What if the structure is a mechanism?

You can follow the procedure and arrive at the system of equations:

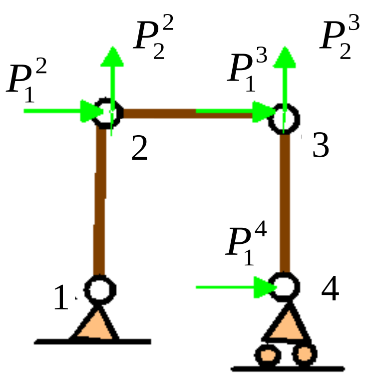
$$[\mathbf{K}]\underline{\mathbf{u}} = \underline{\mathbf{p}}$$

Since nonzero displacements can occur without inducing member forces, matrix **K** will be singular.

•The number of zero eigenvalues corresponds to the degree of indeterminacy: (number of missing members or reactions.

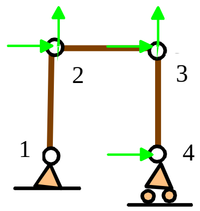
•Null vectors of **K** correspond to the motion allowed by the mechanism

Example



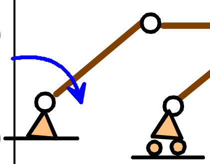
K has two zero eigenvalues;

$$\frac{EA}{L}\begin{bmatrix}1 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ -1 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}u_1^2\\ u_2^2\\ u_1^3\\ u_2^3\\ u_1^4\end{bmatrix}=\begin{bmatrix}P_1^2\\ P_2^2\\ P_1^3\\ P_2^3\\ P_1^4\end{bmatrix}$$

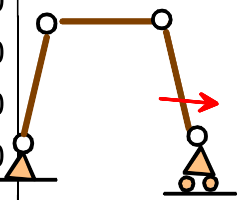


Eigenvalues: 2,1,1,0,0. Null vectors:

$$\begin{bmatrix}u_1^2\\ u_2^2\\ u_1^3\\ u_2^3\\ u_1^4\end{bmatrix}=a_1\begin{bmatrix}1\\ 0\\ 1\\ 0\\ 0\end{bmatrix}$$

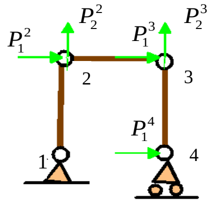


$$\begin{bmatrix}u_1^2\\ u_2^2\\ u_1^3\\ u_2^3\\ u_1^4\end{bmatrix}=a_2\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 1\end{bmatrix}$$



Eigenvectors: Possible, but unstable loads

$$\frac{EA}{L}\begin{bmatrix}1 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ -1 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}u_1^2\\ u_2^2\\ u_1^3\\ u_2^3\\ u_1^4\end{bmatrix}=\begin{bmatrix}P_1^2\\ P_2^2\\ P_1^3\\ P_2^3\\ P_1^4\end{bmatrix}$$



$$\begin{bmatrix}P_1^2\\ P_2^2\\ P_1^3\\ P_2^3\\ P_1^4\end{bmatrix}=c_1\begin{bmatrix}-1\\ 0\\ 1\\ 0\\ 0\end{bmatrix}\qquad \begin{bmatrix}P_1^2\\ P_2^2\\ P_1^3\\ P_2^3\\ P_1^4\end{bmatrix}=c_2\begin{bmatrix}0\\ 1\\ 0\\ 0\\ 0\end{bmatrix}\qquad \begin{bmatrix}P_1^2\\ P_2^2\\ P_1^3\\ P_2^3\\ P_1^4\end{bmatrix}=c_3\begin{bmatrix}0\\ 0\\ 0\\ 1\\ 0\end{bmatrix}$$