# Arches


## Arches ideally are in a state of pure compression

Cable under its own weight

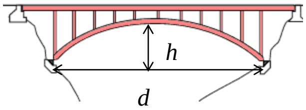
**Arch under its own weight**

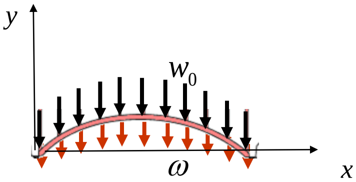
Ideal shape: cosh

## Carrying a constant vertical load


## Arch with dead and self-load

- Suspension cable weight is usually small compared with distributed load it carries.
- Arch self weight $\omega$ is often comparable to constant distributed load $w_0$




Ideal Arch shape equation:

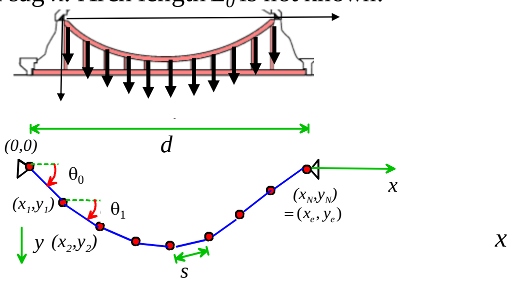
$$\frac{d^2y}{dx^2} = -\frac{w}{R_x} = -\frac{w_0}{R_x} - \frac{\omega}{R_x}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

BC: $y(0)=y(d)=0$, $y(d/2)=h$

# Can solve by reduction of order, but the solution is messy.

Solve for the cable shape by energy minimization.

Given sag $h$. Arch length $L_0$ is not known.



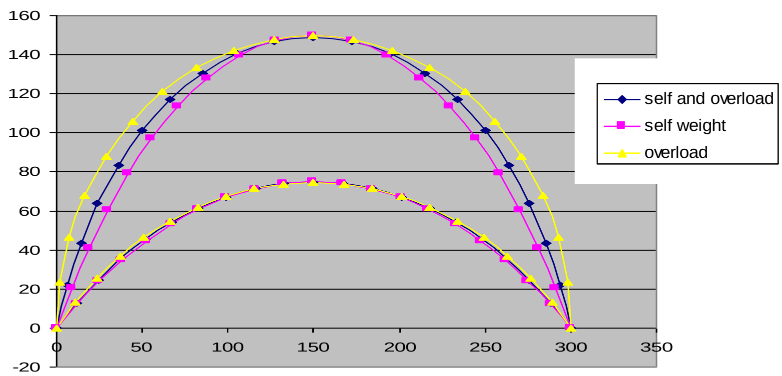

$$V_0=-\frac{1}{2}(\omega s+w_0x_1)y_1 \qquad V_1=-\frac{1}{2}(\omega s+w_0(x_2-x_1))(y_1+y_2)\ .....$$

$$V_i=-\frac{1}{2}(\omega s+w_0(x_{i+1}-x_i))(y_i+y_{i+1})$$

Vary $\theta_0,\theta_1,\ldots\theta_{N-1}$ and segment length $s$ to minimize the total PE $V=\sum_{i=0}^{N-1}V_i$

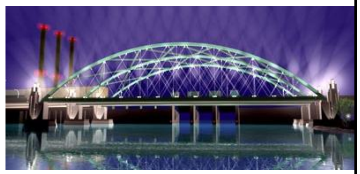
Constraints: $(x_N,y_N)=(d,0)$ and $\max(y_i)=h$

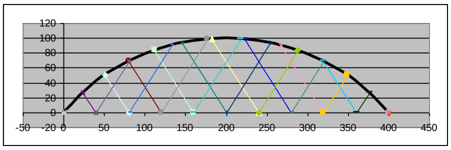
# Ideal Arch Shapes
# Load is $\omega$ (self wt) $+w_0$ (overload)




$\omega L= w_0d$=W/2, $\omega$ L=W, $w_0d$=W

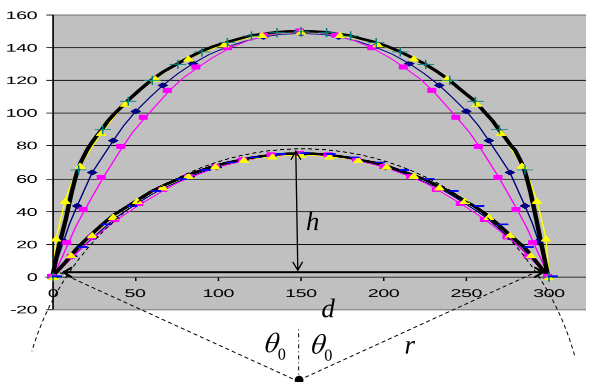
# Circular arches






- Easy to draw
- Easy to fabricate
- Easy to analyze


# Geometric Comparison



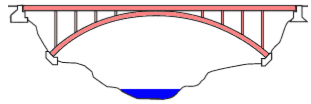

$$r = \frac{1}{2}\left(\frac{d^2}{4h} + h\right), \quad y = \sqrt{r^2 - (x - d/2)^2} - r + h$$

$$|\theta| \le \theta_0 = \tan^{-1}\frac{d}{2(r-h)}$$

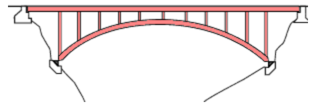
# Some ArchTypes

### Statically Indeterminate

Fixed

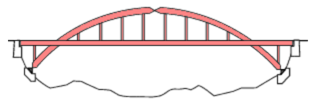


Two-hinge

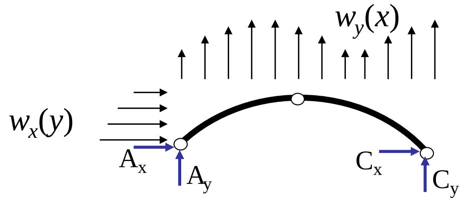


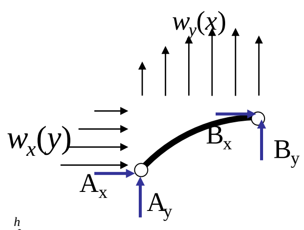
### Statically Determinate

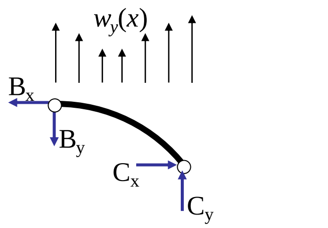
Three-hinge



# Analysis: 3-Hinged




$$A_x + B_x + \int_0^h w_x(\eta)\,d\eta = 0$$

$$A_y + B_y + \int_0^{d/2} w_y(\eta)\,d\eta = 0$$

$$B_y \frac{d}{2} - B_x h - \int_0^h w_x(\eta)\eta\,d\eta + \int_0^{d/2} \eta w_y(\eta)\,d\eta = 0$$

$$C_x - B_x = 0$$

$$C_y - B_y + \int_{d/2}^{d} w_y(\eta)\,d\eta = 0$$

$$C_y \frac{d}{2} + C_x h + \int_{d/2}^{d} (\eta - d/2) w_y(\eta)\eta\,d\eta = 0$$