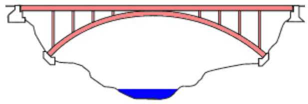
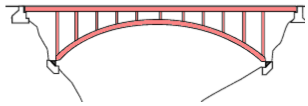
Some ArchTypes

Statically Indeterminate

Fixed

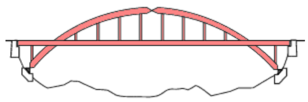


Two-hinge

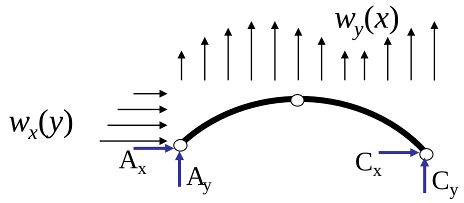


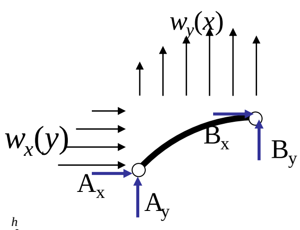
Statically Determinate

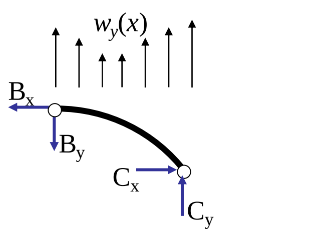
Three-hinge



Analysis: 3-Hinged







$$A_x + B_x + \int_0^h w_x(\eta)d\eta = 0$$

$$A_y + B_y + \int_0^{d/2} w_y(\eta)d\eta = 0$$

$$B_y \frac{d}{2} - B_x h - \int_0^h w_x(\eta)\eta d\eta + \int_0^{d/2} \eta w_y(\eta)d\eta = 0$$

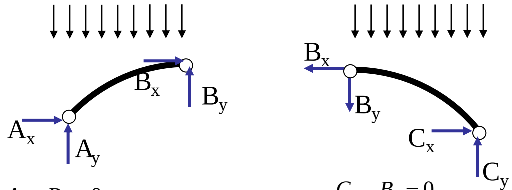
$$C_x - B_x = 0$$

$$C_y - B_y + \int_{d/2}^{d} w_y(\eta)d\eta = 0$$

$$C_y \frac{d}{2} + C_x h + \int_{d/2}^{d} (\eta - d/2) w_y(\eta)\eta d\eta = 0$$

Constant load w_y=-w_0, w_x=0

$-w_y(x)=w_0$



$$A_x + B_x = 0$$

$$A_y + B_y - w_0 \frac{d}{2} = 0$$

$$B_y \frac{d}{2} - B_x h - w_0 \frac{d^2}{8} = 0$$

$$C_x - B_x = 0$$

$$C_y - B_y - w_0 \frac{d}{2} = 0$$

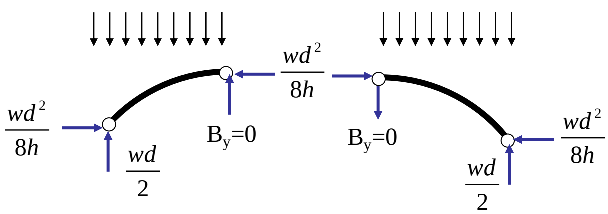
$$C_y \frac{d}{2} + C_x h - w_0 \frac{d^2}{8} = 0$$

$$by = 0,\ cy = \frac{w\,d}{2},\ cx = -\frac{w\,d^2}{8\,h},\ ay = \frac{w\,d}{2},\ bx = -\frac{w\,d^2}{8\,h},\ ax = \frac{w\,d^2}{8\,h}$$

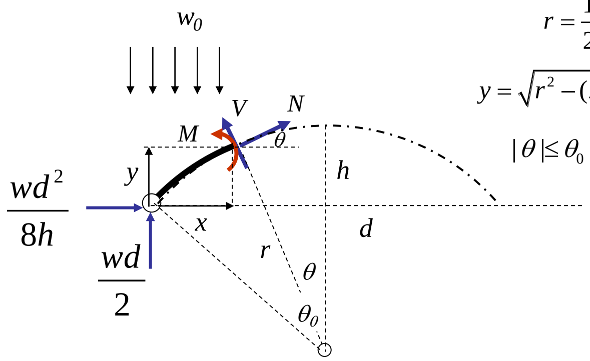
Constant load

$-w_y = w_0$, w_x=0

$-w_y(x)=w_0$



Internal Forces, Moments



$$r = \frac{1}{2}\left(\frac{d^2}{4h} + h\right)$$

$$y = \sqrt{r^2 - (x - d/2)^2} - r + h$$

$$|\theta| \le \theta_0 = \tan^{-1}\frac{d}{2(r-h)}$$

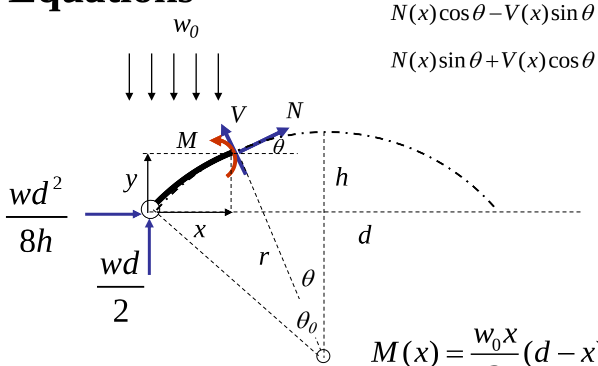
$$x(\theta) = d/2 - r\sin\theta,\ y(\theta) = r\cos\theta - r + h,\ \theta = \tan^{-1}\frac{d/2 - x}{y + r - h}$$

$$\sin\theta = \frac{d/2 - x}{r},\ \cos\theta = \frac{y + r - h}{r}$$

Equations

$$N(x)\cos\theta - V(x)\sin\theta + \frac{w_0 d^2}{8h} = 0$$

$$N(x)\sin\theta + V(x)\cos\theta + w_0(d/2 - x) = 0$$



$$M(x) = \frac{w_0 x}{2}(d - x) - \frac{w_0 d^2}{8h} y$$

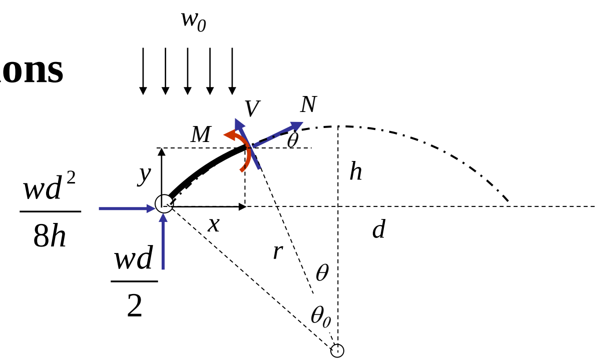
$$y = \sqrt{r^2 - (x - d/2)^2} - r + h$$

$$N(x) = -w_0(d/2 - x)\sin\theta - \frac{w_0 d^2}{8h}\cos\theta$$

$$\sin\theta = \frac{d/2 - x}{r},\ \cos\theta = \frac{y + r - h}{r}$$

$$V(x) = -w_0(d/2 - x)\cos\theta + \frac{w_0 d^2}{8h}\sin\theta$$

Equations



$$r = \frac{1}{2}\left(\frac{d^2}{4h} + h\right)$$

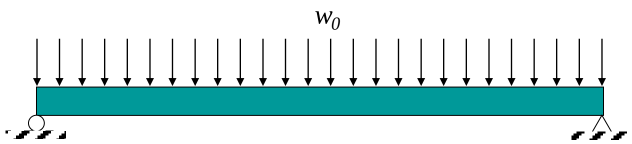
$$M(x) = \frac{w_0 x}{2}(d - x) - \frac{w_0 d^2}{8h}\left(\sqrt{r^2 - (x - d/2)^2} - r + h\right)$$

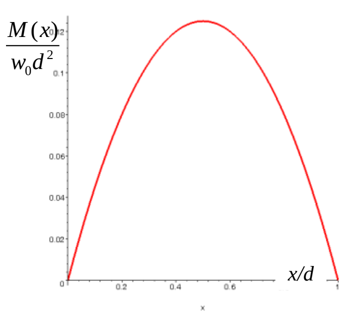
$$N(x) = -w_0 \frac{(d/2 - x)^2}{r} - \frac{w_0 d^2}{8h}\frac{\sqrt{r^2 - (x - d/2)^2}}{r}$$

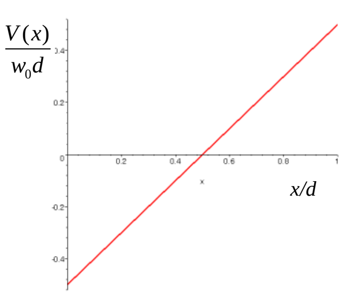
$$V(x) = -w_0(d/2 - x)\frac{\sqrt{r^2 - (x - d/2)^2}}{r} + \frac{w_0 d^2}{8h}\frac{d/2 - x}{r}$$

Simply Supported Straight Beam

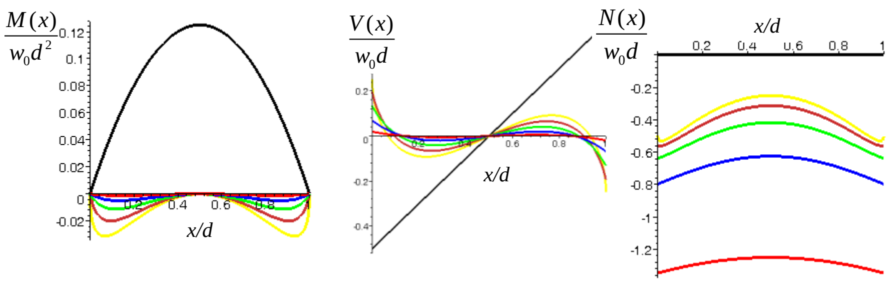
$$M(x) = \frac{w_0 x}{2}(d - x),\ V(x) = -w_0(d/2 - x),\ N(x) = 0$$

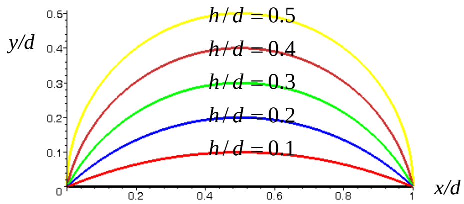






Three hinge arch vs SS beam

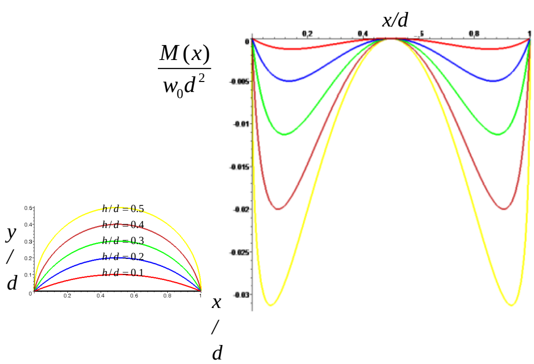




Max Moment in the Arch

$$\frac{M(x)}{w_0 d^2} = \frac{s}{2}(1-s) - \frac{1}{8\eta}\left(\sqrt{\rho^2 - (s-1/2)^2} - \rho + \eta\right)$$

$$s = x/d,\ \eta = h/d,\ \rho = r/d = \frac{1}{2}\left(\frac{1}{4\eta} + \eta\right)$$



$$M_{\max} = w_0 d^2 \frac{1}{8}\left(\frac{h}{d}\right)^2 = \frac{1}{8} w_0 h^2$$

$$x_m / d = \frac{1}{2}\left(1 - \sqrt{1/2 + \eta^2}\right)$$