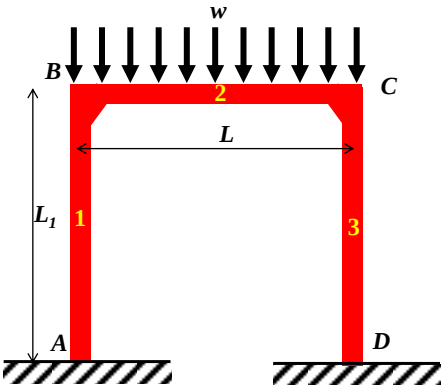
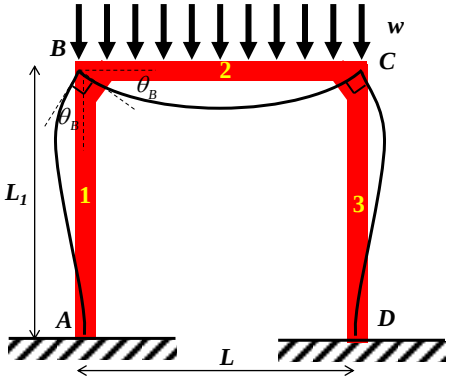
Indeterminate Frame:
Full Moment connections



Unknowns: Moment and forces transmitted through each joint

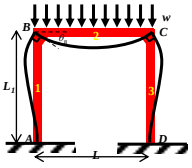
Equations: 3 per member

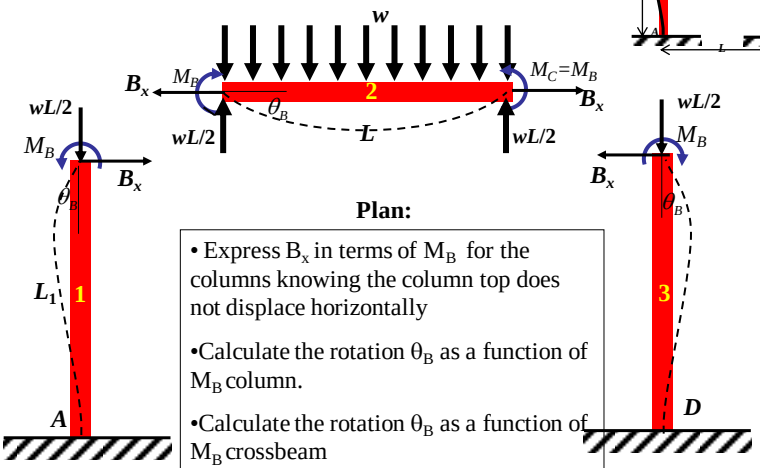
Full Moment connections:
Joint angles preserved



Neglect Axial Deformation: $\mathbf{u}^B=\mathbf{u}^C=\mathbf{0}$

Moment at Joints: M_B

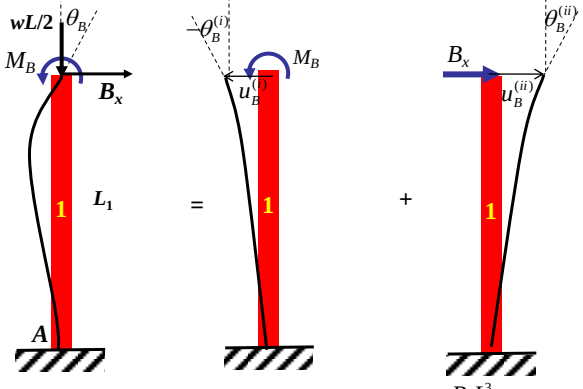




Plan:

- Express B_x in terms of M_B for the columns knowing the column top does not displace horizontally
- Calculate the rotation θ_B as a function of M_B column.
- Calculate the rotation θ_B as a function of M_B crossbeam
- Equate these expressions and solve for M_B

Joint rotation in terms of M_B: Column



No net horizontal displacement at B

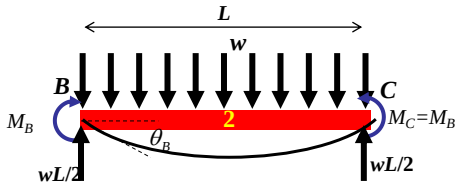
$$u_B^{(i)} = u_B^{(ii)}$$

$$u_B^{(i)} = \frac{M_B L_1^2}{2EI} \qquad u_B^{(ii)} = \frac{B_x L_1^3}{3EI} = u_B^{(i)} \Rightarrow B_x = \frac{3M_B}{2L_1}$$

$$\theta_B^{(i)} = -\frac{M_B L_1}{EI} \qquad \theta_B^{(ii)} = \frac{B_x L_1^2}{2EI} = \frac{3M_B L_1}{4EI}$$

$$\theta_B = \theta_B^{(i)} + \theta_B^{(ii)} = \frac{3M_B L_1}{4EI} - \frac{M_B L_1}{EI} = -\frac{M_B L_1}{4EI}$$

Joint rotation in terms of M_B: Cross Bar



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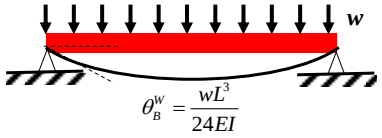

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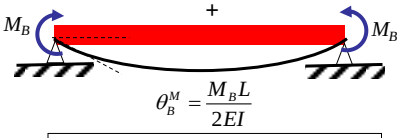
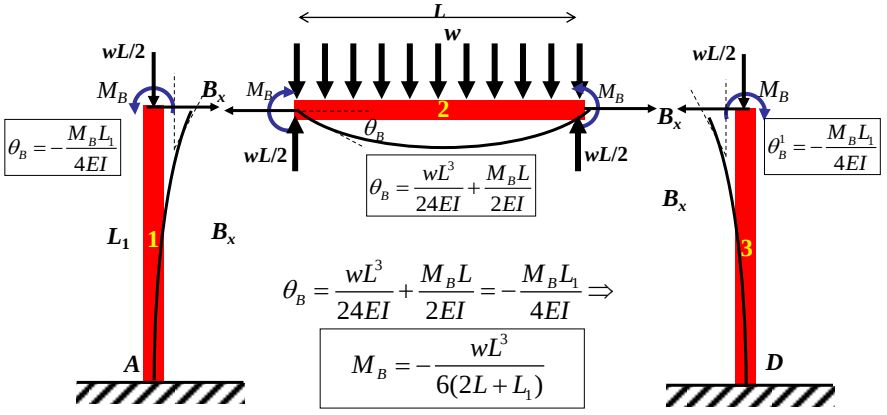


$$\theta_B = \theta_B^M + \theta_B^W = \frac{wL^3}{24EI} + \frac{M_B L}{2EI}$$

Finally



$$\theta_B = \frac{wL^3}{24EI} + \frac{M_B L}{2EI} = -\frac{M_B L_1}{4EI} \Rightarrow$$

$$M_B = -\frac{wL^3}{6(2L + L_1)}$$

$$\theta_B = \frac{wL^3}{24EI(1 + 2L/L_1)}$$

$$B_x = \frac{3M_B}{2L_1} = -\frac{wL^3}{4L_1(2L + L_1)}$$

Even more finally