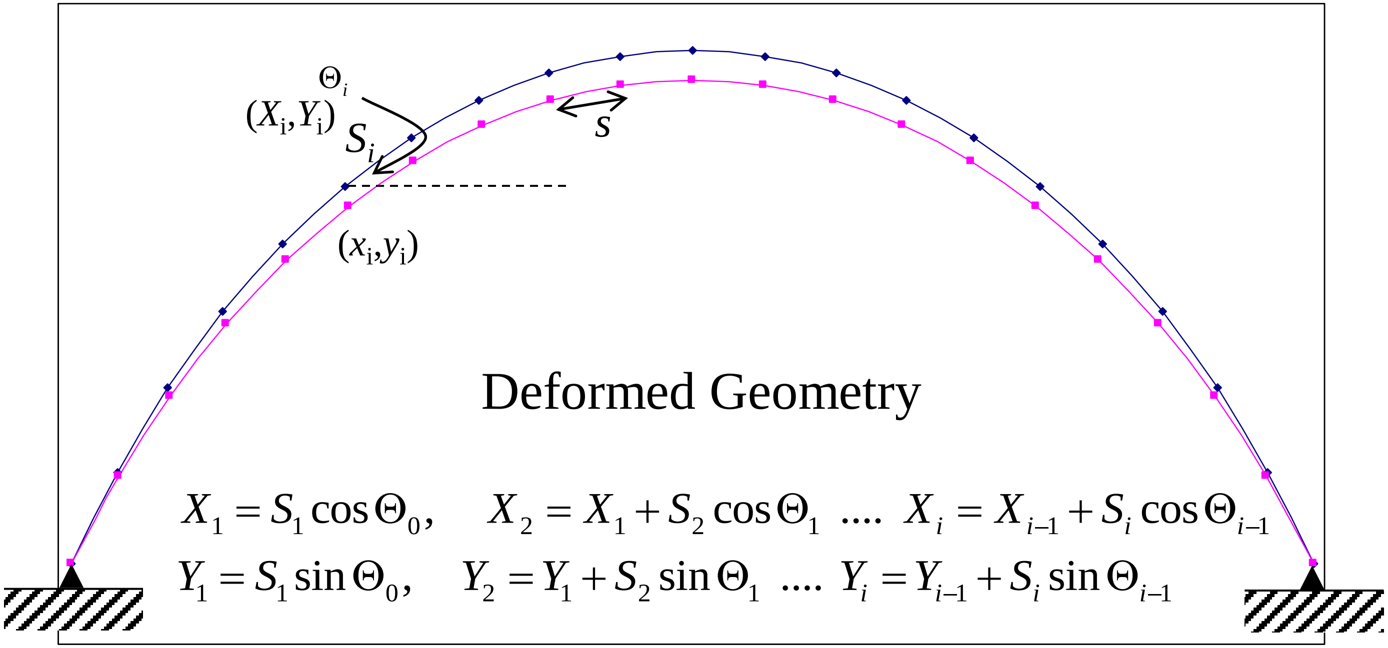
$$X_1 = S_1 \cos\Theta_0, \quad X_2 = X_1 + S_2 \cos\Theta_1 \; .... \; X_i = X_{i-1} + S_i \cos\Theta_{i-1}$$

$$Y_1 = S_1 \sin\Theta_0, \quad Y_2 = Y_1 + S_2 \sin\Theta_1 \; .... \; Y_i = Y_{i-1} + S_i \sin\Theta_{i-1}$$

- Curvature in segment i is $\kappa_i = (\Theta_{i-1} - \Theta_i)/S_i$
- Moment at i is $M_i = EI(\kappa_i - \kappa_0)$ $\kappa_0 = 1/r_0$
- Axial force for segment i is $EA(s - S_i)/s$
- Deflection at point i is $u_y = Y_i - y_i$
- Load at i is W_i



Minimize V, Varying Θ_i, S



$$X_1 = S_1\cos\Theta_0, \quad X_2 = X_1 + S_2\cos\Theta_1 \;\; .... \;\; X_i = X_{i-1} + S_i\cos\Theta_{i-1}$$

$$Y_1 = S_1\sin\Theta_0, \quad Y_2 = Y_1 + S_2\sin\Theta_1 \;\; .... \;\; Y_i = Y_{i-1} + S_i\sin\Theta_{i-1}$$

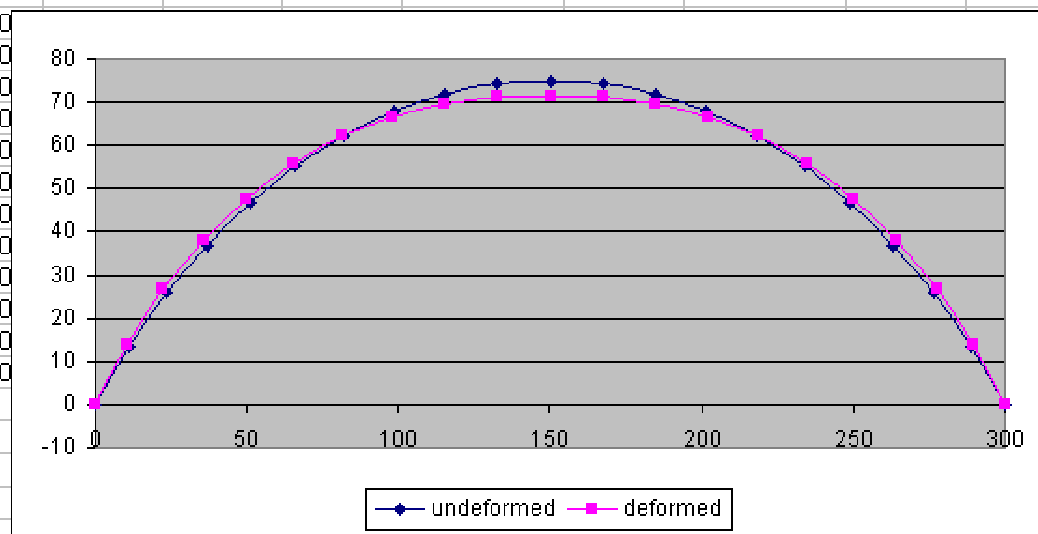
$$V = \frac{1}{2}EI\sum_{i=1}^{N-1} s\left(\frac{1}{r_o} - \kappa_i\right)^2 + \frac{1}{2}EA\sum_{i=0}^{N-1} s\left(\frac{S_i - s}{s}\right)^2 - \sum_{i=0}^{N} W_i u_i$$

Constraints from BC.

Microsoft Excel - archdefl refined.xls

J38

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
4	rise h	75	section i	0.083333		center x	150			numb of segs	20		anglespan	0.927295218
5			area	1		center y	112.5			undef chorleng	17.38056		dtheta0	0.092729522
6														
7		initial geometry			force P	deformed geometry								
8	point	theta	x	y		S	theta	X	Y	curv k	delta k	PE bending	PE stretch	PE load
9	0	0.88093	0	0		17.38041	0.926362	0	0				6.05E-05	
10	1	0.788201	11.06157	13.40617		17.38041	0.822862	10.44122	13.89459	0.005954964	0.000622	0.027984442	6.02E-05	0
11	2	0.695471	23.31699	25.73048		17.38041	0.711795	22.26209	26.63609	0.006390363	0.001057	0.080914604	5.99E-05	0
12	3	0.602742	36.66094	36.86702		17.38041	0.596727	35.42238	37.98887	0.006620534	0.001287	0.119990017	5.97E-05	0
13	4	0.510012	50.97877	46.72012		17.38041	0.481403	49.79909	47.75559	0.006635283	0.001302	0.122755532	5.98E-05	0
14	5	0.417283	66.14745	55.2051		17.38041	0.36958	65.20415	55.80313	0.006433884	0.001101	0.087714734	6.01E-05	0
15	6	0.324553	82.03664	62.24905		17.38041	0.264855	81.41102	62.08134	0.006025444	0.000692	0.034689904	6.04E-05	0
16	7	0.231824	98.50982	67.79146		17.38041	0.17052	98.18539	66.631	0.005427678	9.43E-05	0.000644597	6.04E-05	0
17	8	0.139094	115.4254	71.7847		17.38041	0.089442	115.3137	69.58036	0.004664922	-0.00067	0.032354864	5.96E-05	0
18	9	0.046365	132.6381	74.19444		17.380								0
19	10	-0.04636	150	75	-1	17.380								41621
20	11	-0.13909	167.3619	74.19444		17.380								0
21	12	-0.23182	184.5746	71.7847		17.380								0
22	13	-0.32455	201.4902	67.79146		17.380								0
23	14	-0.41728	217.9634	62.24905		17.380								0
24	15	-0.51001	233.8525	55.2051		17.380								0
25	16	-0.60274	249.0212	46.72012		17.380								0
26	17	-0.69547	263.3391	36.86702		17.380								0
27	18	-0.7882	276.683	25.73048		17.380								0
28	19	-0.88093	288.9384	13.40617		17.380								0
29	20	-0.97366	300	-1.8E-14		17.380								0
30														
31														
32	Total energ	-1.59962												
33														
34														
35														



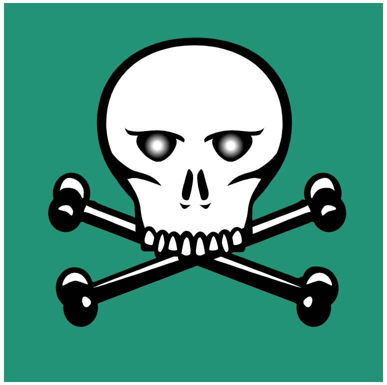
circ | geometry | Moments | Axial Force

Chart Area

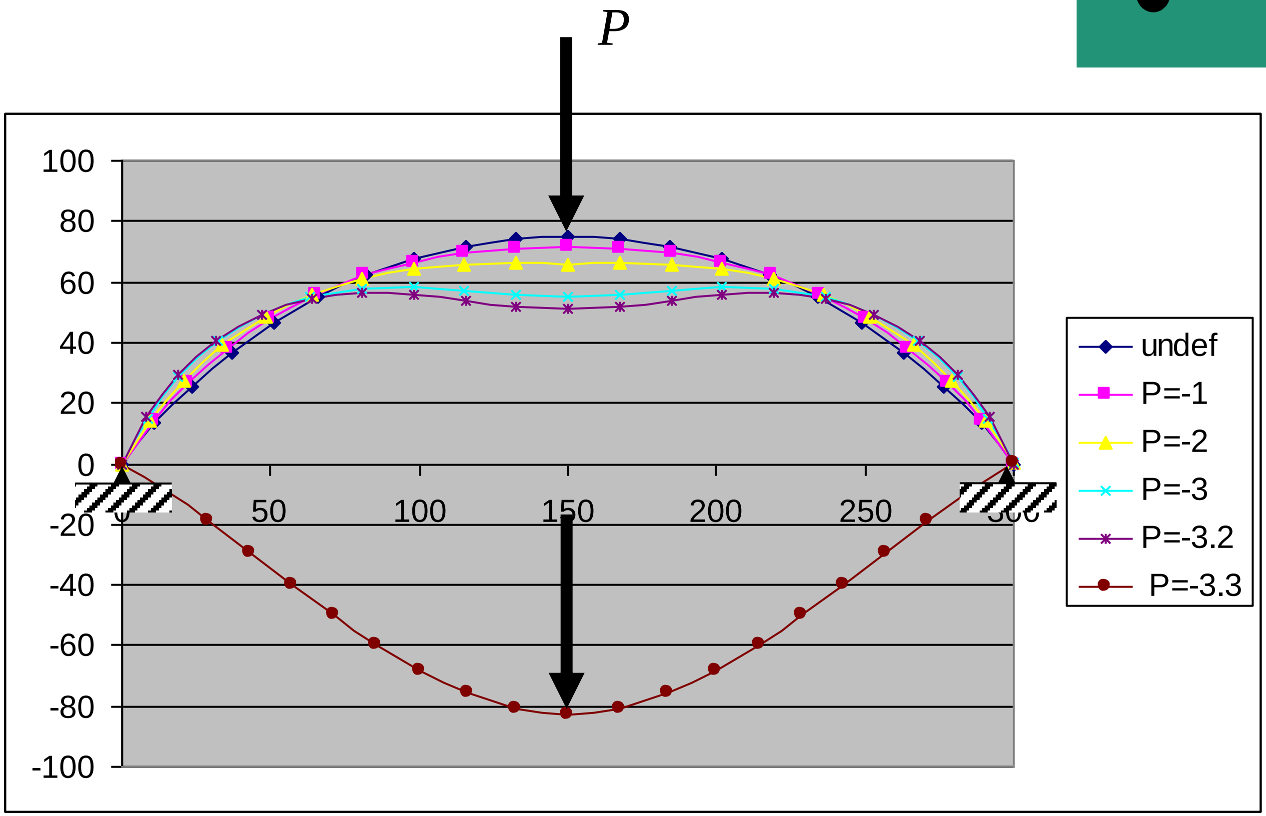
Ready

Don’t Worry, Be Happy

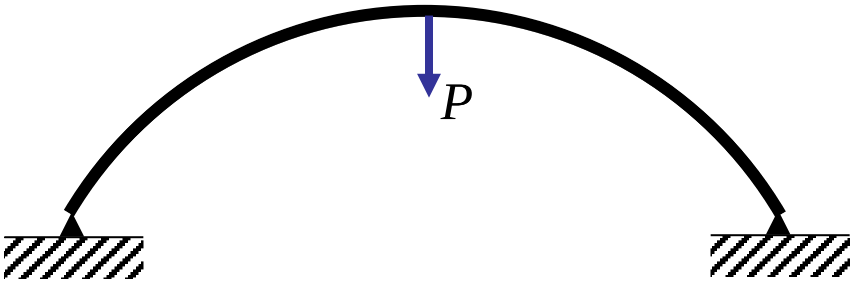
Beware Snap-through Buckling!



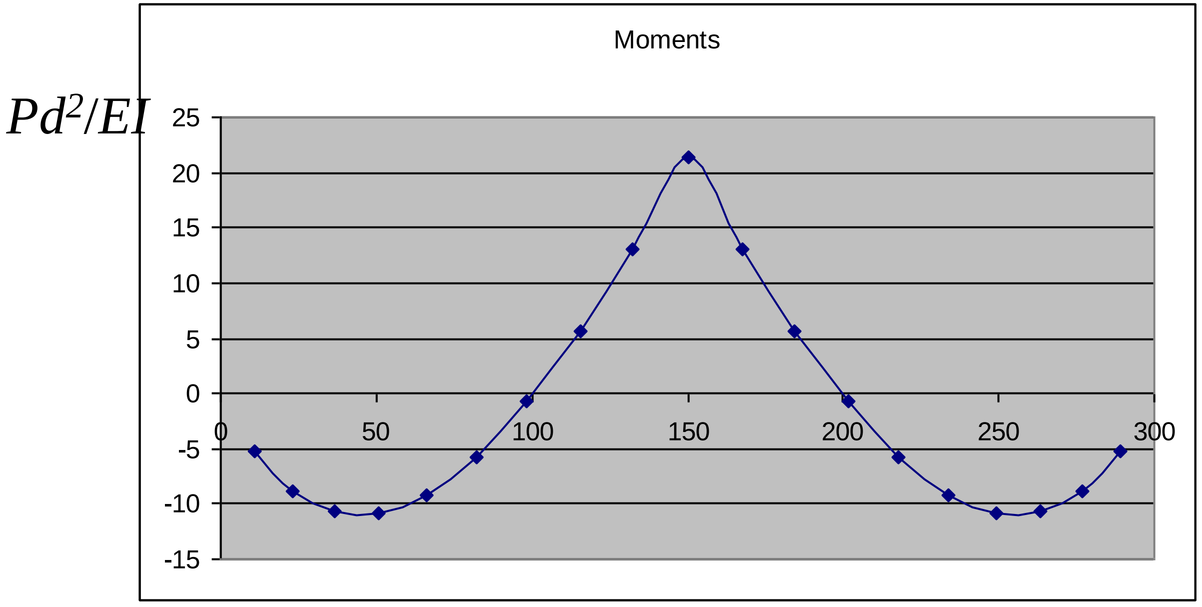
E=10^5, I=1/12, A=1, d=300, h=70



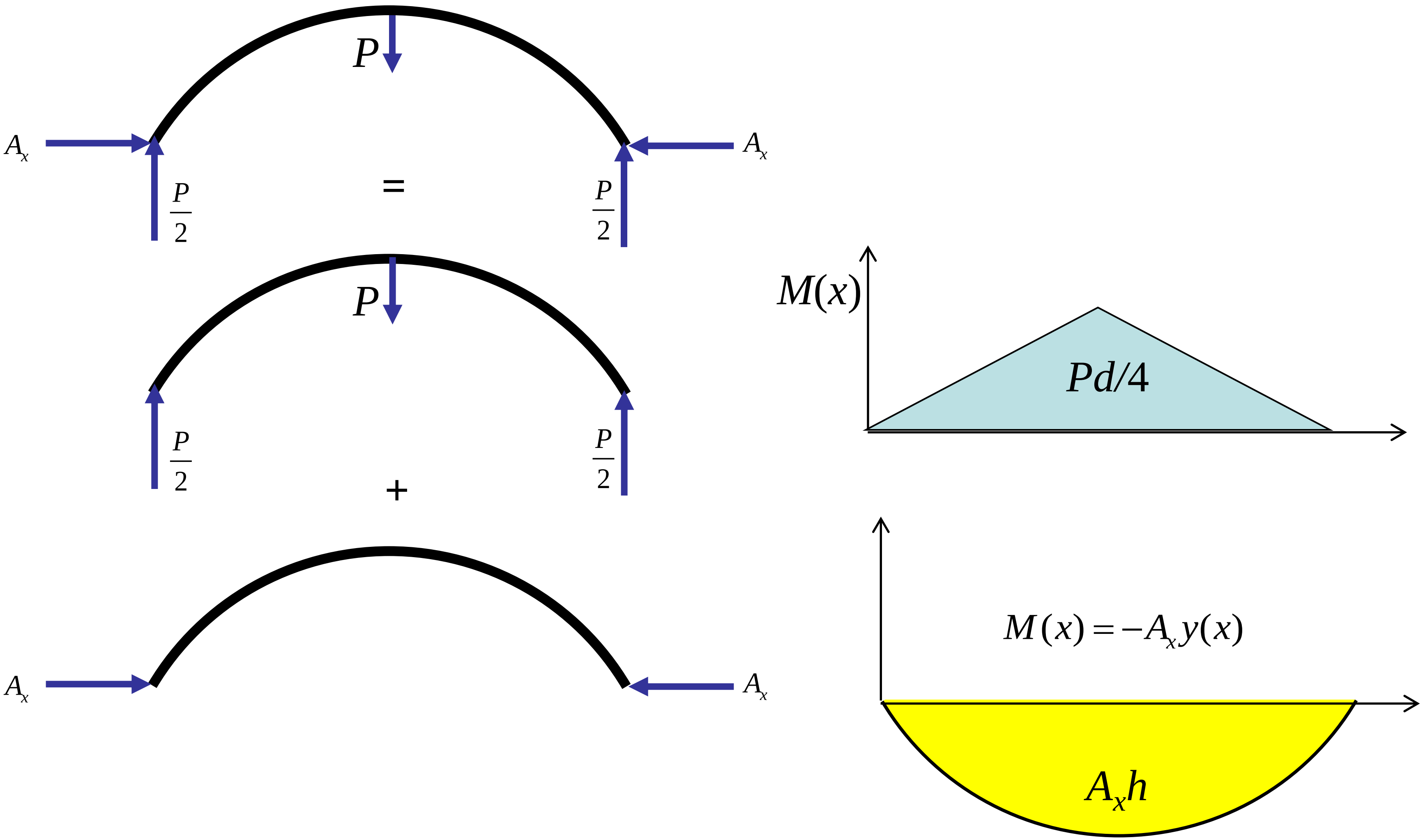
Small deflections not assumed!



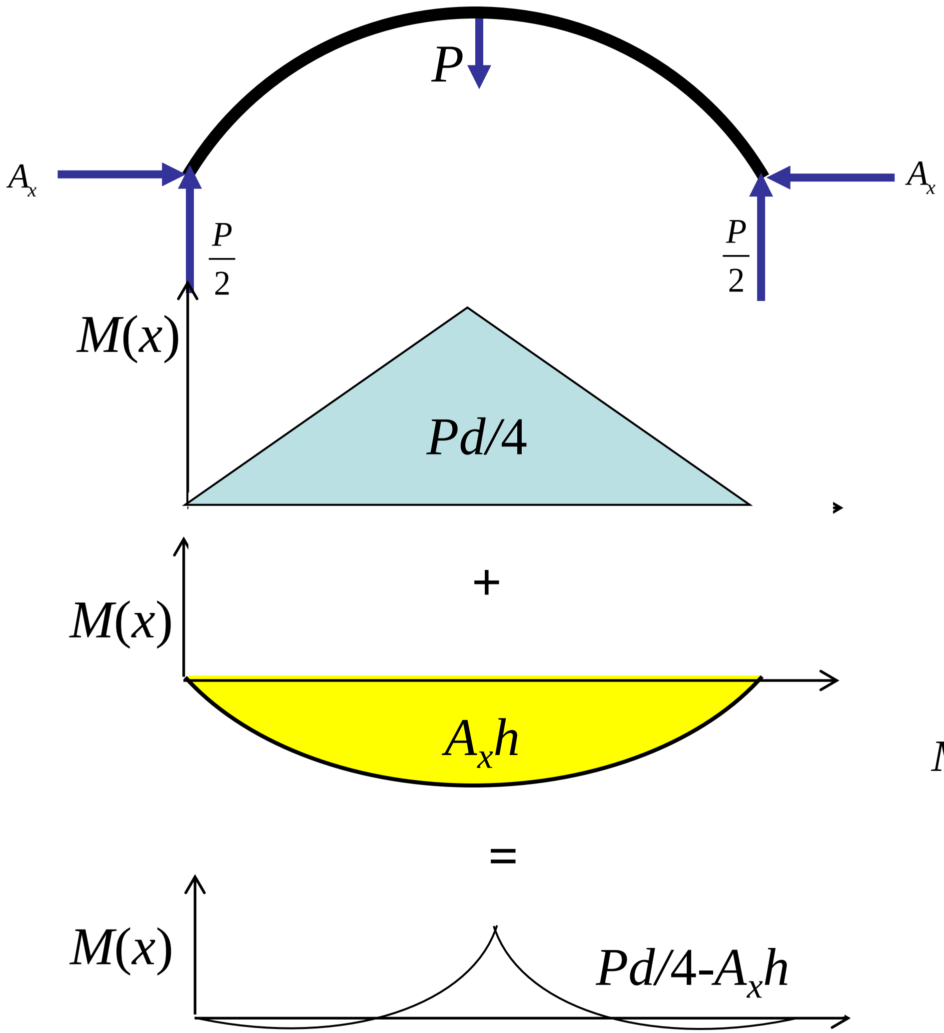
Moments



Approximate Moments

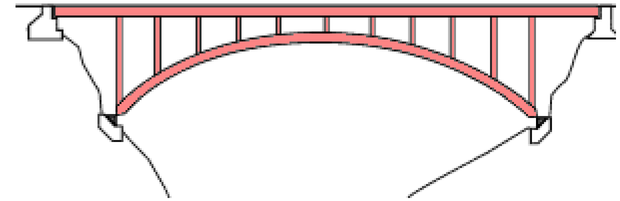


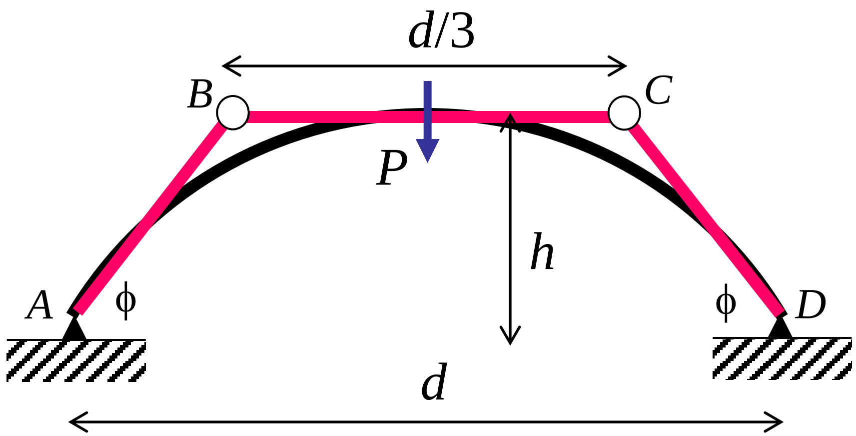
Moments

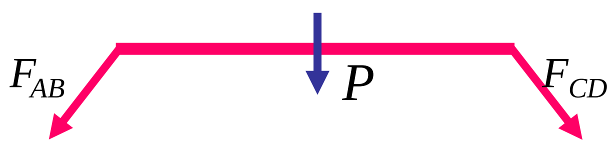


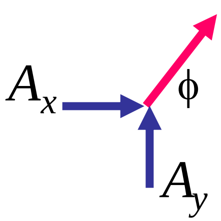
$$M(x) = -A_x y(x)$$

Approximate Analysis









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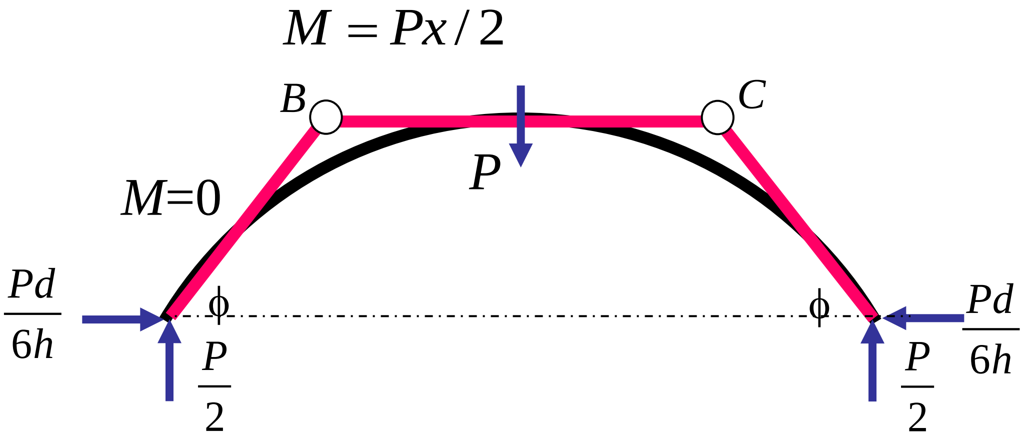
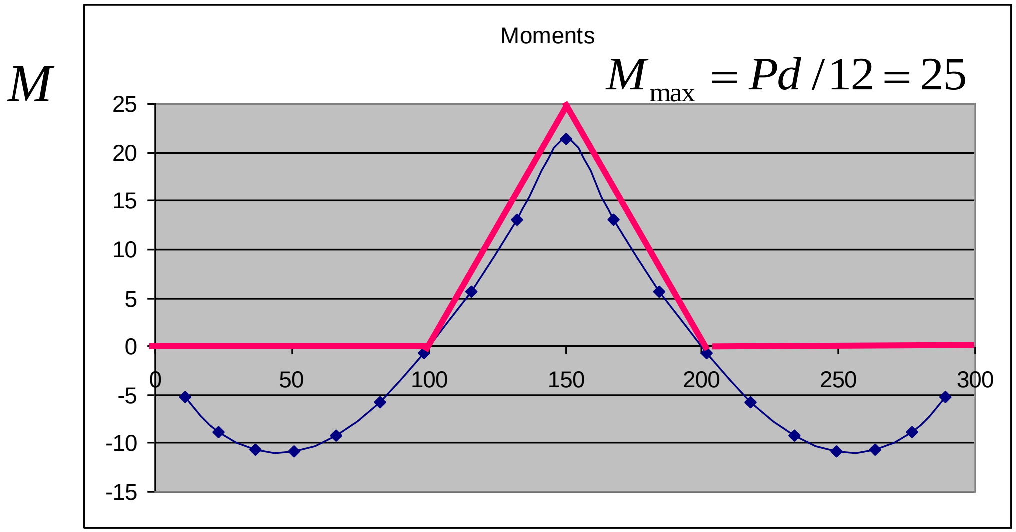
Moments (1)





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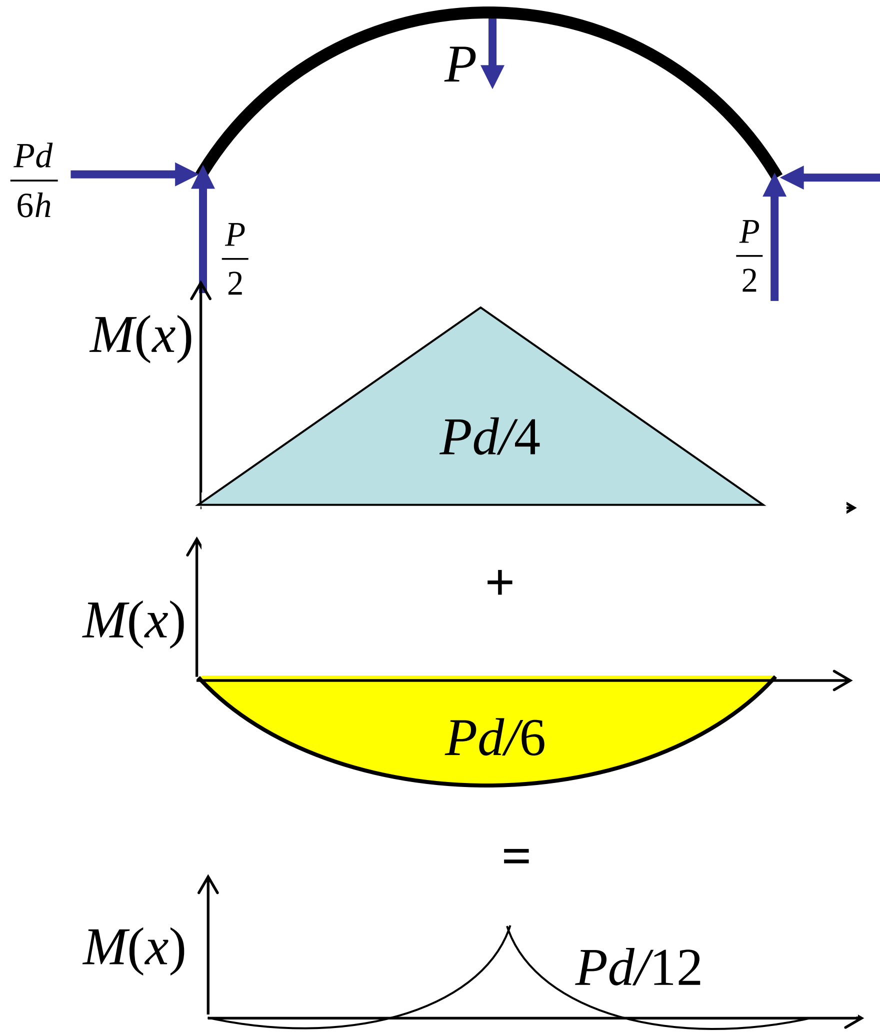
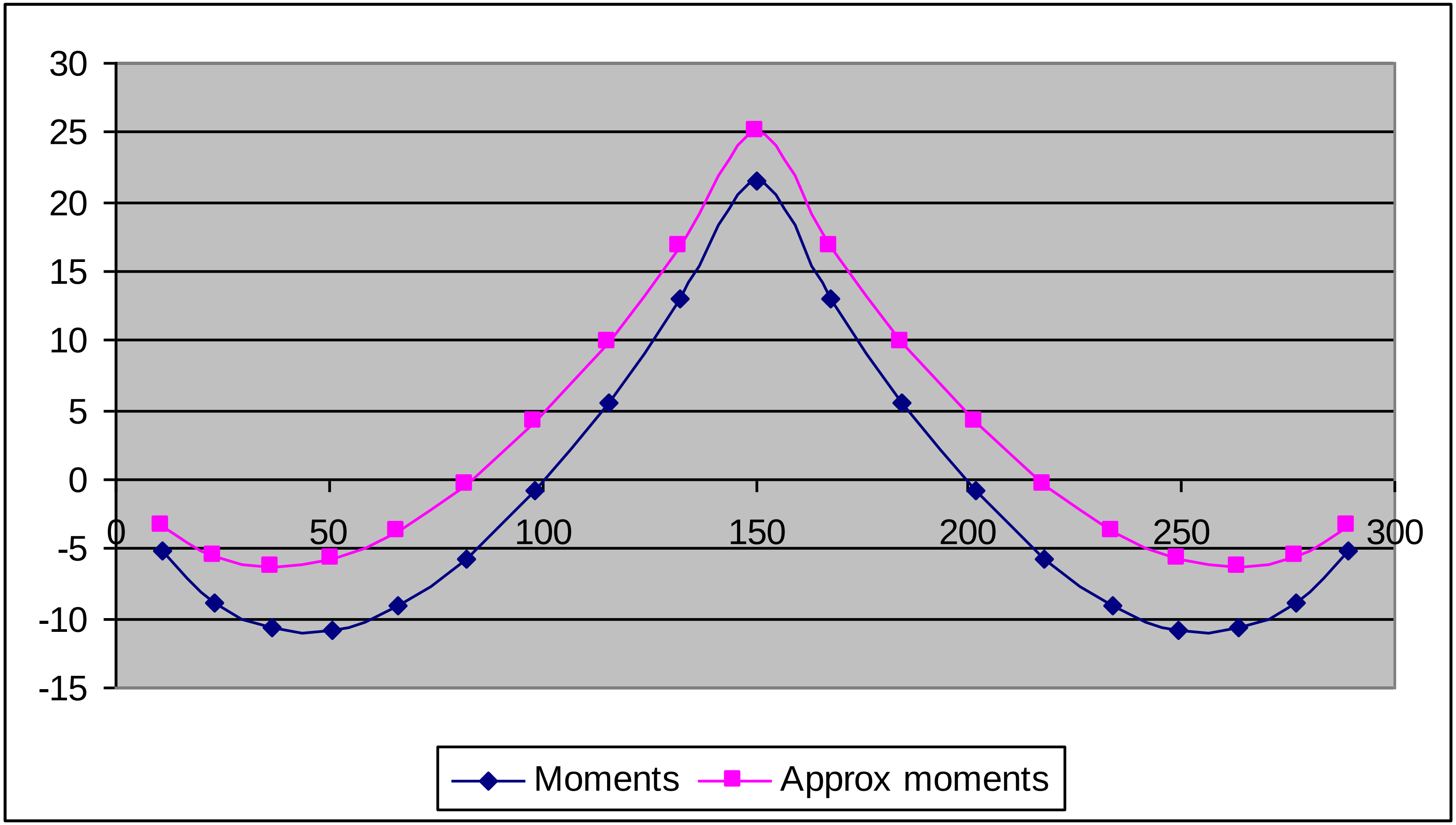
Moments (2)

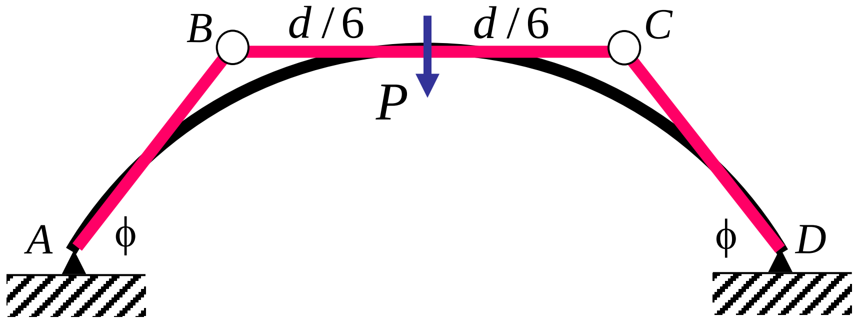


$$M(x) = -\frac{Pd}{6h}\, y(x)$$

P=1, d=300ft, h=75ft



Deflections



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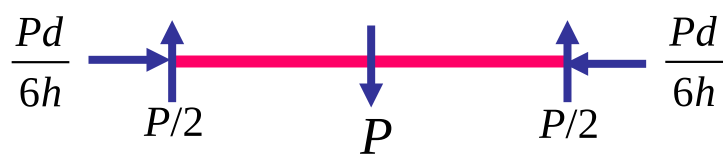

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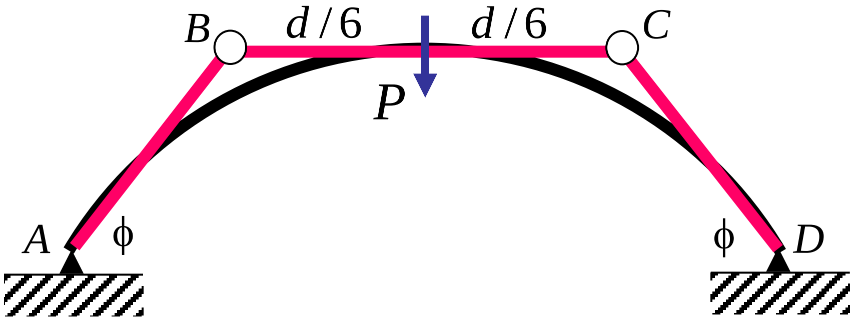
Deflections



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$$\Rightarrow u_y^B = -\frac{PL_{AB}}{2EA\sin^2\phi} - \frac{Pd^2}{36EAh\tan\phi} = -\frac{P(h^2+d^2/9)^{3/2}}{2EAh^2} - \frac{Pd^3}{108EAh^2}$$

$$u_y^P = u_y^B - \frac{P\left(d/3\right)^3}{48EI}$$

For E=10^6, P=1, d=300ft, h=75ft, I=0.0833ft^4, A=1ft^2

$$u_y^P = -0.25\text{ft}$$

Excel gives 0.28 feet