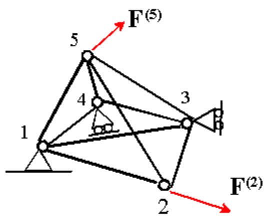
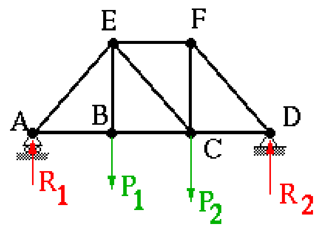
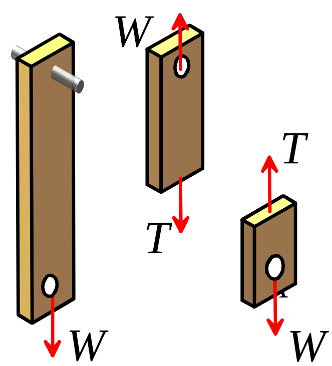
# Review: Trusses

• **Each joint consists of a single pin to which the respective members are connected individually.**
• **No member extends beyond a joint.**
• **Loads are applied only at joints.**
• **Each Member is a 2-force member and carries only axial load**



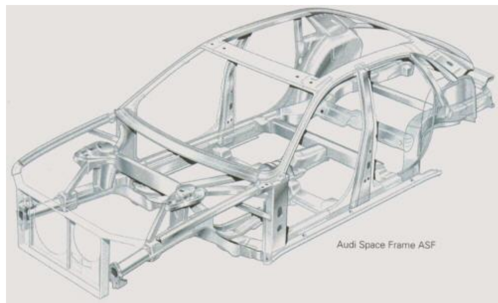

# Internal forces in 2-force members



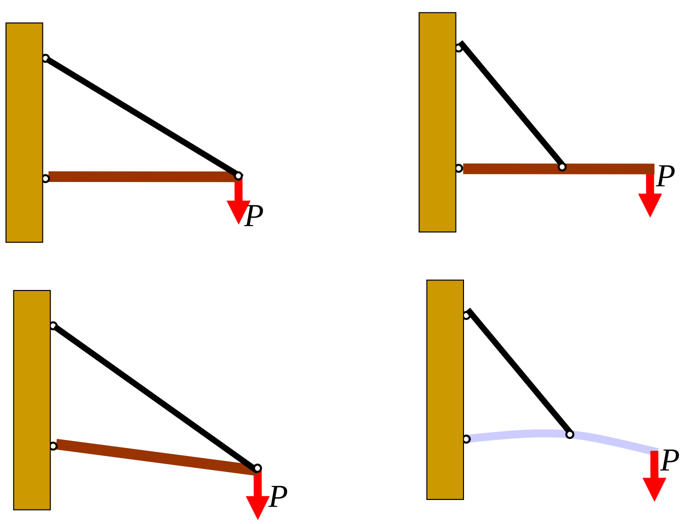

The internal force in a straight 2-force member consists of a force acting normal to the surface of internal surfaces that are perpendicular to the member. Force can be positive or negative. Positive force is tension, negative force is compression.

# Frames

• **Structures consisting of structural members connected in arbitrary ways.**

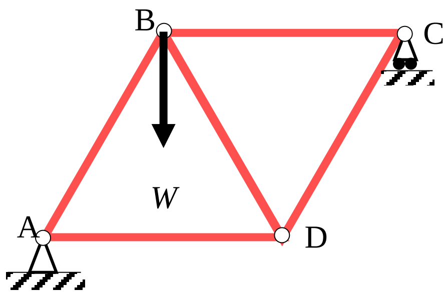




# Truss vs Frames

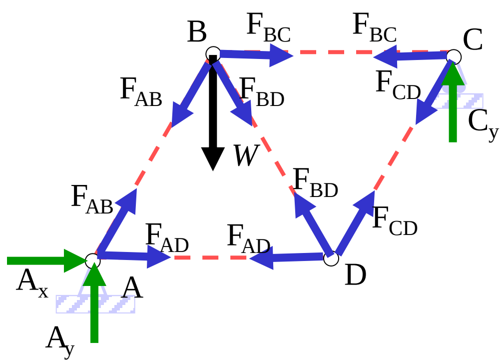


## Calculating forces in structures made of 2-force members

Find the internal force in each member of the idealized bike frame. Assume all joints are hinges; all members have equal length.




*Method of Joints* Net force at each joint is zero.



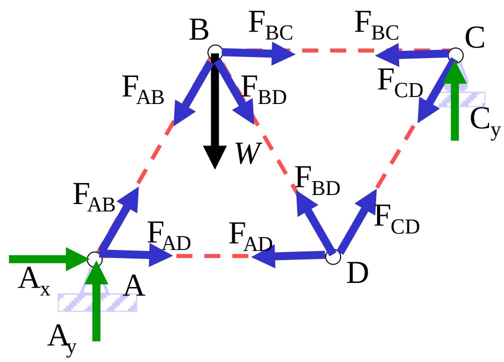

All angels $60^o$

**8 Unknowns**: Force in each member (5): $F_{AB}$, $F_{BC}$, $F_{AD}$, $F_{CD}$, $F_{BD}$
reaction forces (3) $A_x$, $A_y$, $C_y$

**8 Equations:** $\Sigma F_x = \Sigma F_y = 0$ (2 at each of the 4 jonts)

## Equations

All angles 60°




$$\sum F_{Ax} = F_{AD} + F_{AB}\cos 60 + A_x = 0, \sum F_{Ay} = F_{AB}\sin 60 + A_y = 0$$

$$\sum F_{Bx} = F_{BC} + F_{BD}\cos 60 - F_{AB}\cos 60 = 0, \sum F_{By} = -W - F_{AB}\sin 60 - F_{BD}\sin 60 = 0$$

$$\sum F_{Cx} = -F_{BC} + F_{CD}\cos 60 = 0, \sum F_{Cy} = -F_{CD}\cos 60 + C_y = 0$$

$$\sum F_{Dx} = -F_{AD} - F_{BD}\cos 60 + F_{DC}\cos 60 = 0, \sum F_{Dy} = F_{BD}\sin 60 + F_{CD}\sin 60 = 0$$

$$\{ax = 0, cy = \frac{w}{3}, ay = \frac{2w}{3}, fcd = \frac{2\sqrt{3}w}{9}, fad = \frac{2\sqrt{3}w}{9}, fab = -\frac{4\sqrt{3}w}{9}, fbc = -\frac{\sqrt{3}w}{9}, fbd = -\frac{2\sqrt{3}w}{9}\}$$

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```
> restart;t:=60*Pi/180;
```

$$t := \frac{\pi}{3}$$

```
> fax:=fad+fab*cos(t)+ax;fay:=fab*sin(t)+ay;
```

$$fax := fad + \frac{fab}{2} + ax$$

$$fay := \frac{fab\sqrt{3}}{2} + ay$$

```
> fbx:=fbc+fbd*cos(t)-fab*cos(t);fby:=-w-fab*sin(t)-fbd*sin(t);
```

$$fbx := fbc + \frac{fbd}{2} - \frac{fab}{2}$$

$$fby := -w - \frac{fab\sqrt{3}}{2} - \frac{fbd\sqrt{3}}{2}$$

```
> fcx:=-fbc-fcd*cos(t);fcy:=-fcd*sin(t)+cy;
```

$$fcx := -fbc - \frac{fcd}{2}$$

$$fcy := -\frac{fcd\sqrt{3}}{2} + cy$$

```
> fdx:=-fad-fbd*cos(t)+fcd*cos(t);fdy:=fcd*sin(t)+fbd*sin(t);
```

$$fdx := -fad - \frac{fbd}{2} + \frac{fcd}{2}$$

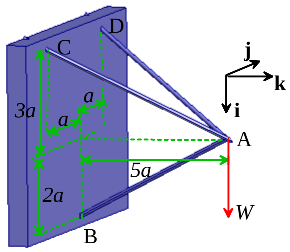
$$fdy := \frac{fcd\sqrt{3}}{2} + \frac{fbd\sqrt{3}}{2}$$

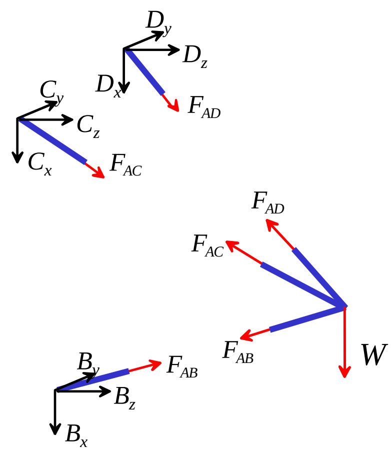
```
> solve({fax,fay,fbx,fby,fcx,fcy,fdx,fdy},{ax,ay,cy,fab,fad,fbd,fbc,fcd});
```

$$\{ax = 0, cy = \frac{w}{3}, ay = \frac{2w}{3}, fad = \frac{2\sqrt{3}w}{9}, fab = -\frac{4\sqrt{3}w}{9}, fcd = \frac{2\sqrt{3}w}{9}, fbd = -\frac{2\sqrt{3}w}{9}, fbc = -\frac{\sqrt{3}w}{9}\}$$

## 3D problem: $\sum F_x = \sum F_y = \sum F_z = 0$

At each joint




**Equations:** $\Sigma \mathbf{F_x} = \Sigma \mathbf{F_y} = \Sigma \mathbf{F_z} = 0$ at each joint (12)
**Unknowns:** Total of 12:
Member forces $F_{AC}$, $F_{AD}$, $F_{AB}$, (3)
Reactions :$B_x$,$B_y$,$B_z$, $C_x$,$C_y$,$C_z$, $D_x$,$D_y$,$D_z$ (9)

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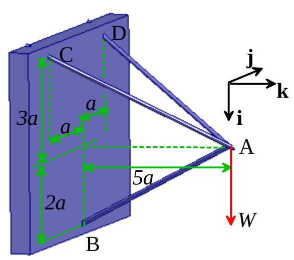
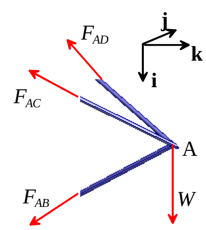
## Forces act parallel to the members




$$F_{AB}\mathbf{n}_{AB} + F_{AC}\mathbf{n}_{AC} + F_{AD}\mathbf{n}_{AD} + W\mathbf{i} = 0$$

| Member | Unit Vector pointing away from Joint A. |
|---|---|
| Member AB | $\mathbf{n}_{AB} = (2a\mathbf{i} - 5a\mathbf{k})/\sqrt{29a^2} = (2\mathbf{i} - 5\mathbf{k})/\sqrt{29}$ |
| Member AC | $\mathbf{n}_{AC} = (-3\mathbf{i} - \mathbf{j} - 5\mathbf{k})\sqrt{35}$ |
| Member AD | $\mathbf{n}_{AD} = (-3\mathbf{i} + \mathbf{j} - 5\mathbf{k})\sqrt{35}$ |

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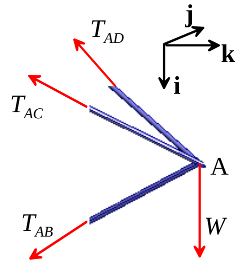
# Joint A




$$F_{AB}\mathbf{n}_{AB} + F_{AC}\mathbf{n}_{AC} + F_{AD}\mathbf{n}_{AD} + W\mathbf{i} = 0$$

$$F_{AB}(2\mathbf{i} - 5\mathbf{k})/\sqrt{29} + F_{AC}(-3\mathbf{i} - \mathbf{j} - 5\mathbf{k})/\sqrt{35} + F_{AD}(-3\mathbf{i} + \mathbf{j} - 5\mathbf{k})/\sqrt{35} + W\mathbf{i} = \mathbf{0}$$

$$\Sigma F_{Ax} = 2F_{AB}/\sqrt{29} - 3F_{AC}/\sqrt{35} - 3F_{AD}/\sqrt{35} + W = 0$$

$$\Sigma F_{Ay} = -F_{AC}/\sqrt{35} + F_{AD}/\sqrt{35} = 0$$

$$\Sigma F_{Az} = -5F_{AB}/\sqrt{29} - 5F_{AC}/\sqrt{35} - 5F_{AD}/\sqrt{35} = 0$$

$$F_{AD} = F_{AC} = (\sqrt{35}/10)W \qquad F_{AB} = -(\sqrt{29}/5)W$$

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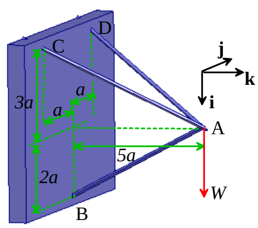
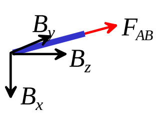
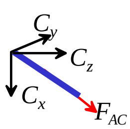
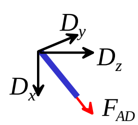
# Joints B,C,D




| | | |
|---|---|---|
| $F_{AB}\mathbf{n}_{BA} + B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} = \mathbf{0}$ | $F_{AC}\mathbf{n}_{CA} + C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} = \mathbf{0}$ | $F_{AD}\mathbf{n}_{DA} + D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k} = \mathbf{0}$ |
| $-F_{AB}\mathbf{n}_{BA} + B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} = \mathbf{0}$ | $-F_{AC}\mathbf{n}_{AC} + C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} = \mathbf{0}$ | $-F_{AD}\mathbf{n}_{AD} + D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k} = \mathbf{0}$ |
| $\sum F_{Bx} = -F_{AB}2/\sqrt{29} + B_x = 0$ | $\sum F_{Cx} = -F_{AC}3/\sqrt{35} + C_x = 0$ | $\sum F_{Dx} = F_{AD}3/\sqrt{35} + D_x = 0$ |
| $\sum F_{By} = B_y = 0$ | $\sum F_{Cy} = F_{AC}/\sqrt{35} + C_y = 0$ | $\sum F_{Dy} = F_{AD}/\sqrt{35} + D_y = 0$ |
| $\sum F_{Bz} = 5F_{AB}/\sqrt{29} + B_z = 0$ | $\sum F_{Cz} = 5F_{AC}/\sqrt{35} + C_z = 0$ | $\sum F_{Dz} = 5F_{AD}/\sqrt{35} + D_z = 0$ |

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