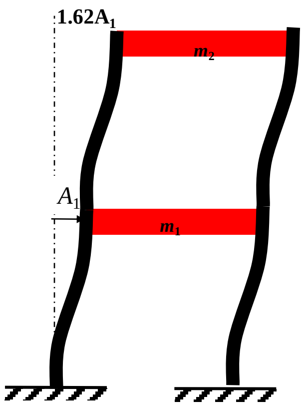
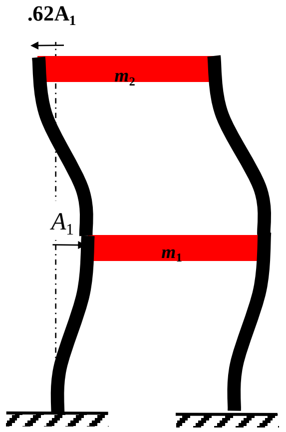
Two Story Building free vibrations

Mode I



Mode II

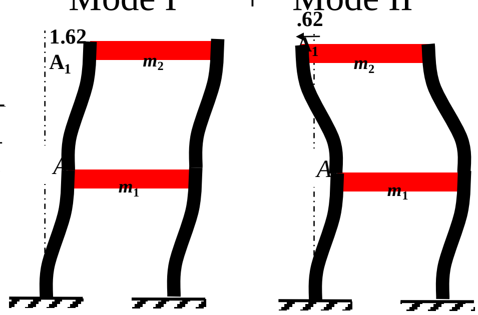


$$\omega_1 = 0.62\sqrt{\frac{k}{m}}$$

$$\omega_2 = 1.62\sqrt{\frac{k}{m}}$$

General Free Vibration Response

Mode I + Mode II



$$\omega_1 = 0.62\sqrt{\frac{k}{m}} \qquad \omega_2 = 1.62\sqrt{\frac{k}{m}}$$

$$u_1(t) = A_1^I \sin\left(\omega_1 t + \phi_1\right) + A_1^{II} \sin\left(\omega_2 t + \phi_2\right)$$

$$u_2(t) = A_1^I\, 0.62 \sin\left(\omega_1 t + \phi_1\right) - A_1^{II}\, 1.62 \sin\left(\omega_2 t + \phi_2\right)$$

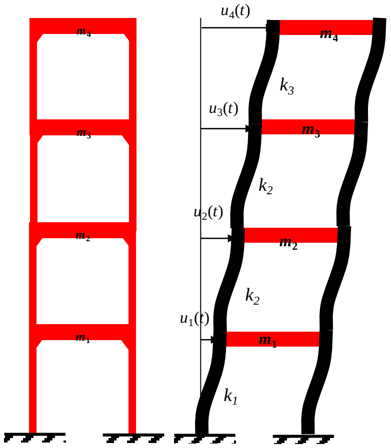
4 unknowns:

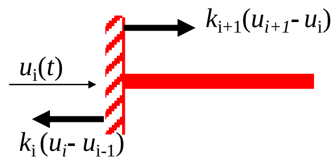
$A_1^I, A_1^{II}, \phi_1, \phi_2$

4 IC

$u_1(0) = u_{10}, \dot{u}_1(0) = v_{10}, u_2(0) = u_{20}, \dot{u}_2(0) = v_{20}$

More floors:

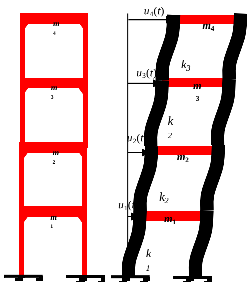




$$m_i \ddot{u}_i + k_i\left(u_i - u_{i-1}\right) - k_{i+1}\left(u_{i+1} - u_i\right) = 0$$

$$\begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & m_4 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \\ \ddot{u}_4 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & & \\ -k_2 & k_2+k_3 & -k_3 & \\ & -k_3 & k_3+k_4 & -k_4 \\ & & -k_4 & k_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Find 4 natural frequencies, Mode shapes

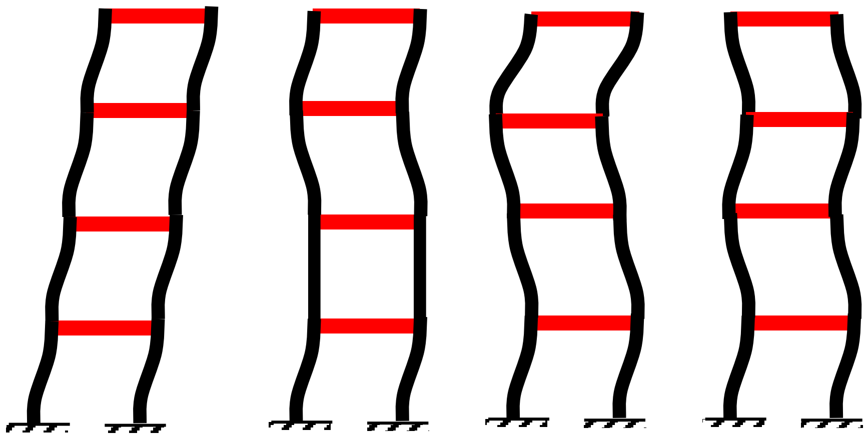


$$\underbrace{\begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & m_4 \end{bmatrix}}_{\text{M: Mass Matrix}} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \\ \ddot{u}_4 \end{bmatrix} + \underbrace{\begin{bmatrix} k_1+k_2 & -k_2 & & \\ -k_2 & k_2+k_3 & -k_3 & \\ & -k_3 & k_3+k_4 & -k_4 \\ & & -k_4 & k_4 \end{bmatrix}}_{\text{K: Stiffness Matrix}} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det\left[-\omega^2 \mathbf{M} + \mathbf{K}\right] = 0, \quad \left[-\omega^2 \mathbf{M} + \mathbf{K}\right] \underline{A} = \underline{0}$$

$$\det\left[-\omega^2 \mathbf{1} + \mathbf{H}\right] = 0, \quad \left[-\omega^2 \mathbf{1} + \mathbf{H}\right] \underline{A} = \underline{0}$$

They look like this (equal *k*’s, equal *m*’s)



$\omega_1 = 0.35\sqrt{\frac{k}{m}}$ $\omega_2 = \sqrt{\frac{k}{m}}$ $\omega_3 = 1.5\sqrt{\frac{k}{m}}$ $\omega_4 = 1.9\sqrt{\frac{k}{m}}$

Mode Shape Orthogonality

$$-\omega_i^2 \mathbf{M}\underline{A}_i + \mathbf{K}\underline{A}_i = \underline{0}$$

$$-\omega_i^2 \underline{A}_j \bullet \mathbf{M}\underline{A}_i + \underline{A}_j \bullet \mathbf{K}\underline{A}_i = 0$$

$$-\omega_i^2 \underline{A}_i \bullet \mathbf{M}^T \underline{A}_j + \underline{A}_i \bullet \mathbf{K}^T \underline{A}_j = 0$$

$$-\omega_i^2 \underline{A}_i \bullet \mathbf{M}\underline{A}_j + \underline{A}_i \bullet \mathbf{K}\underline{A}_j = 0 \qquad (1)$$

$$-\omega_j^2 \mathbf{M}\underline{A}_j + \mathbf{K}\underline{A}_j = \underline{0}$$

$$-\omega_j^2 \underline{A}_i \bullet \mathbf{M}\underline{A}_j + \underline{A}_i \bullet \mathbf{K}\underline{A}_j = 0 \qquad (2)$$

$$\left(\omega_i^2 - \omega_j^2\right) \underline{A}_j \bullet \mathbf{M}\underline{A}_i = 0$$

$$\underline{A}_j \bullet \mathbf{M}\underline{A}_i = 0 \; i \neq j$$