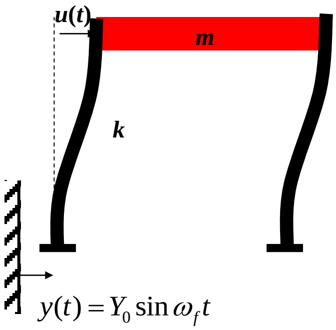
Base Excitation: Undamped



$$m(\ddot{u} - \ddot{y}) + ku = 0$$

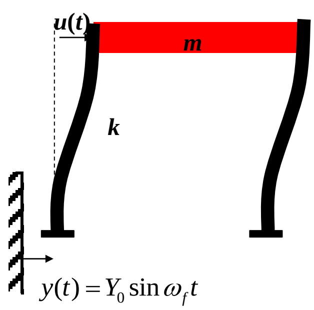
$$\frac{d^2u}{dt^2} + \omega_n^2 u = \frac{ky}{m} = Y_o \omega^2 \sin(\omega t) = \frac{F_o}{m} \sin(\omega t)$$

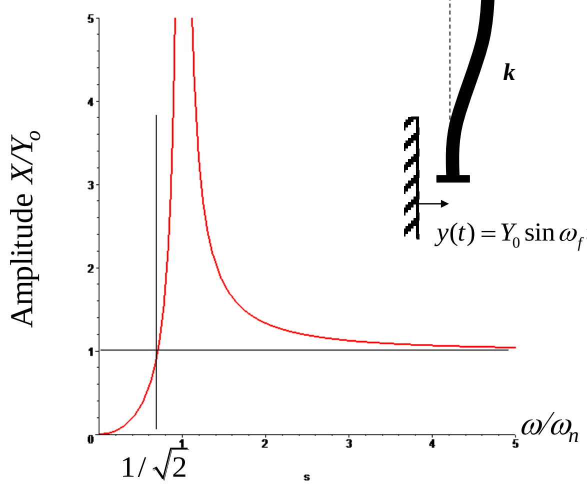
$$F_o = mY_o\omega^2 = kY_o\omega^2 / \omega_n^2$$

$$X = \frac{F_0 / k}{\left|1 - \omega^2 / \omega_n^2\right|} = \frac{Y_o \omega^2 / \omega_n^2}{\left|1 - \omega^2 / \omega_n^2\right|}$$

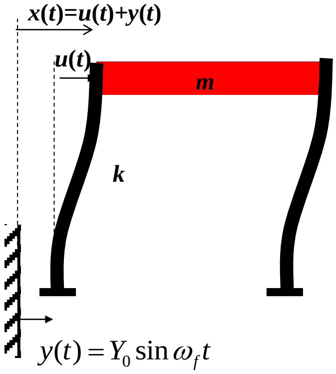
Amplitude of the steady state response

$$\frac{X}{Y_o} = \frac{\omega^2 / \omega_n^2}{\left|1 - \omega^2 / \omega_n^2\right|}$$





The **absolute** motion of the mass



$y(t) = Y_0 \sin \omega_f t$

$$m\frac{d^2x}{dt^2}+k(x-y)=0$$

$$\frac{d^2x}{dt^2}+\omega_n^2 x=\frac{ky}{m}=Y_o\omega_n^2\sin(\omega t)=\frac{F_o}{m}\sin(\omega t)$$

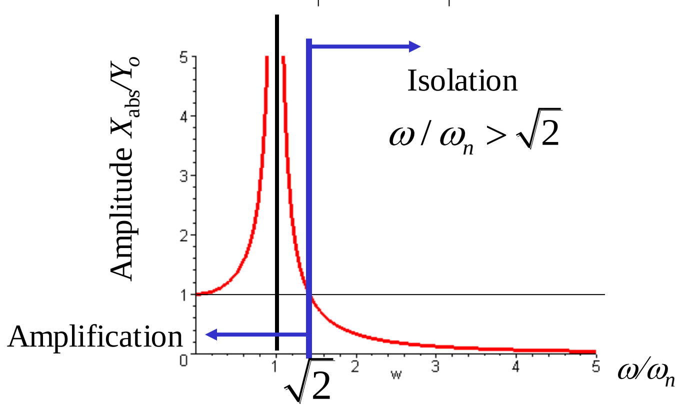
$$F_o = mY_o\omega_n^2 = kY_o$$

$$x(t)=u(t)+y(t)=X_{abs}\sin(\omega t)$$

$$X_{abs}=\frac{F_0/k}{\left|1-\omega^2/\omega_n^2\right|}=\frac{Y_o}{\left|1-\omega^2/\omega_n^2\right|}$$

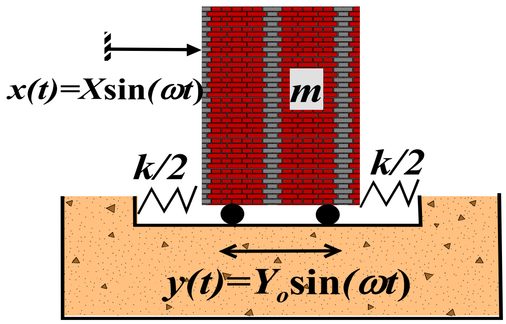
Amplitude of the steady state response

$$X_{abs}=\frac{Y_o}{\left|1-\omega^2/\omega_n^2\right|}$$



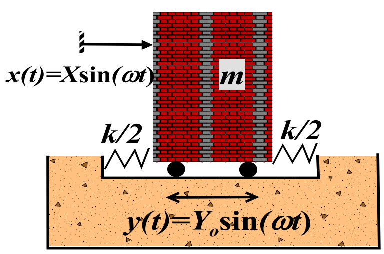
Pick a (soft) spring so that $\omega_n^2=k/m<\omega^2/2$.

Building Base Isolation Design

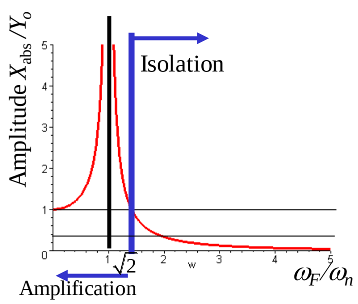


- Building mass m=10^4 kg
- Maximum allowable building motion X_{abs} =2 cm.
- Ground Motion:
  - Max amplitude Y_o=5 cm.
  - Freq range is $f=\omega/2\pi$ =3-5hz

Building Base Isolation Design



F

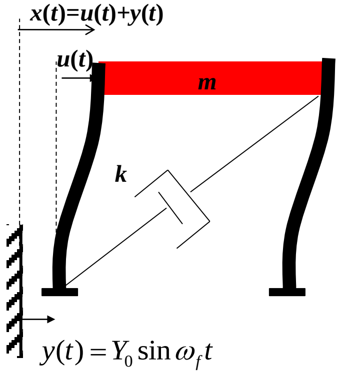


$$\frac{X_{abs}}{Y_o} = \frac{1}{\left|1-\omega_F^2/\omega_n^2\right|} \le \frac{2}{5} \Rightarrow \omega_F^2/\omega_n^2 \ge \frac{7}{2} \Rightarrow \omega_n^2 = \frac{k}{m} \le \frac{2}{7}\omega_F^2$$

$$\Rightarrow \omega_n^2 = \frac{k}{m} \le \frac{2}{7}(6\pi/\sec)^2$$

$$k \le 10^4 kg\,\frac{2}{7}(6\pi/\sec)^2 = 1\text{MN/m}$$

Base Excitation: Effect of damping



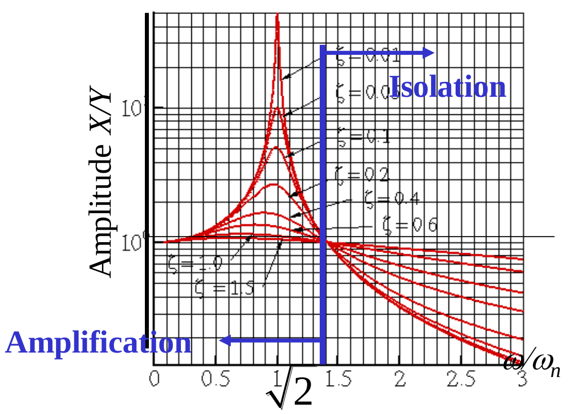
$$m\frac{d^2x}{dt^2}+c(\dot{x}-\dot{y})+k(x-y)=0$$

$$\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^2 x=\omega_n^2 y+2\zeta\omega_n\dot{y}=\omega_n^2 Y\sin(\omega t)+2\zeta Y\omega_n\omega\cos(\omega t)$$

$$\omega_n=\sqrt{\frac{k}{m}} \qquad \zeta=\frac{C}{2m\omega_n}=\frac{C}{2\sqrt{mk}}<1$$

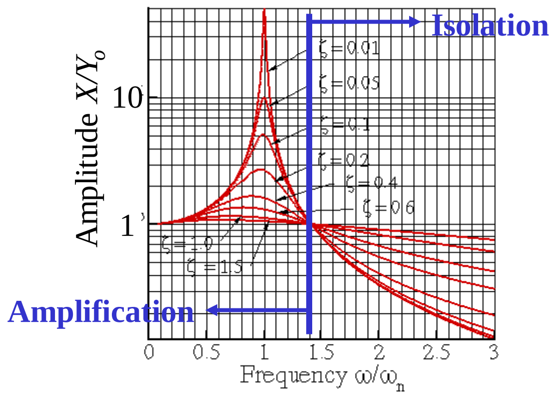
Amplitude of the steady state response

$$X_{\text{abs}}=Y\sqrt{\frac{1+2\zeta r^2}{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}},\ r=\frac{\omega}{\omega_n}$$

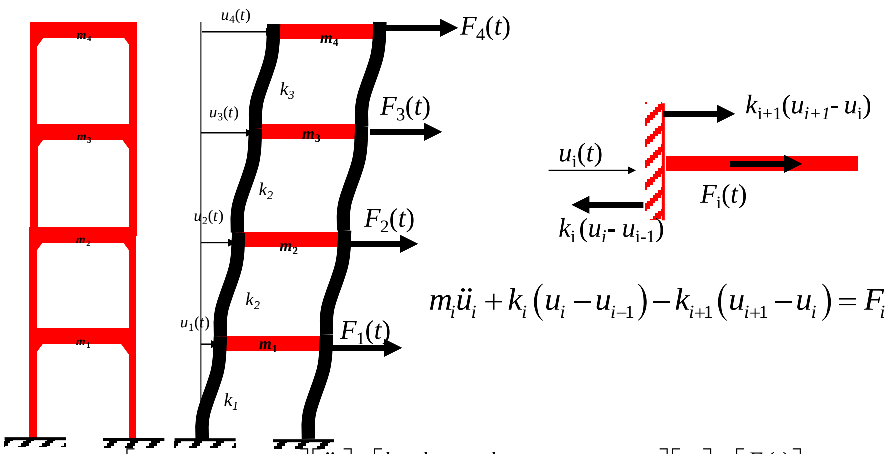


For Good Isolation: $X_{abs} > Y_o$:

- Pick a (soft) spring so that $(\omega_n)^2 = k/m < \omega^2/2$.
- Keep damping low (with caution)



Multi Degree of Freedom forced vibrations:

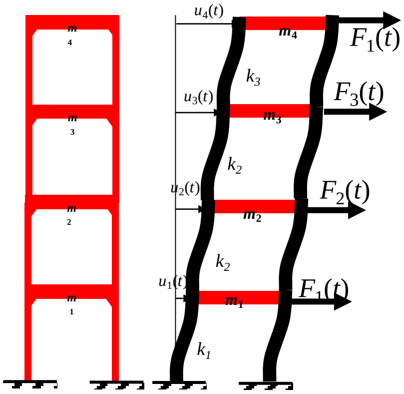


$$m_i \ddot{u}_i + k_i (u_i - u_{i-1}) - k_{i+1}(u_{i+1} - u_i) = F_i(t)$$

$$\begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & m_4 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \\ \ddot{u}_4 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & & \\ -k_2 & k_2 + k_3 & -k_3 & \\ & -k_3 & k_3 + k_4 & -k_4 \\ & & -k_4 & k_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ F_4(t) \end{bmatrix}$$

$$\mathbf{M}\underline{\ddot{u}} + \mathbf{K}\underline{u} = \underline{F}$$

Periodic vibrations: $\underline{F}(t) = \underline{f}\,\sin(\omega t)$



$$\mathbf{M}\underline{\ddot{u}} + \mathbf{K}\underline{u} = \underline{f}\,\sin\omega t$$

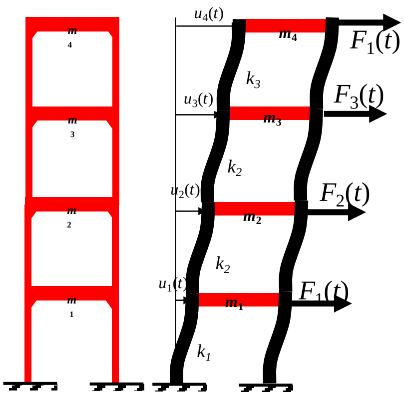
$$\underline{u}(t) = \sin(\omega t)\sum_{i=1}^{n} X_i \underline{A}_i$$

$$\sum X_i\left(-\omega^2\mathbf{M}\underline{A}_i + \mathbf{K}\underline{A}_i\right) = \underline{f}$$

$$\sum X_i\left(-\omega^2 + \omega_i^2\right)\mathbf{M}\underline{A}_i = \underline{f}$$

$$\underline{A}_j \bullet \sum X_i\left(-\omega^2 + \omega_i^2\right)\mathbf{M}\underline{A}_i = \sum X_i\left(-\omega^2 + \omega_i^2\right)\underline{A}_j \bullet \mathbf{M}\underline{A}_i = \underline{A}_j \bullet \underline{f}$$

Periodic vibrations: $\underline{F}(t) = \underline{f}\,\sin(\omega t)$



$$\underline{u}(t) = \sin(\omega t)\sum_{i=1}^{n} X_i \underline{A}_i$$

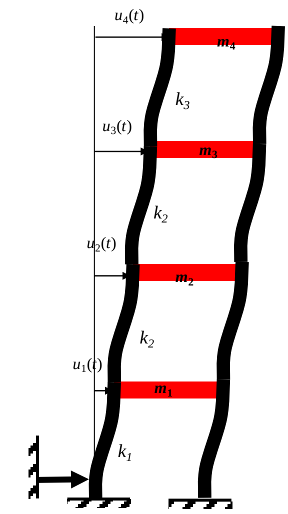
$$\sum X_i\left(-\omega^2 + \omega_i^2\right)\underline{A}_j \bullet \mathbf{M}\underline{A}_i = \underline{A}_j \bullet \underline{f}$$

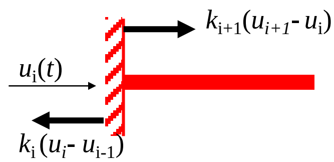
$$\underline{A}_j \bullet \mathbf{M}\underline{A}_i = 0 \quad i \neq j$$

$$X_j = \frac{\underline{A}_j \bullet \underline{f}}{\left(-\omega^2 + \omega_i^2\right)\underline{A}_j \bullet \mathbf{M}\underline{A}_j}$$

Resonance in a given mode if $\omega^2 \approx \omega_i^2$

Base Motion: $y(t) = Y_0 \sin(\omega t)$





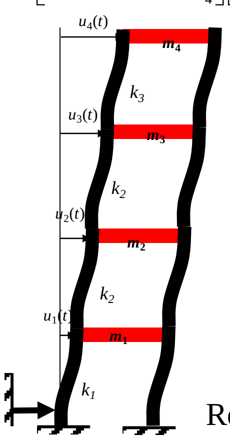
$$m_i\left(\ddot{u}_i + \ddot{u}_g\right) + k_i\left(u_i - u_{i-1}\right) - k_{i+1}\left(u_{i+1} - u_i\right) = 0$$

$$m_i\ddot{u}_i + k_i\left(u_i - u_{i-1}\right) - k_{i+1}\left(u_{i+1} - u_i\right) = F_i(t) = -m_i\ddot{y}$$

$$\begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & m_4 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \\ \ddot{u}_4 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & & \\ -k_2 & k_2+k_3 & -k_3 & \\ & -k_3 & k_3+k_4 & -k_4 \\ & & -k_4 & k_4 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = -\ddot{y}\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix}$$

Base Motion: $y(t) = Y_0 \sin(\omega t)$

$$\begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & m_4 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \\ \ddot{u}_4 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & & \\ -k_2 & k_2+k_3 & -k_3 & \\ & -k_3 & k_3+k_4 & -k_4 \\ & & -k_4 & k_4 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = -\ddot{u}_g\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \omega^2 \sin\omega t\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix}$$



$$\underline{u}(t) = \sin(\omega t)\sum_{i=1}^{n} X_i \underline{A}_i$$

$$X_j = \frac{\underline{A}_j \bullet \underline{f}}{\left(-\omega^2 + \omega_i^2\right)\underline{A}_j \bullet \mathbf{M}\underline{A}_j} \qquad \underline{f} = \omega^2\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix}$$

Resonance in a given mode if $\omega^2 \approx \omega_i^2$