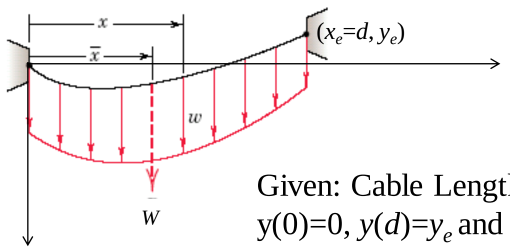
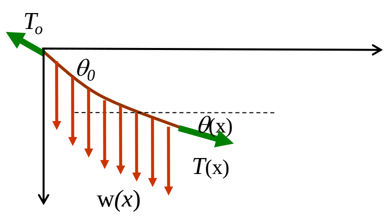
Cable Equations:



$$\frac{d^2y}{dx^2} = -\frac{w}{R_x}$$

Given: Cable Length L_0 , span d Endpoints y(0)=0, $y(d)=y_e$ and force $w(x)$, find $y(x)$ and R_x.



$$L_0 = \int_0^d \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$T(x) = \frac{R_x}{\cos\theta(x)} = R_x\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\theta(x) = \tan^{-1}\frac{dy}{dx}(x)$$

Cable Under Its Own Weight

ω: weight per unit length of cable.

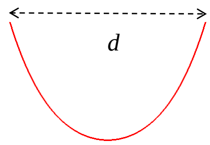
$$\frac{d^2y}{dx^2} = -\frac{w}{R_x} = -\frac{\omega}{R_x}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$y(x) = \frac{R_x}{\omega}\left\{\cosh(c) - \cosh\left(c - \frac{\omega}{R_x}x\right)\right\}$$

Constants c and R_x come from $y(d)=y_e$ and

$$L_0 = \int_0^d \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

Example: $y(d)=0$



$$\Rightarrow \cosh(c) = \cosh\left(c - \frac{\omega}{R_x} d\right) \Rightarrow c = \frac{\omega}{2R_x} d.$$

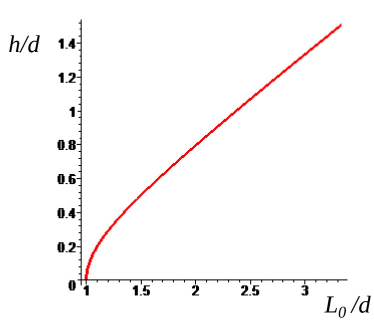
$$y(x)/d = \frac{1}{2c}\left\{\cosh(c) - \cosh\left(c(1-2x/d)\right)\right\}$$

Determine the remaining constant: c

$$y(x)/d = \frac{1}{2c}\left\{\cosh(c) - \cosh\left(c(1-2x/d)\right)\right\}$$

$$L_0 = \int_0^d \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_0^d \cosh\left(c(1-2x/d)\right) dx = \frac{d}{c}\sinh(c)$$

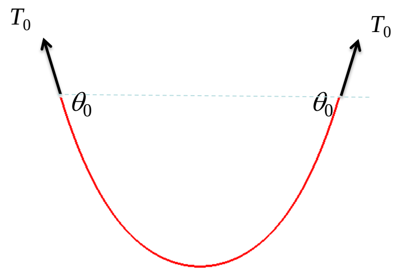
$$\Rightarrow L_0/d = \frac{1}{c}\sinh c$$



$$y(d/2) = h = \frac{d}{2c}\left\{\cosh(c) - 1\right\}$$

$$\Rightarrow \frac{h}{d} = \frac{1}{2c}\left\{\cosh(c) - 1\right\}$$

End Tension



$$2T_0 \sin\theta_0 = wL_0$$

$$\tan\theta_0 = y'(0) = \sinh(c)$$

$$\sin\theta_0 = \tanh(c) \qquad T_0 = \frac{wL_0}{2\sin\theta_0} = \frac{wL_0}{2\tanh(c)} = \frac{wd}{2c}\cosh(c)$$

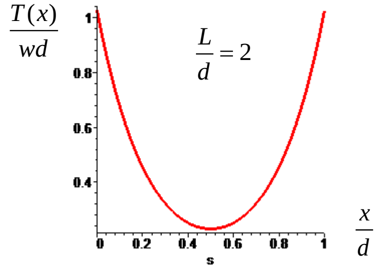
$$L_0 / d = \frac{1}{c}\sinh c$$

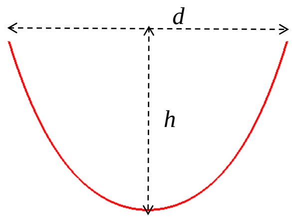
Internal tension

$$T(x) = T_0 \frac{\cos\theta_0}{\cos\theta(x)} = T_0 \cos\theta_0 \sqrt{1+\tan\theta^2(x)} = T_0 \cos\theta_0 \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$T(x) = \frac{wd}{2c}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow \frac{T(x)}{wd} = \frac{1}{2c}\cosh\left(c(1-2x/d)\right)$$





Summary

$$L_0 / d = \frac{1}{c}\sinh c$$

$$\frac{h}{d} = \frac{1}{2c}\left\{\cosh(c) - 1\right\}$$

Get *c* from one of these

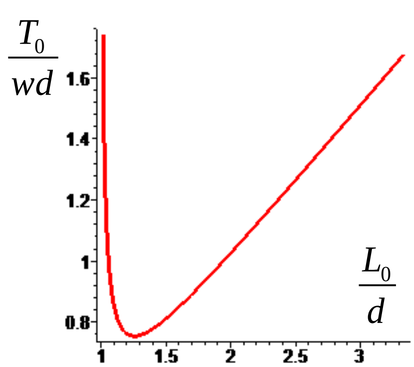
$$y(x)/d = \frac{1}{2c}\left\{\cosh(c) - \cosh\left(c(1-2x/d)\right)\right\}$$

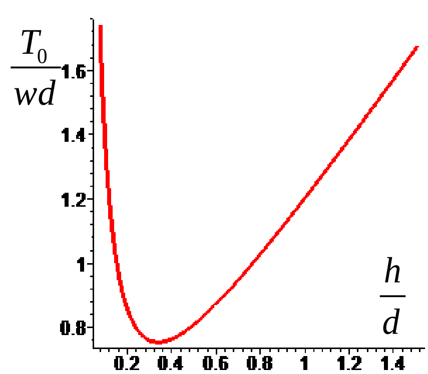
$$\frac{T_0}{wd} = \frac{1}{2c}\cosh(c)$$

$$\frac{T(x)}{wd} = \frac{1}{2c}\cosh\left(c(1-2x/d)\right)$$

Optimal Length to minimize tension, given *d*?

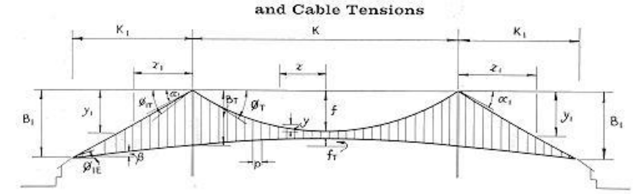
$$L_0 / d = \frac{1}{c}\sinh c \qquad \frac{T_0}{wd} = \frac{1}{2c}\cosh(c) \qquad \frac{h}{d} = \frac{1}{2c}\left\{\cosh(c) - 1\right\}$$





Optimum sag is about 1/3 of the span

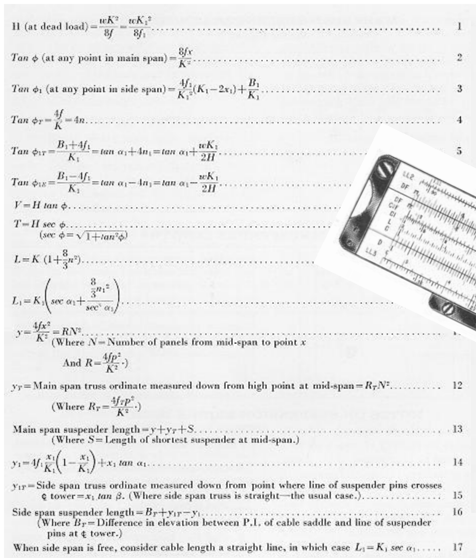
1930's



Approximate Formulas for Determining Cable and Suspender Lengths and Cable Tensions

w = Dead load total weight per foot, including cable and suspenders.
V = Vertical component of cable tension.
H = Horizontal component of cable tension.
T = Cable tension.
ϕ = Angle with horizontal of tangent to cable curve.
α = Angle with horizontal of chord line joining ends of span.
$n=\frac{f}{K}$, $n_1=\frac{f_1}{K_1}$ (sag ratio).
L = Length of cable in main span.
L_1 = Length of cable in side span.

Slide Rules Rule!



H (at dead load) $= \frac{wK^2}{8f} = \frac{wK_1^2}{8f_1}$ 1

$Tan\ \phi$ (at any point in main span) $= \frac{8fx}{K^2}$ 2

$Tan\ \phi_1$ (at any point in side span) $= \frac{4f_1}{K_1^2}(K_1 - 2x_1) + \frac{B_1}{K_1}$ 3

$Tan\ \phi_T = \frac{4f}{K} = 4n$ 4

$Tan\ \phi_{1T} = \frac{B_1 + 4f_1}{K_1} = tan\ \alpha_1 + 4n_1 = tan\ \alpha_1 + \frac{wK_1}{2H}$ 5

$Tan\ \phi_{1S} = \frac{B_1 - 4f_1}{K_1} = tan\ \alpha_1 - 4n_1 = tan\ \alpha_1 - \frac{wK_1}{2H}$ ....

$V = H\ tan\ \phi$ ....

$T = H\ sec\ \phi$ ....
($sec\ \phi = \sqrt{1 + tan^2\phi}$)

$L = K\ (1 + \frac{8}{3}n^2)$ ....

$L_1 = K_1\left(sec\ \alpha_1 + \frac{\frac{8}{3}n_1^2}{sec^3\ \alpha_1}\right)$ ....

$y = \frac{4fx^2}{K^2} = RN^2$ ....
(Where N = Number of panels from mid-span to point x
And $R = \frac{4fp^2}{K^2}$.)

y_T = Main span truss ordinate measured down from high point at mid-span $= R_T N^2$ 12
(Where $R_T = \frac{4f_T p^2}{K^2}$.)

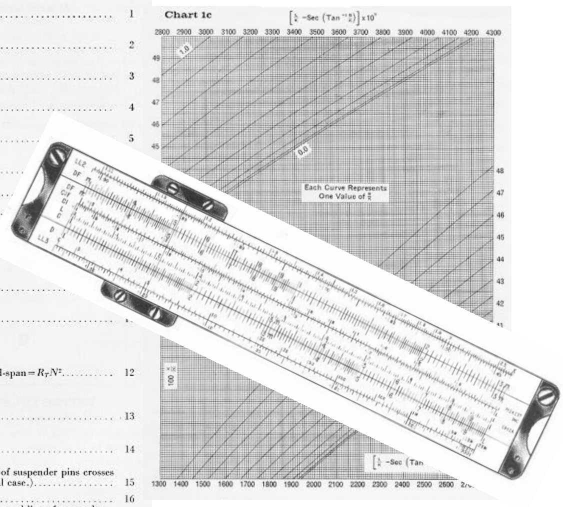
Main span suspender length $= y + y_T + S$ 13
(Where S = Length of shortest suspender at mid-span.)

$y_1 = 4f_1 \frac{x_1}{K_1}\left(1 - \frac{x_1}{K_1}\right) + x_1\ tan\ \alpha_1$ 14

y_{1T} = Side span truss ordinate measured down from point where line of suspender pins crosses ℄ tower $= x_1\ tan\ \beta$. (Where side span truss is straight—the usual case.) 15

Side span suspender length $= B_T + y_{1T} - y_1$ 16
(Where B_T = Difference in elevation between P.I. of cable saddle and line of suspender pins at ℄ tower.)

When side span is free, consider cable length a straight line, in which case $L_1 = K_1\ sec\ \alpha_1$ 17



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