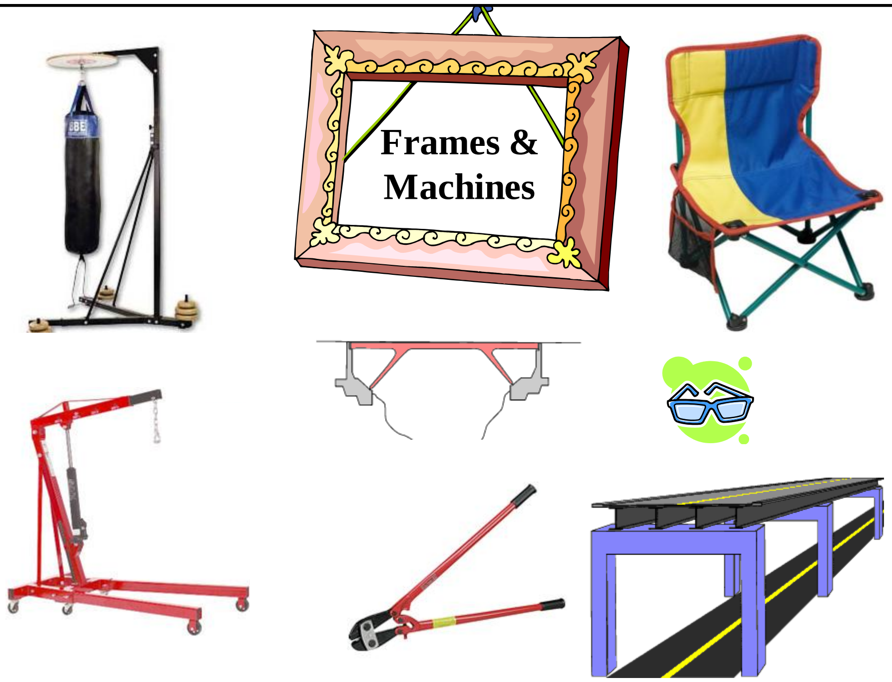
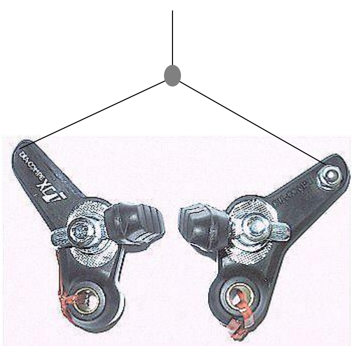
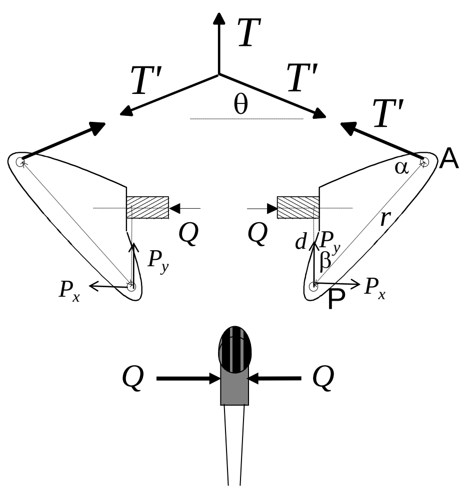
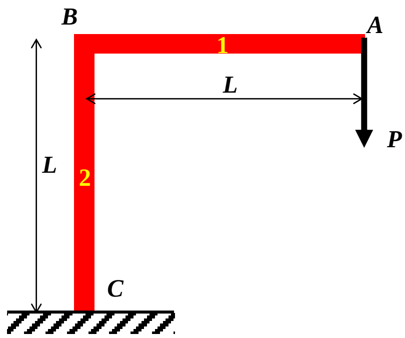


Statically Determinate: Reactions and internal forces can be found by static equilibrium

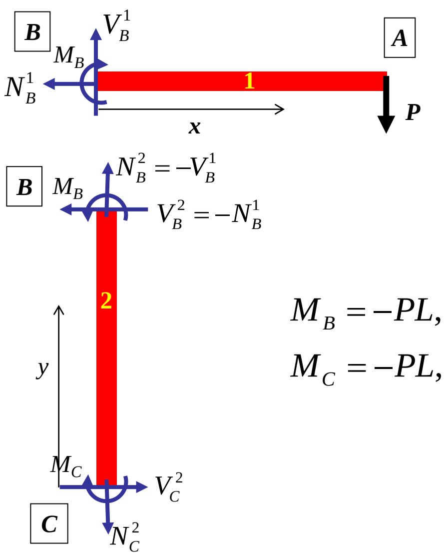




Example: Jib Crane



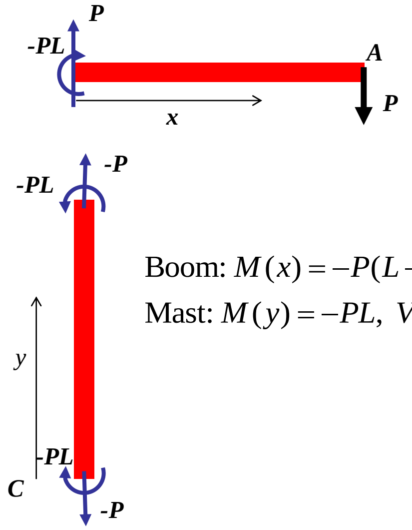
Joints?



$$M_B = -PL,\ V_B^1 = P,\ N_B^1 = 0$$

$$M_C = -PL,\ V_C^2 = 0,\ N_C^2 = -P$$

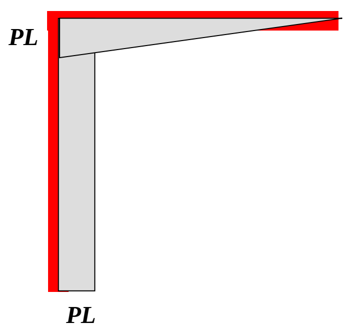
Internal Forces, Moments?



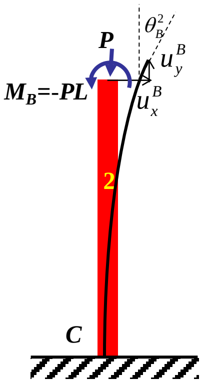
Boom: $M(x) = -P(L-x),\ V(x) = P,\ N(x) = 0$

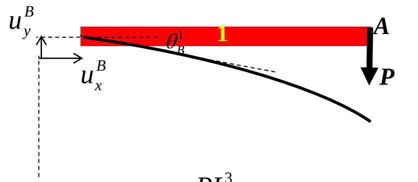
Mast: $M(y) = -PL,\ V(y) = 0,\ N(x) = -P$

Moment Diagram



Deflections?





$$u_x^1(x) = u_x^B = \frac{PL^3}{2EI}$$

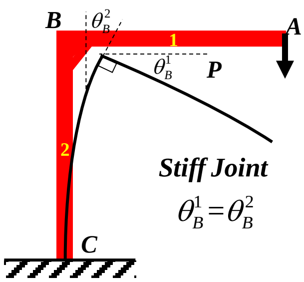
$$u_y^1(x) = \underbrace{-\frac{Px^2}{6EI}(3L - x)}_{\text{Cantilever beam}} \; \underbrace{-\theta_B^1 x}_{\text{Effect of End rotation}} \; \underbrace{-\frac{PL}{EA}}_{\text{Effect of end deflection}}$$

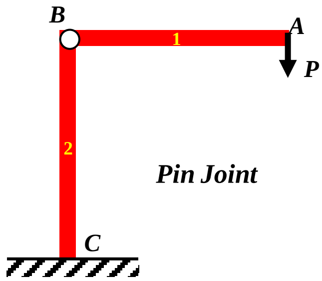
$$u_x^B = \frac{PL^3}{2EI}, \theta_B^2 = \frac{PL^2}{EI}, u_y^B = -\frac{PL}{EA}$$

$$u_x^A = \frac{PL^3}{2EI}$$

$$u_y^A = -\frac{PL^3}{3EI} - \frac{PL}{EA} - \theta_B^1 L$$

Relating θ_B^1 and θ_B^2





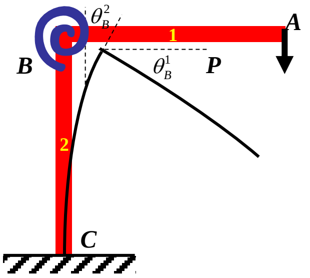
$$u_y^A = -\frac{5PL^3}{6EI} - \frac{PL}{EA}$$

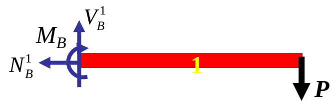
Mechanism

Full moment connection

Intermediate Joint

Torsional spring, stiffness Γ (Force×Length)





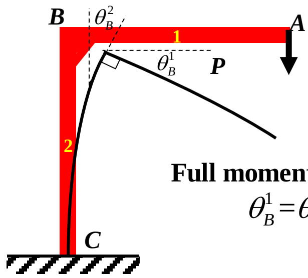
$$M_B = \Gamma\left(\theta_B^2 - \theta_B^1\right)$$

$$M_B = -PL = \Gamma\left(\frac{PL^2}{EI} - \theta_B^1\right)$$

$$\Rightarrow \theta_B^1 = \frac{PL}{\Gamma} + \frac{PL^2}{EI}$$

$$u_y^A = -\frac{5PL^3}{6EI} - \frac{PL}{EA} - \frac{PL^2}{\Gamma}$$

Bending dominates



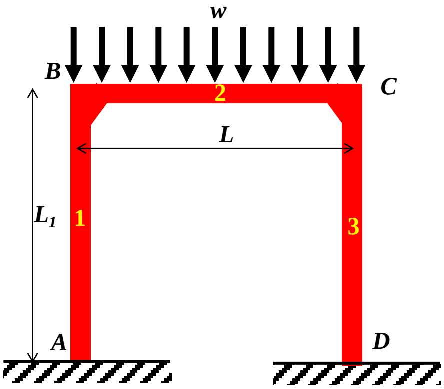
Full moment connection

$\theta_B^1 = \theta_B^2$

$$u_y^A = -\frac{5PL^3}{6EI} - \frac{PL}{EA} = -\frac{5PL^3}{6EI}\left(1 + \frac{6I}{5AL^2}\right) \approx -\frac{5PL^3}{6EI}$$

$\frac{6I}{5AL^2} \ll 1$ **If the beam is much longer than its cross section dimension**

Indeterminate Frame:
Full Moment connections



Unknowns: Moment and forces transmitted through each joint

Equations: 3 per member