Structural Vibrations

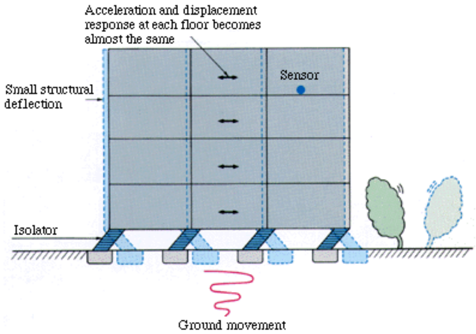
Big Trouble!

Earthquake-induced vibrational failure: Kobe, Japan 1995

http://www.strimoo.com/video/17063414/Kobe-Earthquake-Metacafe.html

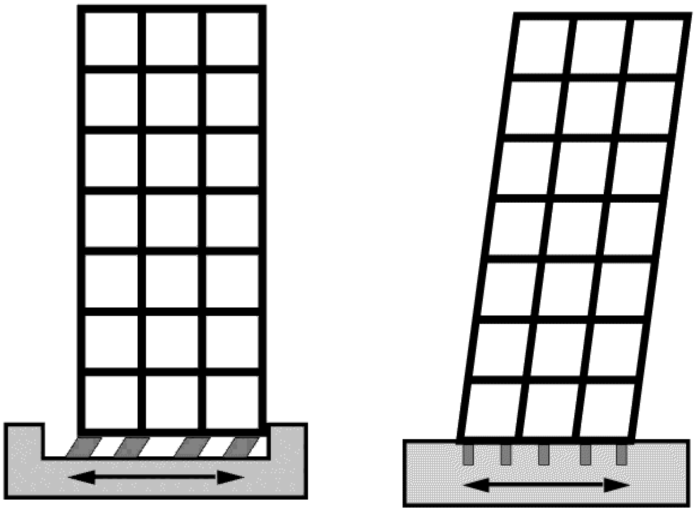
Hanshin Expressway

Safeguards: Base Isolation Keep Structure intact by providing soft supports

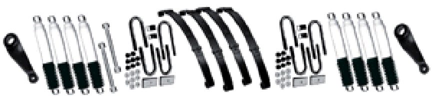


Rubber pads being installed under a building

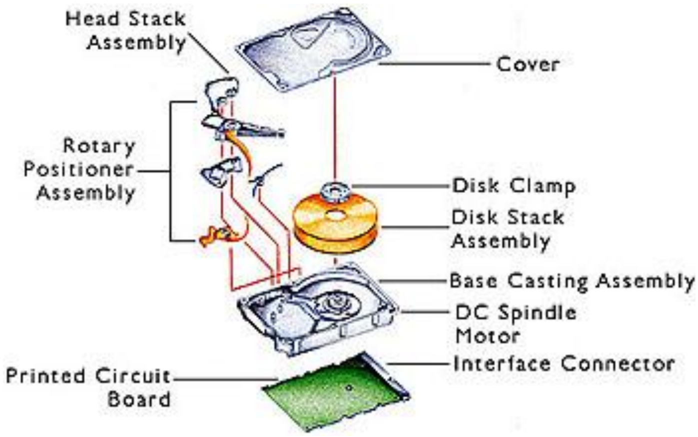
With and without



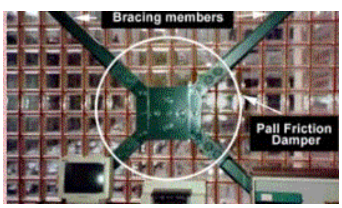
Other Suspension Systems (same idea)

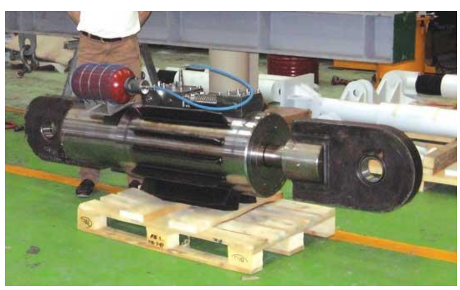


Disk Drive

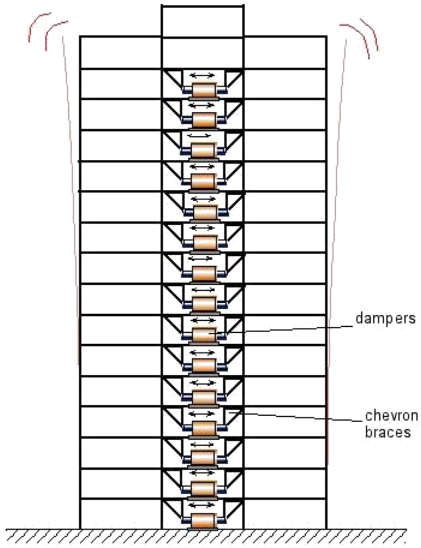


Energy Dissipation: Passive Damping

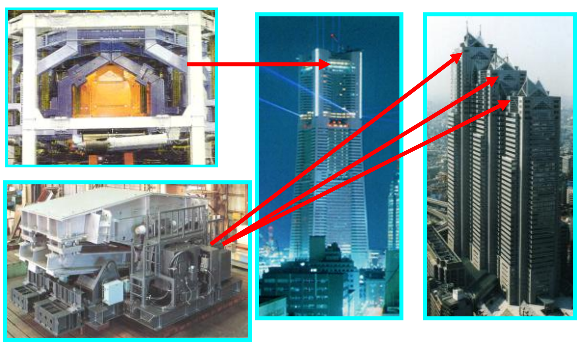


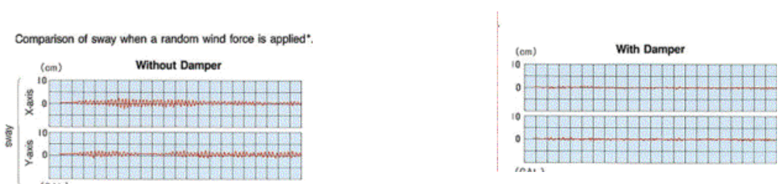


Active Viscous Damping: Rheomagnetic fluid

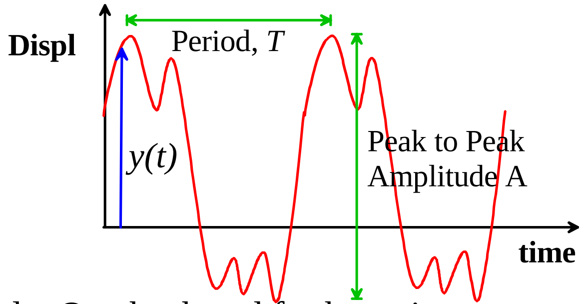


Active Mass Damping





Typical General Vibration Response



Cycle: One back and forth motion

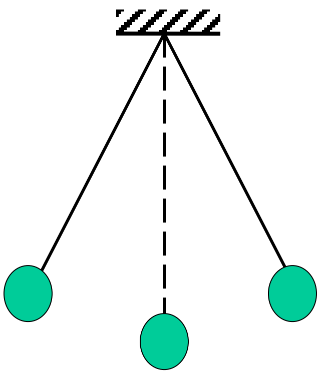
Period: T Time to complete one cycle

Amplitude: X Max departure from equilbrium=2A

Frequency: $f = \frac{1}{T}$ (Hertz, or cycles per sec),

Angular Frequency $\omega = 2\pi f = \frac{2\pi}{T}$ (radians per sec)

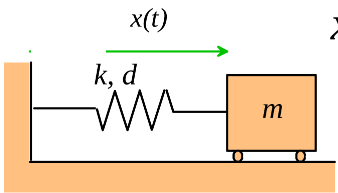
Common Features of Vibrations

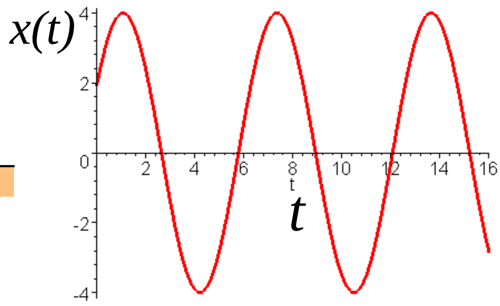


As systems vibrate, they typically pass through their equilibrium state.

Displacement typically follows a sinusoidal variation with time

$$x(t) = X_0 \sin\left(\omega t + \phi\right)$$





Parameters in the typical response

$$x(t) = X_0 \sin\left(\omega t + \phi\right)$$

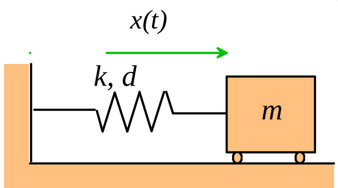
Angular Frequency: ω (radians per unit time)

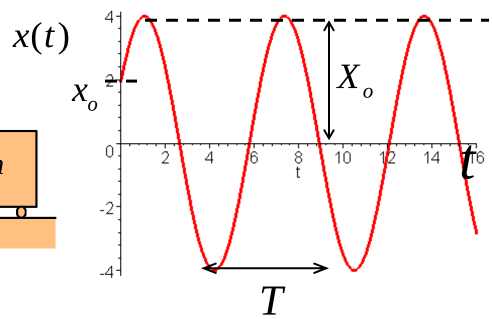
Frequency: $f = \frac{\omega}{2\pi}$ (cycles per unit time, eg Hertz=cycles/sec),

Period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$ (Time to complete one cycle)

Amplitude: X_o (Maximum displacement from equilibrium)

Phase: $\phi = \sin^{-1}\left(x_o/X_o\right)$ (Describes ratio of initial displ $x(0)$ to the amp X_0)



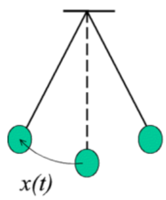


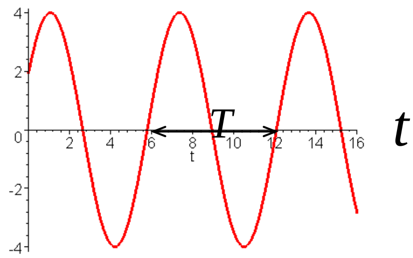
Example: Pendulum

Period: $T = 2\pi\sqrt{\frac{l}{g}}$ seconds

Frequencies $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ Hertz,

$\omega = \sqrt{\frac{g}{l}}$ radians per sec





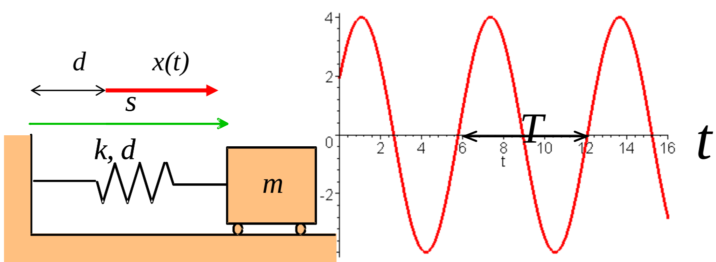
$$x(t) = X_0 \sin(\omega t - \phi)$$

Example: Spring-Mass

Period: $T = 2\pi\sqrt{\frac{m}{k}}$ seconds

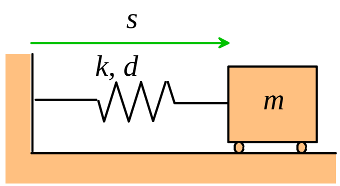
Frequencies $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ Hertz,

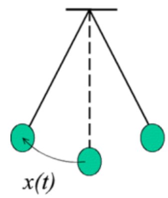
$\omega = \sqrt{\frac{k}{m}}$ radians per sec



$$x(t) = s(t) - d = X_0 \sin(\omega t + \phi)$$

Frequency and Period are **System Dependent**





$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{seconds}$$

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{seconds}$$

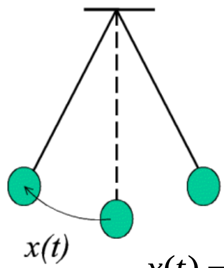
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{Hertz,}$$

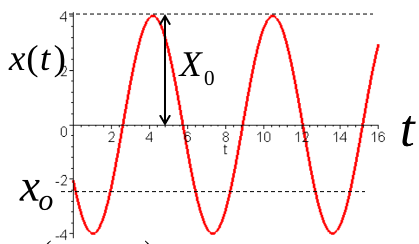
$$\omega = \sqrt{\frac{k}{m}} \quad \text{radians per sec}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \quad \text{Hertz,}$$

$$\omega = \sqrt{\frac{g}{l}} \quad \text{radians per sec}$$

Amplitude and Phase depend on Initial Conditions: $x(0)=x_o$ and $v(0)=v_o$





$$x(t) = X_0 \sin(\omega t + \phi)$$

Amplitude: Maximum displacement from static equilib. position

$$X_0 = \sqrt{x_o^2 + v_o^2/\omega^2}$$

Phase: Describes the ratio of the initial displacement x_0 to the amplitude of vibration X_0

$$\phi = \sin^{-1}\left(\frac{\omega x_o}{X_o}\right) \quad \text{(radians)}$$