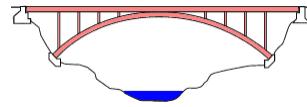


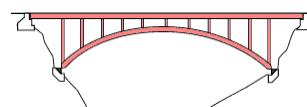
Some ArchTypes

Statically Indeterminate

Fixed

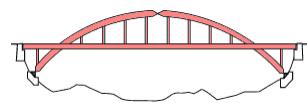


Two-hinge



Statically Determinate

Three-hinge



Analysis: 3-Hinged

Diagram illustrating the analysis of a three-hinged arch bridge under horizontal and vertical loads.

The bridge is shown with three hinge points labeled A, B, and C. Horizontal forces $w_x(y)$ and vertical loads $w_y(x)$ are applied at various points along the arch.

Free body diagrams show the reaction forces at the supports:

- At support A: Horizontal force A_x to the right, vertical force A_y upwards.
- At support B: Horizontal force B_x to the left, vertical force B_y upwards.
- At support C: Horizontal force C_x to the right, vertical force C_y upwards.

Equilibrium equations for the horizontal force balance at each support are:

$$A_x + B_x + \int_0^h w_x(\eta) d\eta = 0$$

$$A_y + B_y + \int_0^{d/2} w_y(\eta) d\eta = 0$$

$$B_y \frac{d}{2} - B_x h - \int_0^h w_x(\eta) \eta d\eta + \int_0^{d/2} \eta w_y(\eta) d\eta = 0$$

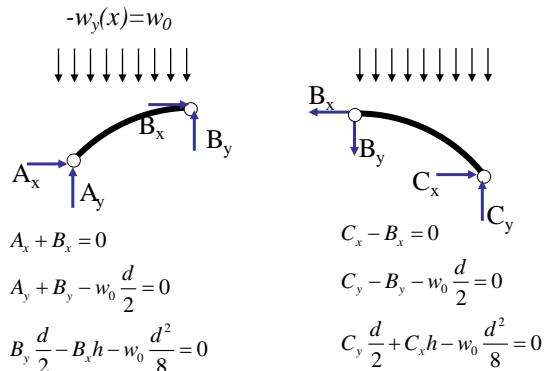
$$C_x - B_x = 0$$

$$C_y - B_y + \int_{d/2}^d w_y(\eta) d\eta = 0$$

$$C_y \frac{d}{2} + C_x h + \int_{d/2}^d (\eta - d/2) w_y(\eta) \eta d\eta = 0$$



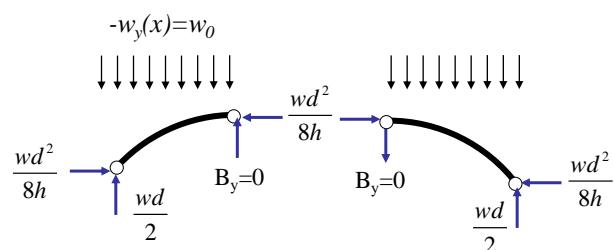
Constant load $w_y = -w_0, w_x = 0$



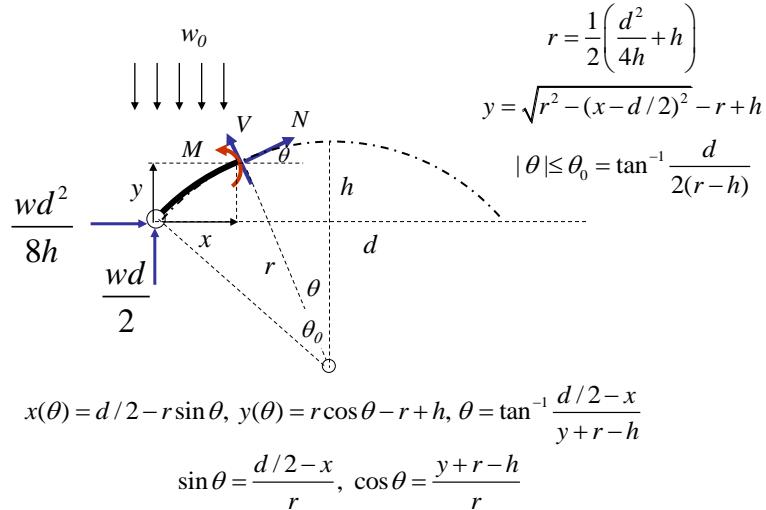
$$by = 0, cy = \frac{wd}{2}, cx = -\frac{wd^2}{8h}, ay = \frac{wd}{2}, bx = -\frac{wd^2}{8h}, ax = \frac{wd^2}{8h}$$



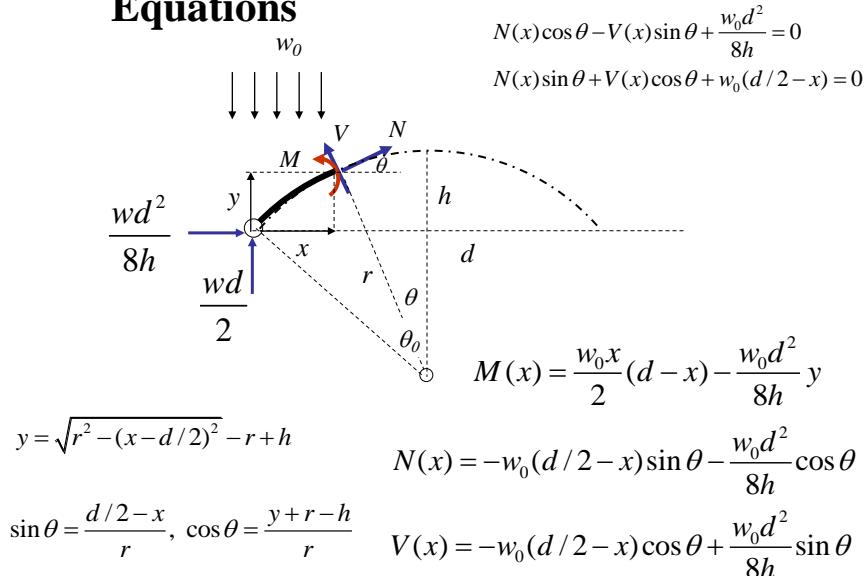
Constant load
 $-w_y = w_0, w_x = 0$



Internal Forces, Moments

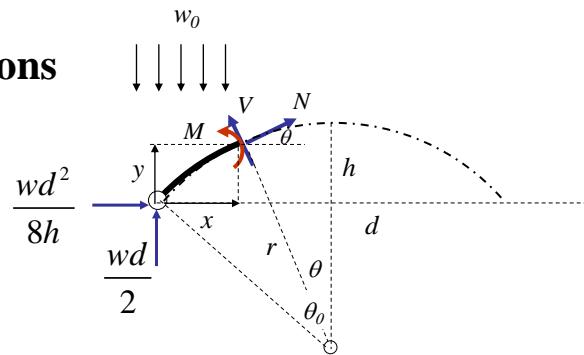


Equations



Equations

$$r = \frac{1}{2} \left(\frac{d^2}{4h} + h \right)$$



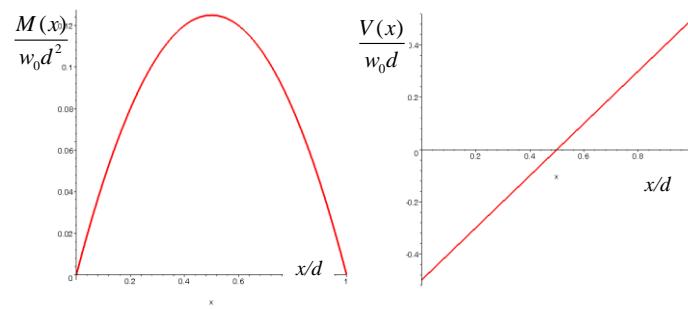
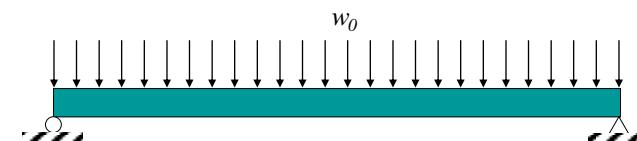
$$M(x) = \frac{w_0 x}{2} (d - x) - \frac{w_0 d^2}{8h} \left(\sqrt{r^2 - (x - d/2)^2} - r + h \right)$$

$$N(x) = -w_0 \frac{(d/2 - x)^2}{r} - \frac{w_0 d^2}{8h} \frac{\sqrt{r^2 - (x - d/2)^2}}{r}$$

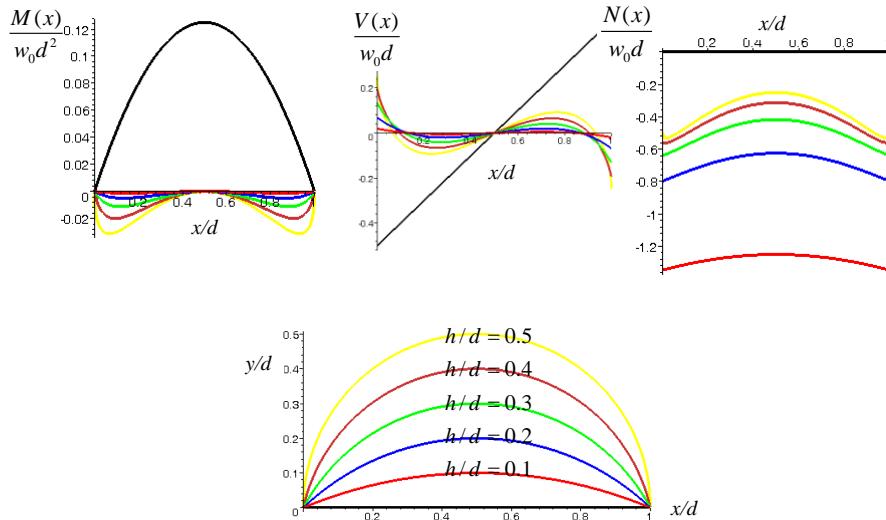
$$V(x) = -w_0 (d/2 - x) \frac{\sqrt{r^2 - (x - d/2)^2}}{r} + \frac{w_0 d^2}{8h} \frac{d/2 - x}{r}$$

Simply Supported Straight Beam

$$M(x) = \frac{w_0 x}{2} (d - x), \quad V(x) = -w_0 (d/2 - x), \quad N(x) = 0$$



Three hinge arch vs SS beam



Max Moment in the Arch

$$\frac{M(x)}{w_0 d^2} = \frac{s}{2}(1-s) - \frac{1}{8\eta} \left(\sqrt{\rho^2 - (s-1/2)^2} - \rho + \eta \right)$$

$$s = x/d, \eta = h/d, \rho = r/d = \frac{1}{2} \left(\frac{1}{4\eta} + \eta \right)$$

