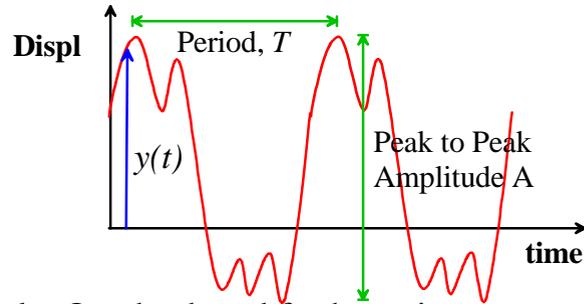


## Typical General Vibration Response



Cycle: One back and forth motion

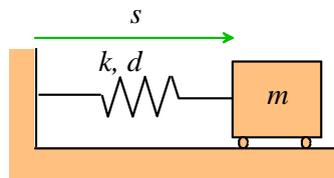
Period:  $T$  Time to complete one cycle

Amplitude:  $X$  Max departure from equilibrium  $= 2A$

Frequency:  $f = \frac{1}{T}$  (Hertz, or cycles per sec),

Angular Frequency  $\omega = 2\pi f = \frac{2\pi}{T}$  (radians per sec)

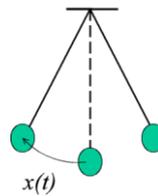
## Frequency and Period are System Dependent



$$T = 2\pi \sqrt{\frac{m}{k}} \text{ seconds}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \text{ Hertz,}$$

$$\omega = \sqrt{\frac{k}{m}} \text{ radians per sec}$$

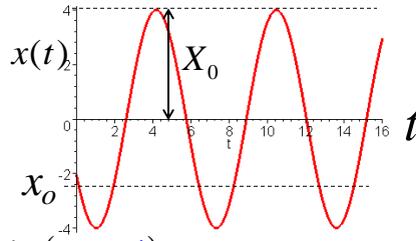
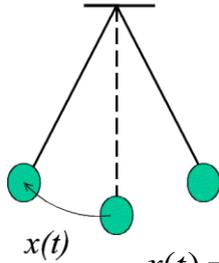


$$T = 2\pi \sqrt{\frac{l}{g}} \text{ seconds}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \text{ Hertz,}$$

$$\omega = \sqrt{\frac{g}{l}} \text{ radians per sec}$$

**Amplitude and Phase depend on Initial Conditions:  $x(0)=x_0$  and  $v(0)=v_0$**



$$x(t) = X_0 \sin(\omega t + \phi)$$

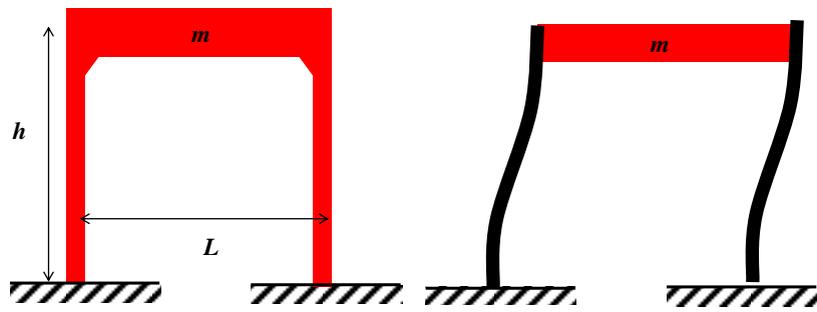
**Amplitude:** Maximum displacement from static equilib. position

$$X_0 = \sqrt{x_0^2 + v_0^2 / \omega^2}$$

**Phase:** Describes the ratio of the initial displacement  $x_0$  to the amplitude of vibration  $X_0$

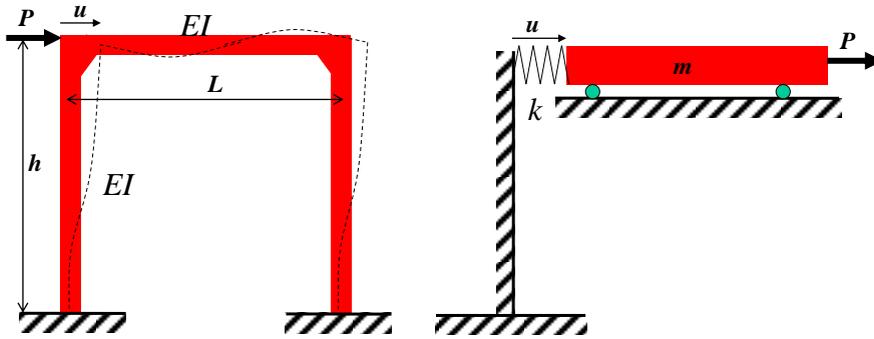
$$\phi = \sin^{-1}\left(\frac{\omega x_0}{X_0}\right) \text{ (radians)}$$

**Free Vibration of a single story frame**



## Spring Mass system, stiffness K

$$P = ku$$

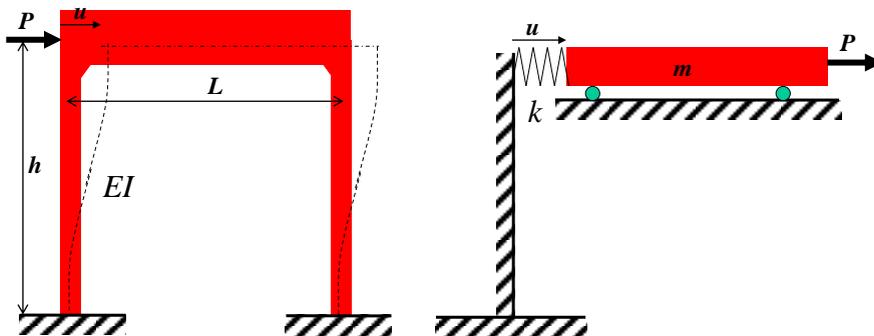


$$k = \frac{P}{u} = \frac{h^3}{12EI} \frac{(3+2L/h)}{(6+L/h)}$$

Hw 9

## Spring mass system, stiffness K

$$P = ku$$

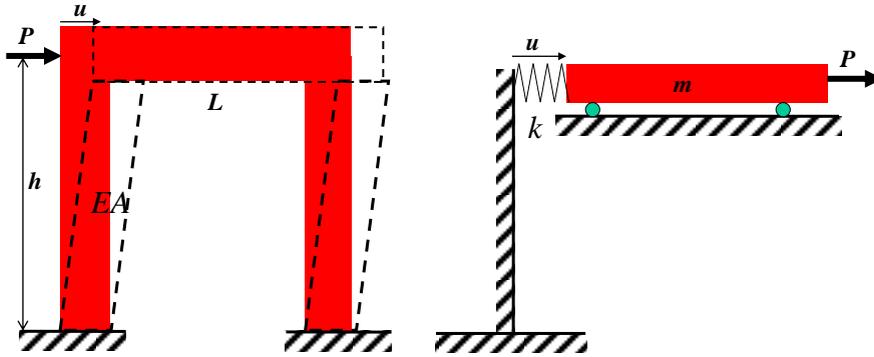


Column Bending, Stiff crossbeam

$$k = \frac{P}{u} = \frac{24EI}{h^3}$$

## Spring mass system, stiffness K

$$P = ku$$

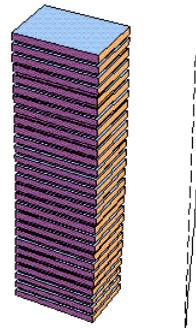


Shearing, stiff floor

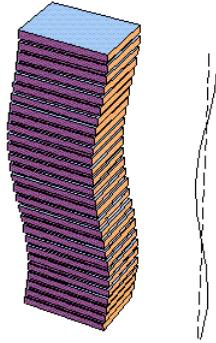
$$k = \frac{P}{u} = \frac{2EA}{h}$$

## Vibration of a tall building

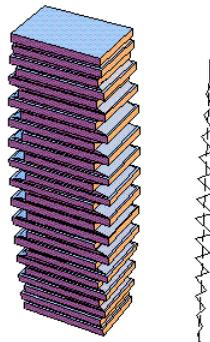
Mode 1 For a tall building



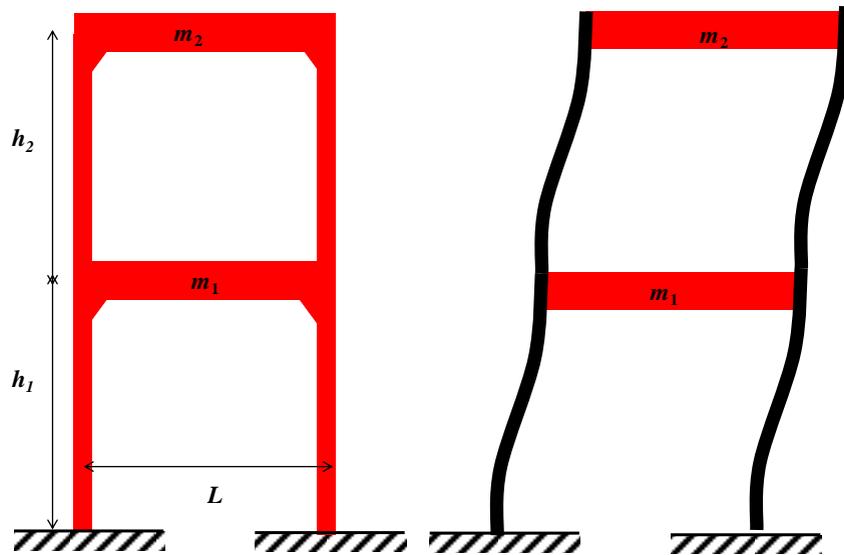
### Another Mode (Mode 3)



### Mode 28



### Free Vibration of a Multi-Degree of freedom structure



### Represent walls as lateral springs, stiffness $k_i$

