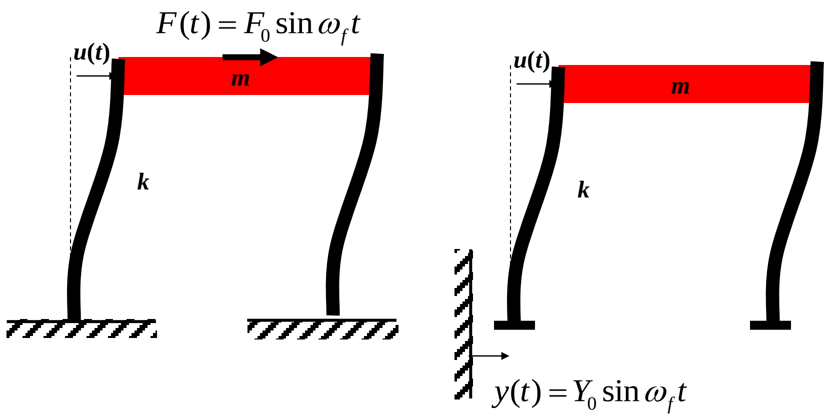
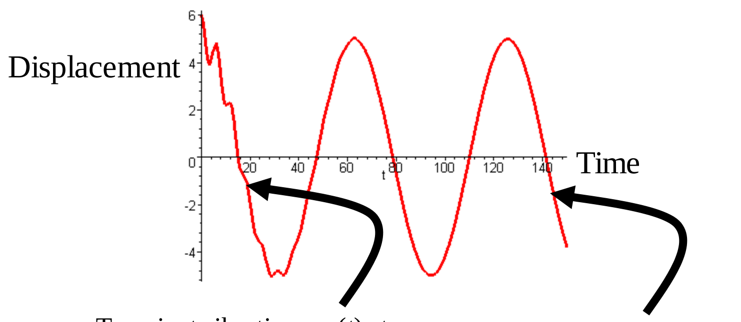
Forced Vibration (Periodic)



External Forcing

Base Excitation

Typical Response



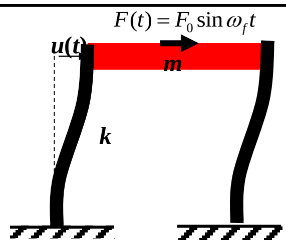
Transient vibrations $u_h(t)$ at the natural frequency. Depend on I.C. These decay exponentially with time.

Steady-state Vibrations $u_p(t)$ at the forcing frequency Independent of I.C. These do not decay.

Total response: $u(t)=u_h(t)+u_p(t)$

Amplitude of the steady-state vibrations is (very) large if the forcing frequency is at or near the system natural frequency.

The Steady State Response: Undamped



$$\frac{d^2u}{dt^2} + \omega_n^2 u = \frac{F_o}{k}\sin(\omega t)$$

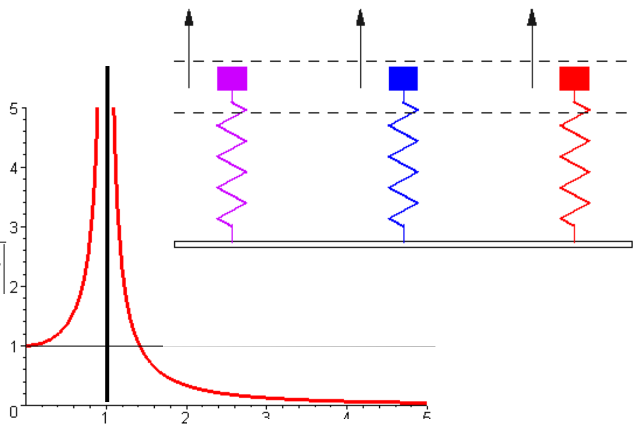
$$u(t) = X\sin\left(\omega t\right) \qquad X = \frac{F_0/k}{\left|1-\omega^2/\omega_n^2\right|}$$

Amplitude $X \to \infty$ as $\omega \to \omega_n$

The Steady State Response

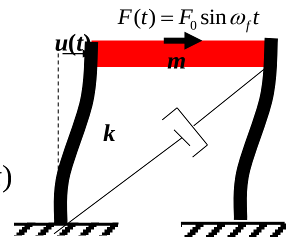
$$\frac{\underline{\text{Amplitude}}}{\text{static disp}}$$

$$\frac{X}{F_0/k} = \frac{1}{\left|1-\omega^2/\omega_n^2\right|}$$



forcing freq/nat. freq: ω/ω_n

The Steady State Response: Damped

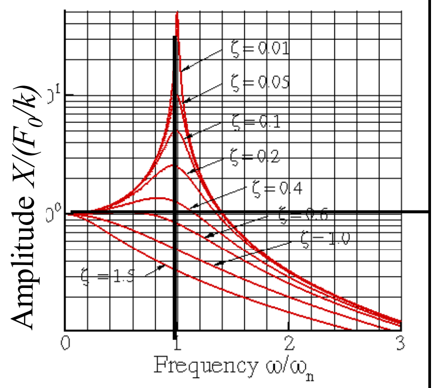


$$\frac{d^2u}{dt^2} + 2\varsigma\omega_n \frac{du}{dt} + \omega_n^2 u = \omega_n^2 \frac{F_o}{k}\sin(\omega t)$$

$$u_p(t) = X \sin\left(\omega t + \phi\right)$$

$$X = \frac{F_0/k}{\left\{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\varsigma\omega/\omega_n\right)^2\right\}^{1/2}}$$

$$\phi = +\tan^{-1}\frac{-2\varsigma\omega/\omega_n}{1-\omega^2/\omega_n^2}$$

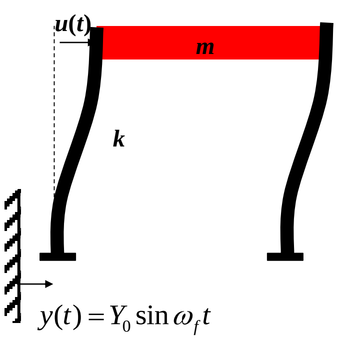


Slow forcing: $X \to F_0/k$ for $\omega \ll \omega_n$

Resonance: X gets large as $\omega/\omega_n \to 1$

Fast forcing: $X \to 0$ for $\omega \gg \omega_n$

Base Excitation: Undamped



$$m(\ddot{u} - \ddot{y}) + ku = 0$$

$$\frac{d^2u}{dt^2} + \omega_n^2 u = \frac{ky}{m} = Y_o\omega^2 \sin(\omega t) = \frac{F_o}{m}\sin(\omega t)$$

$$F_o = mY_o\omega^2 = kY_o\omega^2/\omega_n^2$$

$$X = \frac{F_0/k}{\left|1-\omega^2/\omega_n^2\right|} = \frac{Y_o\omega^2/\omega_n^2}{\left|1-\omega^2/\omega_n^2\right|}$$

Amplitude of the steady state response

$$X = \frac{F_0 / k}{\left|1 - \omega^2 / \omega_n^2\right|} = \frac{Y_o \omega^2 / \omega_n^2}{\left|1 - \omega^2 / \omega_n^2\right|}$$