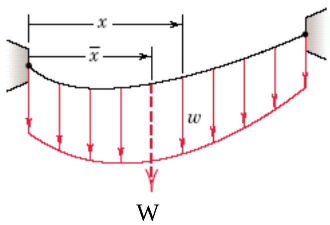
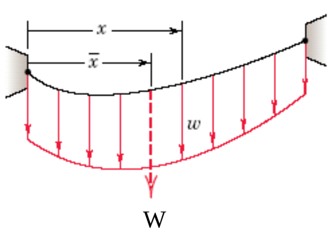
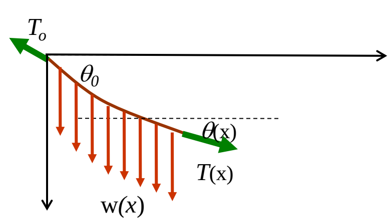
Cable Structures

- Cables can withstand only tensile internal forces: no internal moments or shear.



Differential Equations





$$\theta(x) = \tan^{-1}\frac{dy}{dx}(x)$$

$$\sum F_x = -T_0\cos\theta_0 + T(x)\cos\theta(x) = 0 \Rightarrow \boxed{T(x) = T_0\frac{\cos\theta_0}{\cos\theta(x)} = \frac{R_x}{\cos\theta(x)}}$$

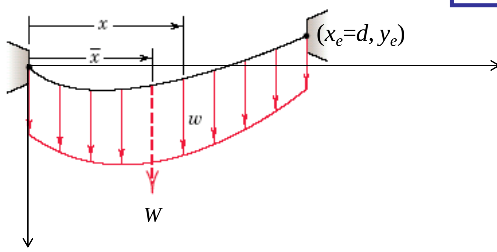
$$\sum F_y = -T_0\sin\theta_0 + T(x)\sin\theta(x) + \int_0^x w(\xi)d\xi = -T_0\sin\theta_0 + R_x\tan\theta(x) + \int_0^x w(\xi)d\xi = 0$$

$$-T_0\sin\theta_0 + R_x\frac{dy}{dx} + \int_0^x w(\xi)d\xi = 0$$

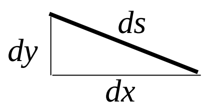
Differentiate:

$$-T_0 \sin\theta_0 + R_x \frac{dy}{dx} + \int_0^x w(\xi)d\xi = 0 \Rightarrow$$

$$\boxed{\frac{d^2y}{dx^2} = -\frac{w}{R_x}}$$

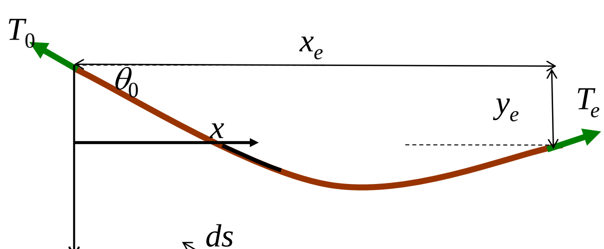


Given: Cable Length L_0 , span d Endpoints y(0)=0, $y(x_e)=y_e$ and force $w(x)$
Find $y(x)$ and R_x.

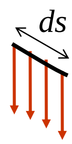


$$L_0 = \int_0^{x_e=d} ds = \int_0^{x_e=d} \sqrt{dx^2+dy^2} = \int_0^{x_e=d} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Cable Under Its Own Weight



$w(x)dx=\omega\, ds(x)$



$$ds = \sqrt{dx^2+dy^2} = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\frac{d^2y}{dx^2} = -\frac{w}{R_x} = -\frac{\omega}{R_x}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Solution by Reduction of Order

$$\frac{d^2y}{dx^2} = -\frac{w}{R_x} = -\frac{\omega}{R_x}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

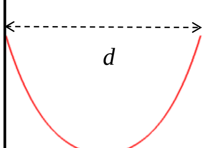
$$\frac{dy}{dx} = p: \quad \frac{dp}{dx} = -\frac{\omega}{R_x}\sqrt{1+p^2} \Rightarrow \int_{p_0}^{p(x)} \frac{dp}{\sqrt{1+p^2}} = -\frac{\omega}{R_x}x$$

$$\sinh^{-1} p(x) = -\frac{\omega}{R_x}x + \underbrace{\sinh^{-1} p_0}_{c} \Rightarrow p(x) = \frac{dy}{dx} = \sinh\left(c - \frac{\omega}{R_x}x\right)$$

$$y(x) = \frac{R_x}{\omega}\left\{\cosh(c) - \cosh\left(c - \frac{\omega}{R_x}x\right)\right\}$$

Two Constants… c and R_x

$$y(x) = \frac{R_x}{\omega}\left\{\cosh(c) - \cosh\left(c - \frac{\omega}{R_x}x\right)\right\}$$



$y(d)=y_e$.   Example: $y(d)=0$.

$$\Rightarrow \cosh(c) = \cosh\left(c - \frac{\omega}{R_x}d\right) \Rightarrow c = \frac{\omega}{2R_x}d.$$

$$y(x)/d = \frac{1}{2c}\left\{\cosh(c) - \cosh\left(c(1-2x/d)\right)\right\}$$