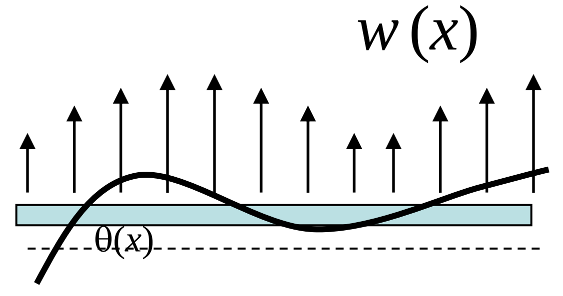
Beam Deflections: Minimum Potential Energy

Potential Energy in a bending beam under a load $w(x)$

$$V[u] = \int_0^{L_0} \frac{1}{2} EI \left(\kappa(x)\right)^2 dx - \int_0^{L_0} w(x)u(x)dx$$



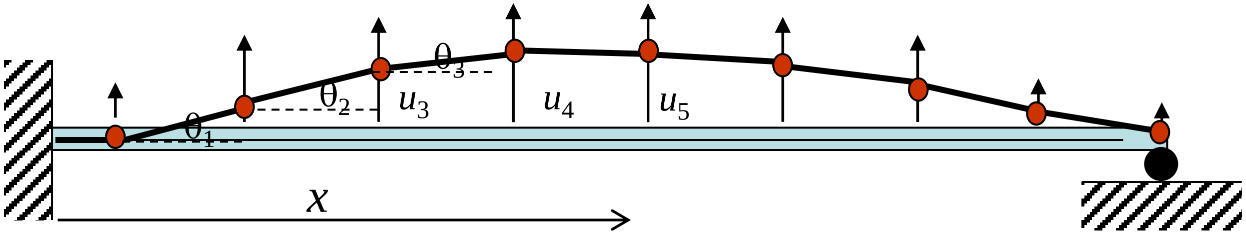
Where κ is the beam curvature:

$$\kappa = \frac{d\theta}{ds} \quad \theta = \tan^{-1}\frac{du}{ds}.$$

For $\left|\frac{du}{ds}\right| \ll 1,\ s \approx x,\ \theta \approx \frac{du}{dx}$ and $\kappa \approx \frac{d^2u}{dx^2}$

Excel

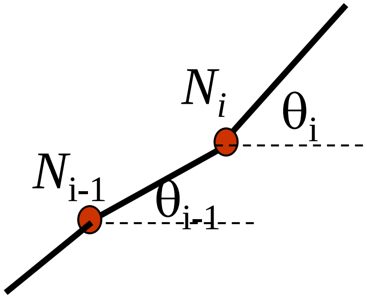
- Divide the beam into N segments, each of length $s=L_0/N$



- Beam deflections are defined by the orientation angles θ_i:

$$x_1 = s\cos\theta_0, \quad x_2 = x_1 + s\cos\theta_1 \;\; .... \;\; x_i = x_{i-1} + s\cos\theta_{i-1}$$
$$u_1 = s\sin\theta_0, \quad u_2 = u_1 + s\sin\theta_1 \;\; .... \;\; u_i = u_{i-1} + s\sin\theta_{i-1}$$

- Curvature is $\kappa = d\theta / ds$

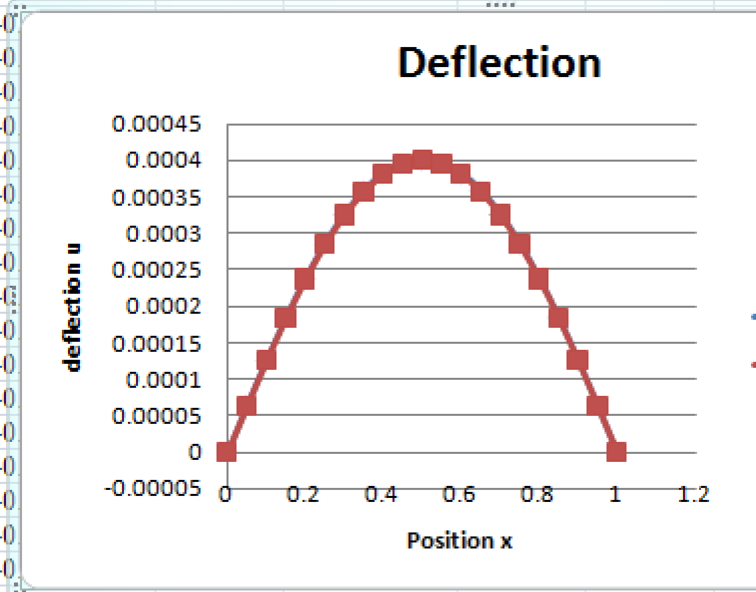


- For the i[th] segment, the curvature is $\kappa_i = (\theta_i - \theta_{i-1})/s$
- Elastic energy for a segment is $V_i = \frac{s}{2} EI\kappa_i^2$
- Load is $W_i = w(x_i)s$ (i=1,2,…N-1), $W_0 = w(0)s/2$, $W_N = w(L_0)s/2$
- Total energy is $$V = \frac{1}{2}EIs\sum_{i=1}^{N}\frac{(\theta_i - \theta_{i-1})^2}{s^2} - \sum_{i=0}^{N} W_i u_i$$
- Minimize V, varying the angles θ_i, subject to any constraints from the BC. Here, $u_0=0$ $u_N=0$. $\theta_0=0$

Clipboard | Font | Alignment | Number | Styles | Cells

Chart 3 | fx

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	Beam length	1		Youngs' Mod	1.50E+09		total PE	-0.012798239				
3	Load paramete	100		Cross sect I	2.17E-06							
4							xsect b	0.0127				
5	number of segı	20		seg lenght	0.05		xsect h	0.127				
6												
7	point	angle	xi	ui	curvature	strain energy	load at i	load energy	xe	beam theory		
8	0	0.00127783	0	0			2.5	0	0	0		
9	1	0.00124132	0.05	6.38916E-05	-0.00073	4.33652E-05	5	-0.000319458	0.05	6.37546E-05		
10	2	0.00117212	0.1	0.000125957	-0.00138	0.000155682	5	-0.000629787	0.1	0.000125699		
11	3	0.0010741	0.15	0.000184564	-0.00196	0.000312446	5	-0.				
12	4	0.00095109	0.2	0.000238269	-0.00246	0.000492033	5	-0.				
13	5	0.00080694	0.25	0.000285823	-0.00288	0.000675705	5	-0.				
14	6	0.00064549	0.3	0.00032617	-0.00323	0.000847605	5	-0.				
15	7	0.00047059	0.35	0.000358445	-0.0035	0.00099476	5	-0.				
16	8	0.00028608	0.4	0.000381974	-0.00369	0.001107073	5	-0.				
17	9	9.5799E-05	0.45	0.000396278	-0.00381	0.001177349	5	-0.				
18	10	-9.64E-05	0.5	0.000401068	-0.00384	0.001201254	5	-0.				
19	11	-0.0002867	0.55	0.000396248	-0.00381	0.001177349	5	-0.				
20	12	-0.0004712	0.6	0.000381914	-0.00369	0.001107076	5	-0.				
21	13	-0.0006461	0.65	0.000358355	-0.0035	0.000994758	5	-0.				
22	14	-0.0008075	0.7	0.00032605	-0.00323	0.000847605	5	-0.				
23	15	-0.0009517	0.75	0.000285673	-0.00288	0.000675705	5	-0.				
24	16	-0.0010747	0.8	0.000238088	-0.00246	0.000492033	5	-0.				
25	17	-0.0011727	0.85	0.000184353	-0.00196	0.000312446	5	-0.				
26	18	-0.0012419	0.9	0.000125716	-0.00138	0.000155682	5	-0.				
27	19	-0.0012784	0.95	6.36206E-05	-0.00073	4.33652E-05	5	-0.				
28	20	-0.0012784	1	-3.01198E-07	-4.3E-10	1.50615E-17	2.5	7.52995E-07	1	0		
29												



SS const | pin pin const | pin pin elastic | SS pt load | SS pt load elastic | clamp clamp pt

Ready | 100%