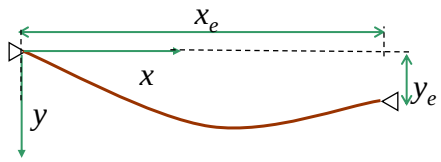
Cable by Energy Minimization

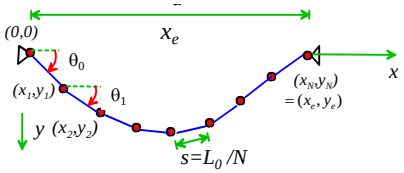


Given cable weight density ω and length L_0

Given endpoints $(x_e y_e)$

Find the cable shape that minimizes the total potential potential energy of the cable

Divide the cable into N segments



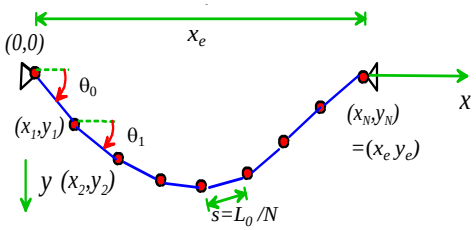
$$x_1 = s\cos\theta_0, \quad x_2 = x_1 + s\cos\theta_1 \;\ldots\; x_i = x_{i-1} + s\cos\theta_{i-1}$$
$$y_1 = s\sin\theta_0, \quad y_2 = y_1 + s\sin\theta_1 \;\ldots\; y_i = y_{i-1} + s\sin\theta_{i-1}$$
$$x_N = x_e \quad y_N = y_e$$

Potential energy of the ith segment connecting points (x_i,y_i) and (x_{i+1},y_{i+1})

$$V_0 = -\frac{1}{2}\omega s y_1 \qquad V_1 = -\frac{1}{2}\omega s(y_1 + y_2) \;\ldots\; V_i = -\frac{1}{2}\omega s(y_i + y_{i+1})$$

Total PE $\quad V = \sum_{i=0}^{N-1} V_i$

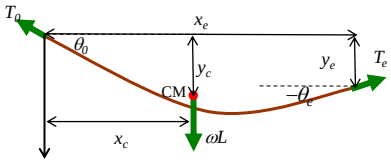
Find the angles θ_i (i=0,1,…,N-1) that minimize the potential energy



$$V=\sum_{i=0}^{N-1}V_i=-\sum_{i=0}^{N-1}\frac{\omega s}{2}\left(y_i+y_{i+1}\right)$$

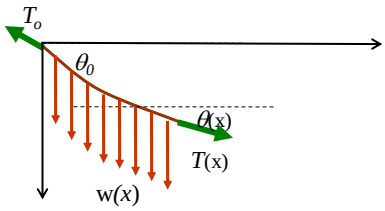
Constraint: $(x_N,y_N)=(x_e,y_e)$

Tension in each segment



$$\sum F_y=-T_0\sin\theta_0+T_e\sin\theta_e+\omega L=0$$
$$\sum F_x=-T_0\cos\theta_0+T_e\cos\theta_e=0$$

$$\theta_e=\theta_{N-1}$$
$$T_0=\frac{\omega L}{\sin\theta_0-\cos\theta_0\tan\theta_e}$$



$$T(x)=T_0\frac{\cos\theta_0}{\cos\theta(x)}$$

$$T_i=T_0\frac{\cos\theta_0}{\cos\theta_i}$$

Defined at segment midpoint:
$x=(x_i+x_{i+1})/2$