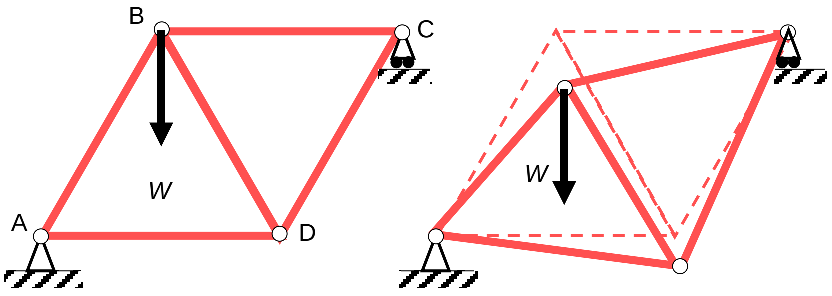
Principle of Stationary Potential Energy

For a system in stable static equilibrium,
the potential energy of the structure is minimized



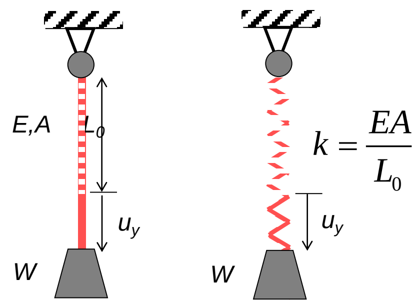
Find the values of the joint displacements for which the potential energy of the structure and its applied loads is a minimum.

PE consists of elastic energy stored in members + energy due to applied loads.

1

Stationary Potential Energy

$$\mathbf{F} = \mathbf{0} \Rightarrow -\frac{dV}{d\mathbf{x}} = \mathbf{0}.$$



Total Potential Energy as function of deflection u_y:

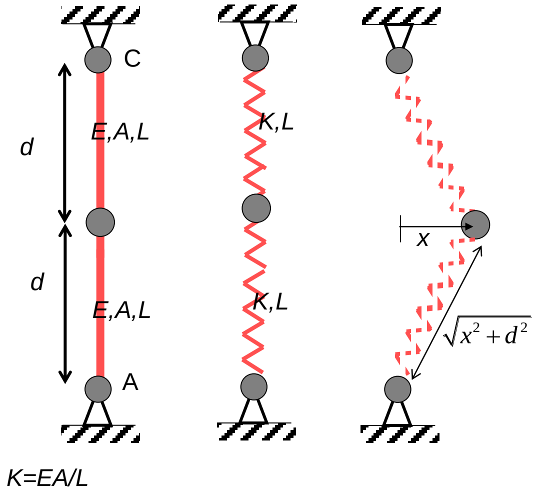
$$V(u_y) = -Wu_y + \frac{1}{2}ku_y^2$$

$$V'(u_y) = -W + ku_y$$

Potential Energy is a minimum when $u_y=W/k$. $F=ku_y=W$

3

Another



$$V(x)=2\left(\frac{1}{2}K\left(\sqrt{x^2+d^2}-L\right)^2\right)$$

$$V'(x)=2K\frac{x\left(\sqrt{x^2+d^2}-L\right)}{\sqrt{x^2+d^2}}$$

$$V'(x)=0\Rightarrow$$

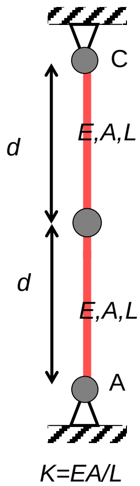
$$x=0,\text{ or }L=\sqrt{x^2+d^2}\text{ if }\left(L>d\right)$$

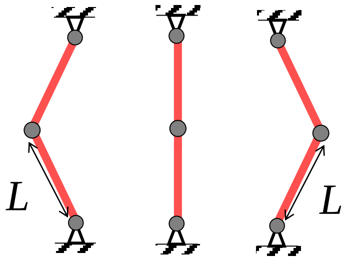
Which one?

$$V'(x)=0\Rightarrow x=0\ \text{ or } x=\pm\sqrt{L^2-d^2}\text{ if }\left(L>d\right)$$

If *L<d:* equilibrium at *x=0* only

If *L>d: 3* equilibria!





Stable Equilibrium: Minimum PE

$$V(x) = 2\left(\frac{1}{2}K\left(\sqrt{x^2+d^2}-L\right)^2\right)$$

$$V'(x) = 0 \Rightarrow x = 0 \text{ or } \sqrt{x^2+d^2} = L \ \left(L > d\right)$$