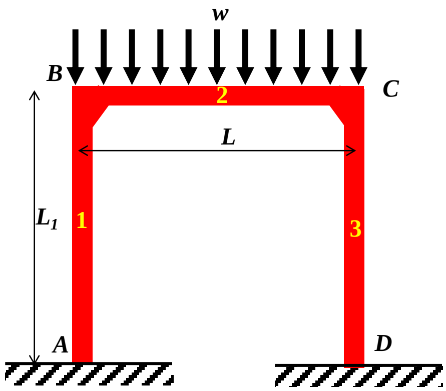
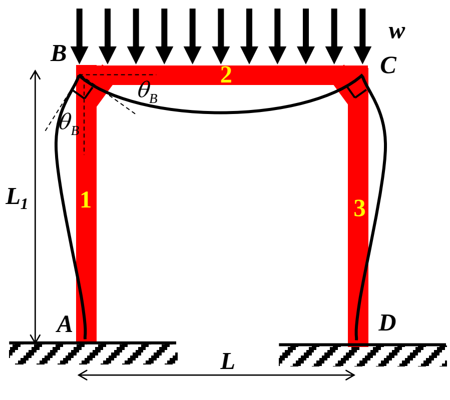
Indeterminate Frame:
Full Moment connections



Unknowns: Moment and forces transmitted through each joint

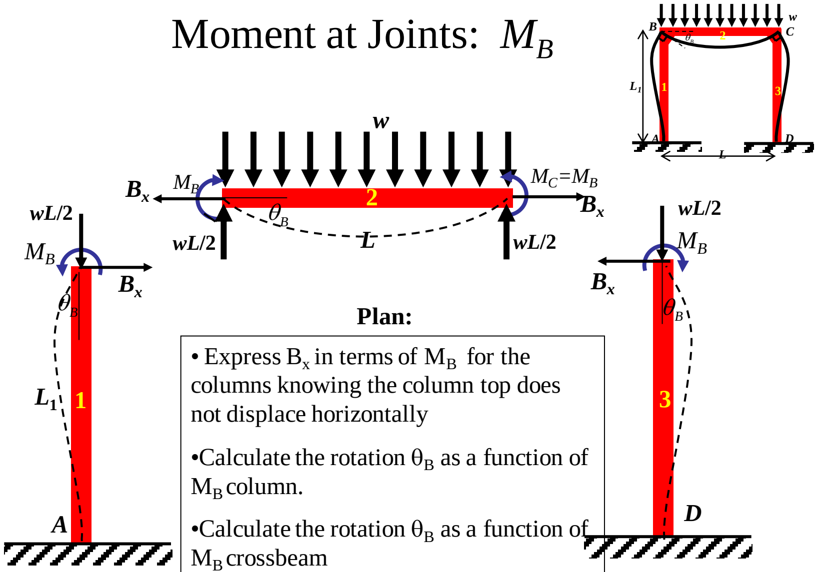
Equations: 3 per member

Full Moment connections:
Joint angles preserved



Neglect Axial Deformation: $\mathbf{u}^B = \mathbf{u}^C = \mathbf{0}$

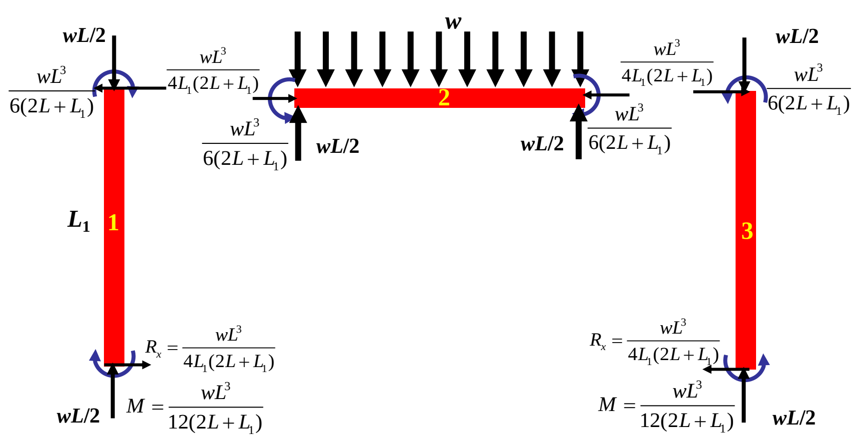
Moment at Joints: M_B



Plan:

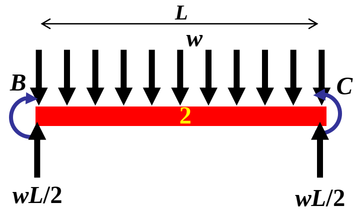
- Express B_x in terms of M_B for the columns knowing the column top does not displace horizontally
- Calculate the rotation θ_B as a function of M_B column.
- Calculate the rotation θ_B as a function of M_B crossbeam
- Equate these expressions and solve for M_B

Finally



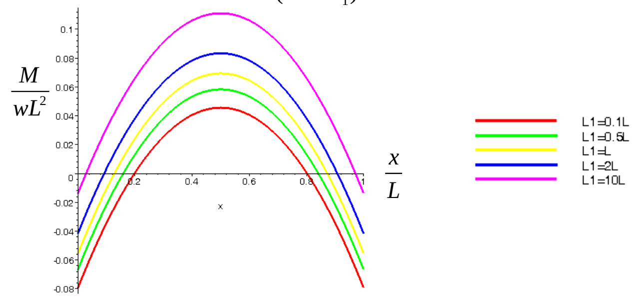
Moments in the cross beam

$$M_B = -\frac{wL^3}{6(2L+L_1)}$$



$$M_B = -\frac{wL^3}{6(2L+L_1)}$$

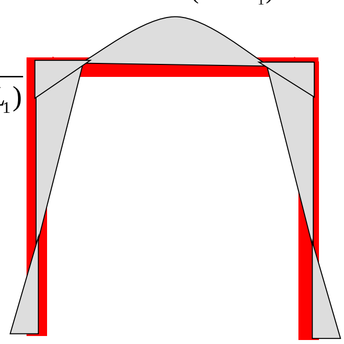
$$M(x) = \frac{wx}{2}(L-x) - \frac{wL^3}{6(2L+L_1)}$$



Moment Diagram

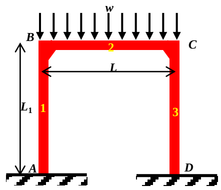
$$\frac{wL^2}{8} - \frac{wL^3}{6(2L+L_1)}$$

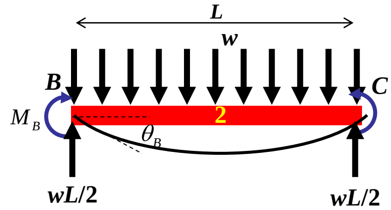
$$\frac{wL^3}{6(2L+L_1)} \qquad \frac{wL^3}{6(2L+L_1)}$$



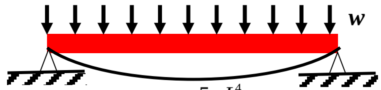
$$\frac{wL^3}{12(2L+L_1)} \qquad \frac{wL^3}{12(2L+L_1)}$$

Deflection at Midspan



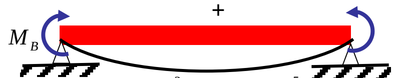


$$M_B = \frac{-wL^3}{6(2L+L_1)}$$



$$u_{\text{mid}}^{\text{W}} = -\frac{5wL^4}{384EI}$$

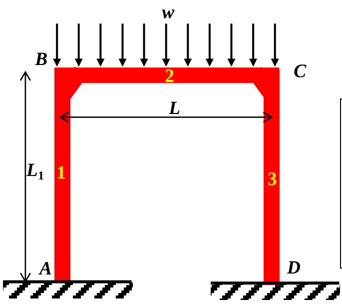
+



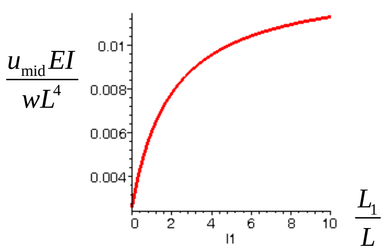
$$u_{\text{mid}}^{\text{M}} = \frac{M_B L^2}{8EI} = -\frac{wL^5}{48EI(2L+L_1)}$$

$$u_{\text{mid}} = u_{\text{mid}}^{\text{w}} + u_{\text{mid}}^{\text{M}_\text{B}} = -\frac{5wL^4}{384EI} - \frac{wL^5}{48EI(2L+L_1)}$$

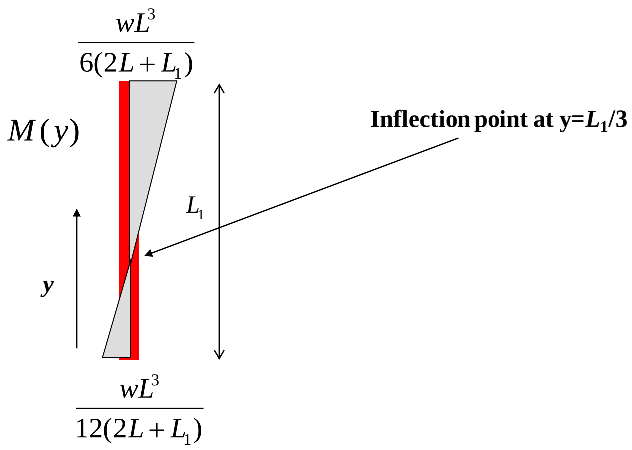
Deflection at Midspan



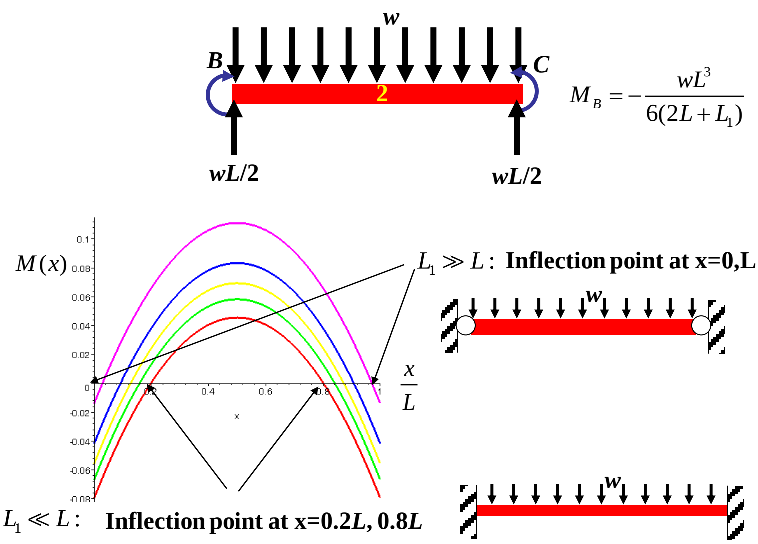
$$u_{\text{mid}} = -\frac{5wL^4}{384EI} - \frac{wL^5}{48EI(2L+L_1)} = -\frac{5wL^3}{384EI}\left(1+\frac{8}{5(2+L_1/L)}\right)$$



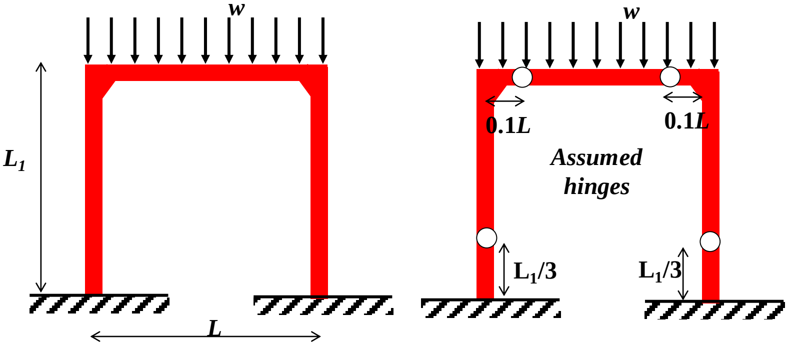
Inflection Points: Columns



Inflection Points: Cross Beam



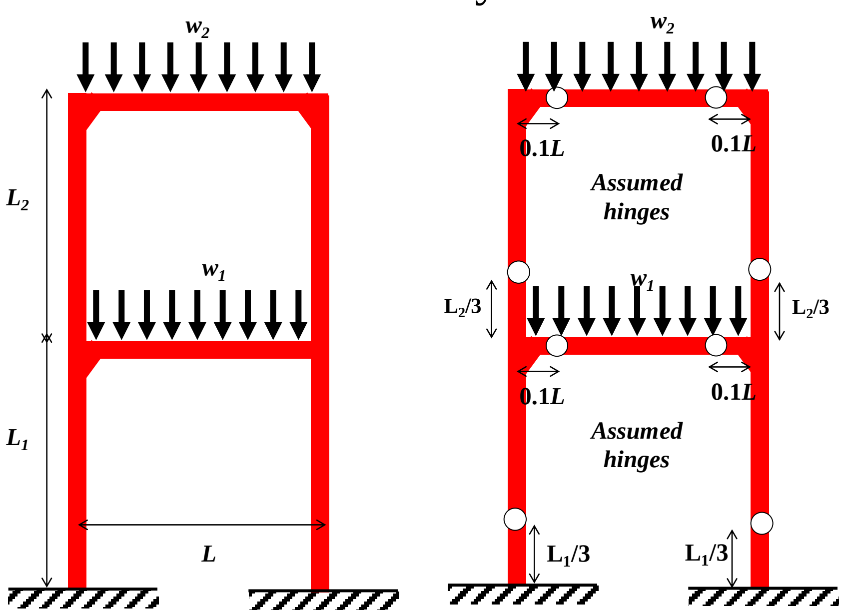
Approximate Analysis:
Place inflection points at x=0.1L, 0.9L, $y=L_1/3$



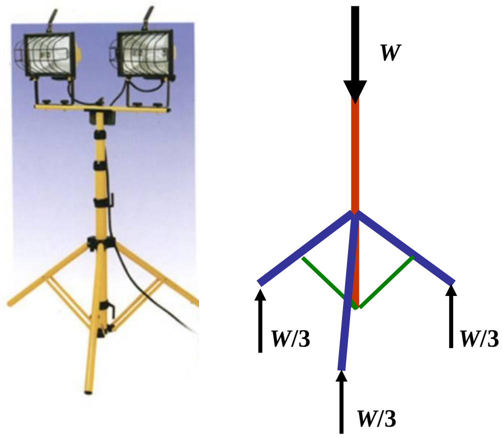
May be used to approximate situations with:

- **$EI_{columns} \neq EI_{crossbeam}$**
- **No full moment connections at the corners**
- **Multi-unit structures**
- **Vertical loads relatively close to those shown above (no horizontal loads)**

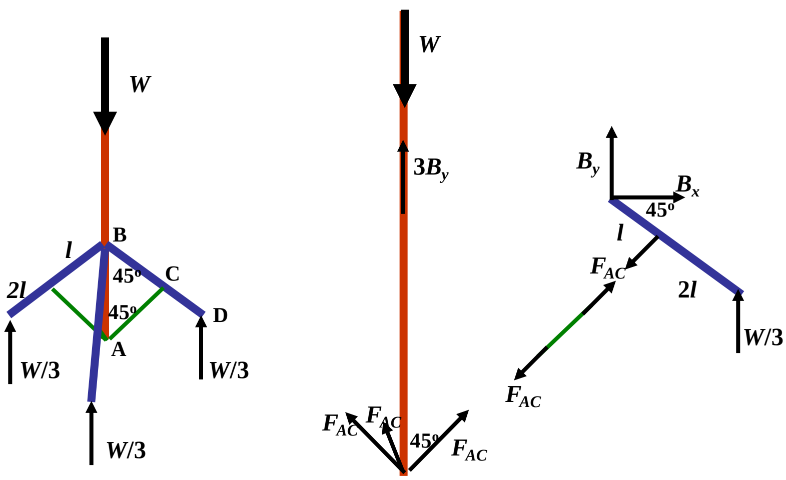
Multistory



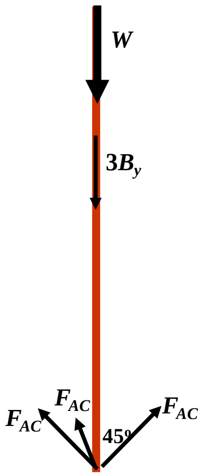
Example: Folding Stand

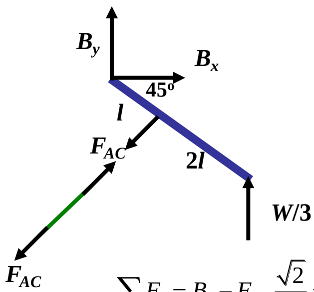


Some Disassembly Required!



Example





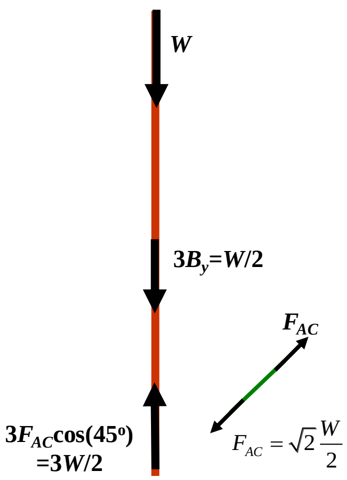
$$\sum F_x = B_x - F_{AC}\frac{\sqrt{2}}{2} = 0,$$

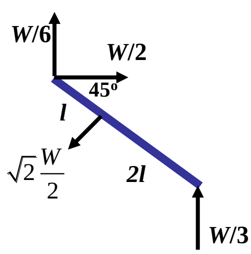
$$\sum F_y = \frac{W}{3} + B_y - F_{AC}\frac{\sqrt{2}}{2} = 0,$$

$$\sum M_B = -F_{AC}l + 3l\frac{\sqrt{2}}{2}\frac{W}{3} = 0$$

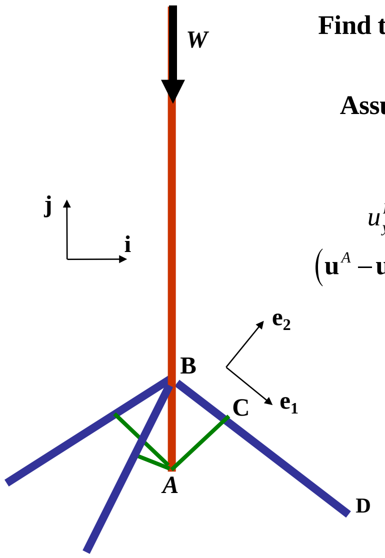
$$\Rightarrow F_{AC} = \sqrt{2}\frac{W}{2},\ B_x = \frac{W}{2},\ B_y = \frac{W}{6}$$

Internal forces:





Deflections?



Find the deflection of the poles due to bending of the legs.

Assume no axial extension in any member.

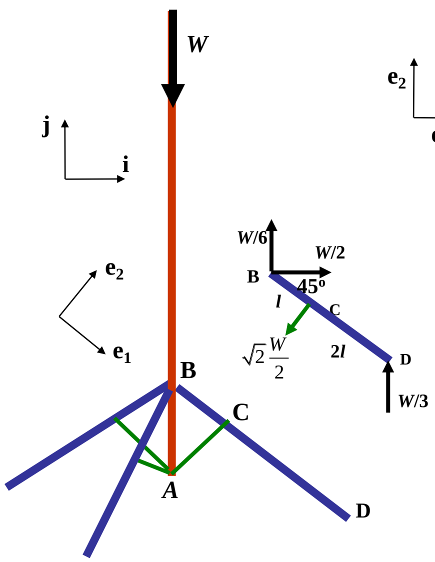
$$u_x^A = u_x^B = 0 \text{ (symmetry)}$$

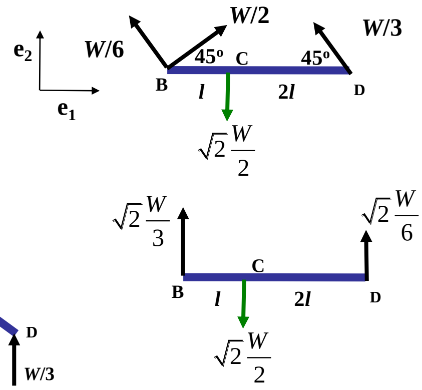
$$u_y^B = u_y^A \text{ (no axial deformation of the pole)}$$

$$\left(\mathbf{u}^A - \mathbf{u}^C\right)\bullet\mathbf{n}^{CA} = 0 \text{ (no axial deformation of the strut)}$$

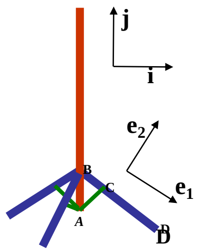
$$u_y^D = 0 \text{ (Support at the base)}$$

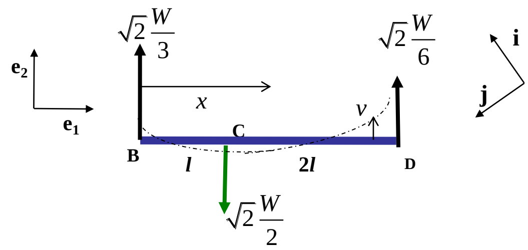
A Leg





A Leg





$$M(x) = \sqrt{2}\frac{W}{3}x,\ (0 < x < l),$$

$$M(x) = \sqrt{2}\frac{W}{6}(l - x)\ (l < x < 3l)$$

$$\frac{d^2v}{dx^2} = \frac{M(x)}{EI}$$

What are the Boundary Conditions on *v*?

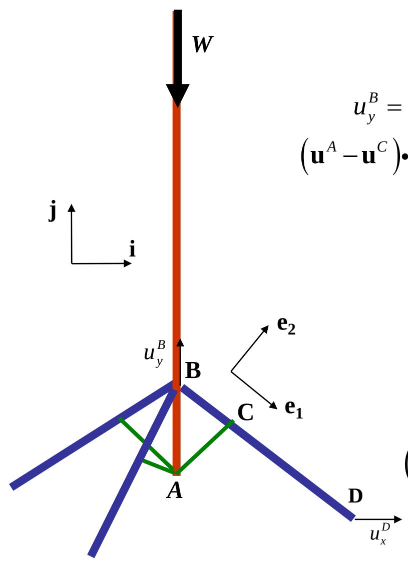
These...

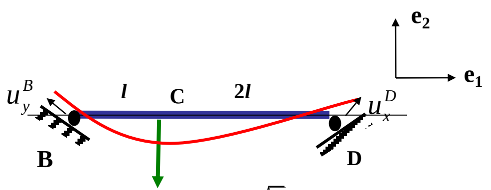
$$u_x^A = u_x^B = 0 \ \text{(symmetry)}$$

$$u_y^B = u_y^A \ \text{(no axial deformation of the pole)}$$

$$\left(\mathbf{u}^A - \mathbf{u}^C\right)\bullet\mathbf{n}^{CA} = 0 \ \text{(no axial deformation of the strut)}$$

$$u_y^D = 0 \ \text{(Support at the base)}$$

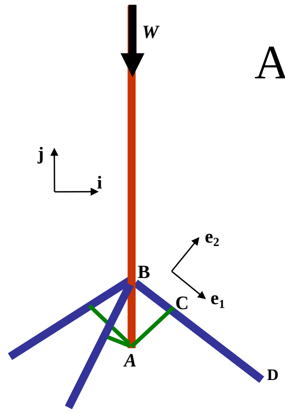


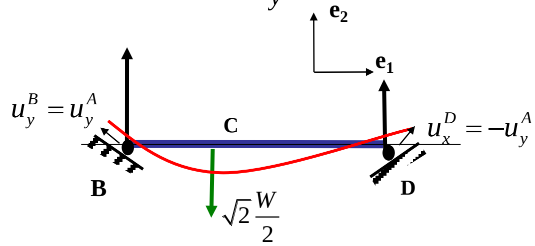


$$\left(\mathbf{u}^A - \mathbf{u}^C\right)\bullet\mathbf{e}_2 = 0 \Rightarrow u_y^A\frac{\sqrt{2}}{2} = u_2^C = v(l)$$

$$u_y^B = -u_x^D$$

All we want is u_y^A



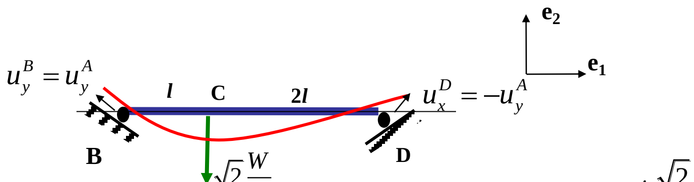


$$M(x) = \sqrt{2}\frac{W}{3}x, \ (0 < x < l),$$

$$M(x) = \sqrt{2}\frac{W}{6}(l-x) \ (l < x < 3l)$$

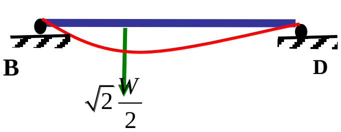
$$\frac{d^2v}{dx^2} = \frac{M(x)}{EI} \qquad v(0) = v(l) = -v(3l) = u_y^A\frac{\sqrt{2}}{2}$$

How?

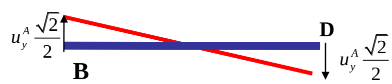


$$v(0) = v(l) = -v(3l) = u_y^A\frac{\sqrt{2}}{2}$$

Simply supported beam deflection + ***Rigid Rotation about the center***

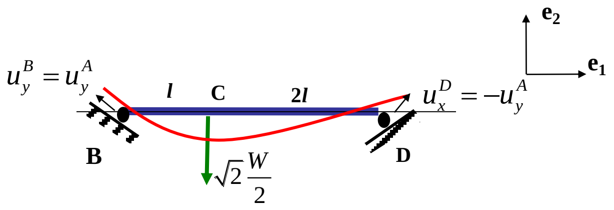


+



$$v(x) = v_{ss}(x) - u_y^A\frac{\sqrt{2}}{3l}\left(x - \frac{3}{2}l\right)$$

Why?



$$v_c = v(l) = v_{ss}(l) - u_y^A \frac{\sqrt{2}}{3l}\left(l - \frac{3}{2}l\right) = v_{ss}(l) + u_y^A \frac{\sqrt{2}}{6} = u_y^A \frac{\sqrt{2}}{2}$$

$$v_{ss}(l) = u_y^A \frac{\sqrt{2}}{3} = -\frac{4}{9}\frac{Pl^3}{EI} \Rightarrow u_y^A = -2\sqrt{2}\frac{Pl^3}{3EI}$$

Vertical deflection is $u_y^A = -2\sqrt{2}\frac{Pl^3}{3EI}$