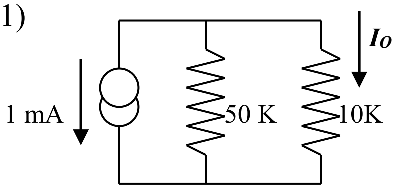
Engineering 1620 -- Spring 2020

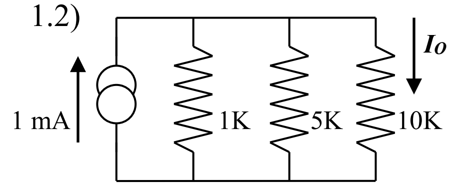
Homework Assignment 1: Passive Circuits and Some Non-linearity

1.) For the following four circuits, find the marked voltage or current. All circuits are DC. As you go through this and the next two problems, try to visually recognize series and parallel elements, voltage division, current division, and Thevenin/Norton sources, rather than always going directly to Kirchoff's laws. You should be able to do them all without resorting to those laws.

1.1)



1.2)



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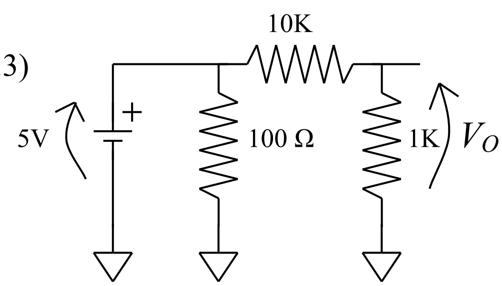

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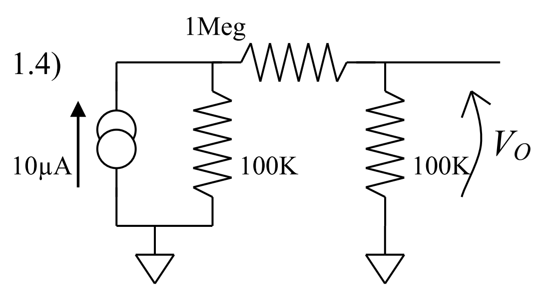
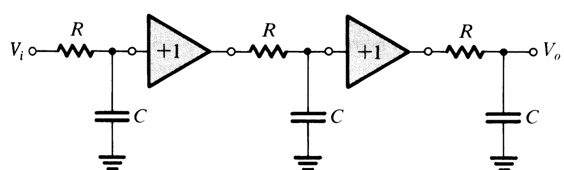


2.) Here is a unity gain circuit that contains three amplifiers. Each of those amplifiers has a gain, G = +1 and infinite input impedance and zero output impedance.

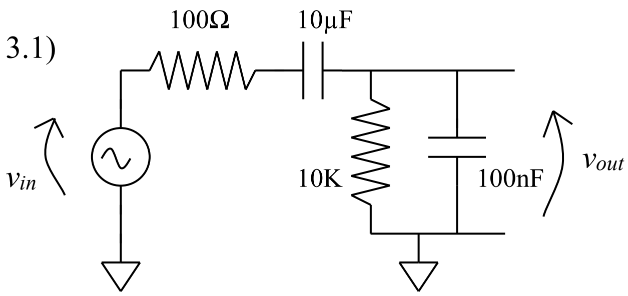
2.1) At what frequency is the gain of the overall circuit = -3dB?

2.2) If R = 1 K and C = 16 nfd, sketch a Bode plot of the overall circuit. Make this a neat hand sketch rather than a computer driven plot. Cover the frequency range from 1/10 the frequency as calculated with the result of (2.1) to 10 times the 3 dB cutoff frequency of a single RC section. To save some effort with the phase part of the plot, you may substitute a table giving the phase shifts at the end points of the plot, the -3dB point, and the cutoff frequency of the single RC section.

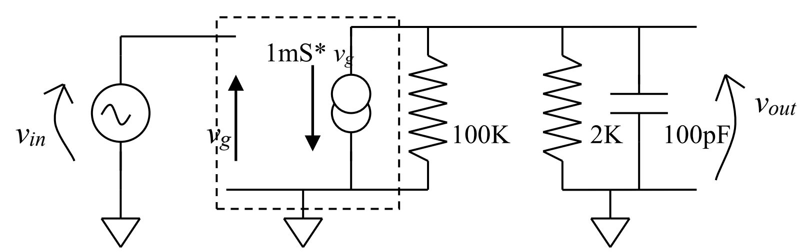


3.) For each of these three circuits sketch the gain, that is, the magnitude of $\left|H(s)\right|$, marking flat gain regions, corner frequencies and slopes of asymptotes. You should be able to do this by Thevenin-Norton transforms, combining components and even approximating when the error is under 2 %.

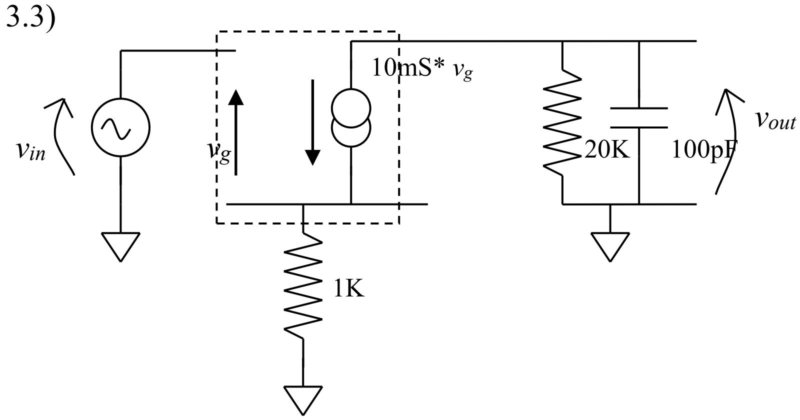
3.1)



3.2) Note: the box encloses a voltage controlled current source. The siemens is the unit of conductivity. In English the same word is both singular and plural. Multiplying a voltage by some number of siemens results in a current in amperes.



3.3)



4.) Just down the hall from my office is a refrigerator, microwave oven, and coffee maker so the graduate students can prepare lunch or midnight snacks. (They notice the leg irons less if allowed to feed regularly.) One of the pleasures of the place is to return to my office to the smell of hot Chinese food. ABET wishes us to include some economics in our courses, so here is my effort at letting you think about the probable utility operations that feed the grad students. A significant part of Rhode Island's electricity is imported from the hydroelectric plants of northern Quebec near James Bay about 1500 km away. It travels over 345 KV high-tension lines to northern Rhode Island whence it is distributed to the Franklin Square substation in Providence near the Point Street Bridge and on to Brown. While it is impossible to separate the power flow into the system from multiple sources – including from the Manchester Street power plant that is on the same fenced in area as the substation itself – let us suppose that some lunchtime when the microwave oven (600 watts) is turned on, it is serviced entirely from Canada. In effect Brown rents a part of that long transmission line for the cooking time.

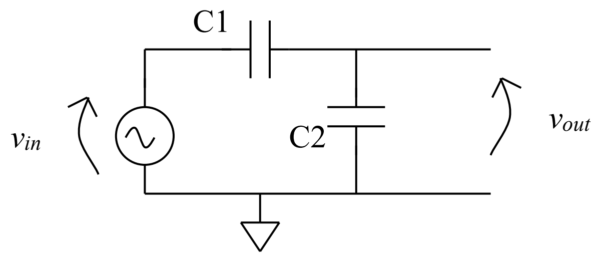
4.1) Power distribution companies try to hold transmission losses overall to less than 7 %. Allowing for a third of that loss along the main line (345 KV in northern RI) from Canada to Burrillville, how large is the cross sectional area of the wire Brown rents? It is small enough a fraction of the whole wire, that we can assume its cross section is square. What is the edge size? (For reasons of weight and cost, that line is probably aluminum with a resistivity of 2.8 micro-ohm cm.)

4.2) The spot metal price of aluminum on the NY market last night was $ 0.775 per pound. Assuming that the material cost is only one fifth of the cost of the line, what capital investment does just that line have? (After all, the line required towers, support cable, insulators, construction, etc. Moreover each wire has a steel cable core to provide sufficient strength and yield resistance to hang in long sections.) A return on capital investment rate of 15 % over a year of continuous use to cover maintenance, overhead, idle time, and profit might be reasonable. How much must Brown pay for this rental in the five minutes it takes to nuke a pack of Green Giant Chinese veggies? On average, a kilowatt hour costs around $ 0.12 for the University. How much did the veggies cost the University and what fraction of that went for the transmission line?

4.3) A compromise between thermal management and electrical insulation limits generators to about 20 KV output. Power enters Prince Lab for the whole campus on a 12,000 volt line under the parking lot to Hope St. No 345 KV line comes into Franklin Square. What is the minimum number of transformers required in the system and what are their turns ratios? (If one voltage along the way is unknown, call it X and compute turns ratios symbolically.)

4.4) Make a block diagram of the system, showing transformer placement, turns ratios, voltages, etc.

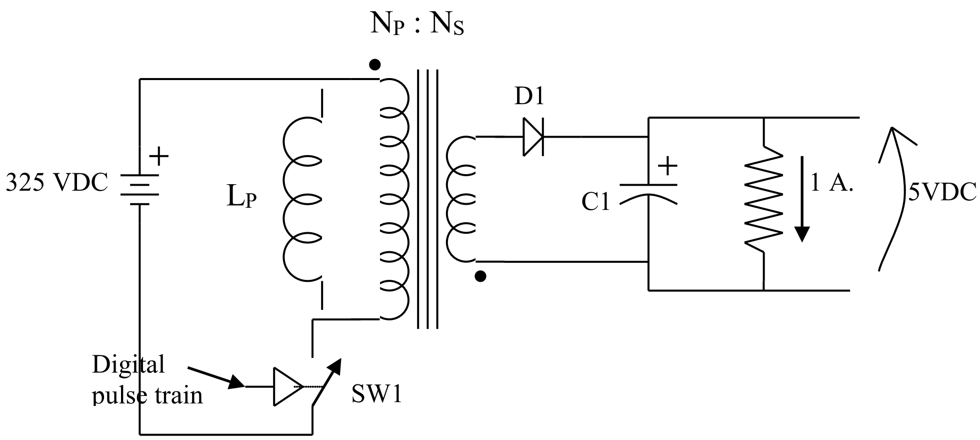
5.) Find the transfer function, Thevenin equivalent circuit and Norton equivalent circuit for this two-capacitor circuit. What is its amplitude response at DC and as frequency goes to infinity. Are either of these results a surprise?



6.) When my family and I visited England a few years ago, we needed something to charge the batteries in our phone and Kindle. We popped into an electricians shop and bought a simple USB charger with an English plug. The output was rated at 5 VDC and 1 ampere. Ever the engineer, I fell to speculating about how that device might be designed.

All devices that go into the hands of consumers must have all conductive surfaces in contact with people isolated from the mains by a transformer. England has 230 VAC power mains and that means a simple rectifier (next homework) will give 325 volts DC shown as a battery DC source below. I want you to work out quantitatively how this circuit works. I have left out a lot of details and just want you to concentrate on how you can get DC through a transformer with a minimum number of inexpensive and compact parts.

I have shown the transformer as fitting the same model I used in class with a primary magnetizing inductance separate from an ideal transformer. We have not talked yet about diodes (coming soon) so here we just see it as an object that lets current through in the direction of the arrowhead and not in the opposite direction at all. The black dots at the ends of the windings mark the direction in which they are wound relative to one another. If the primary dot is the positive end of that winding, then the black dot on the secondary is the positive end. The resistor with 1 A going through it is the appliance with its battery being charged. The output voltage is set by changing the switch on-time.



6.1) Suppose SW1 is open and there is no current in L_P and I close SW1. Current will begin to flow into the primary magnetizing inductance and there will be a voltage across the ideal transformer. Does any current flow in the secondary winding into the output of the system? Why?

6.2) Suppose SW1 closes at $t = 0$. What is the current as a function of time for this condition? [Hint: basic I-V for an inductor.] Write this in terms of the 325 volts and leave the inductance as L_P. Sketch the waveform.

6.3) After 1 microsecond, the switch opens. What is the primary current when it opens? How much energy is stored in the magnetic field in the core at that moment? Again write this expression for stored energy in terms of just L_P.

6.4) When the switch opens, the current in L_P cannot change abruptly so the voltage across the winding flies up until the voltage at the dot end is negative. (This action of 'flying up' gives the circuit its name – this is a flyback converter.) The secondary is also negative so current can flow in D1 into the capacitor and load. I will leave the switch open for 1.5 microseconds before I close it again. In that time, all the energy stored in L_P will be delivered to the load. For equilibrium, that stored energy has to match what the load uses between switch closures. How much energy does the load use in this time? What is the numerical value of L_P?

6.5) If you choose the turns ratio to be the ratio between the output voltage and the VDC supply, then the time for the secondary current to get to zero will be the same as the time it took to store the energy. (As a challenge to the bored, try to prove this!) What is the peak secondary current just after the switch opens? [Hint: there are a couple of ways to do this but the simplest is to use the average current time 5 volts to match the output power. The average is over a triangular pulse one microsecond on base in a period of 2.5 microseconds.]

6.6) If I want two turns on the secondary, how many turns do I need on the primary?

7.) I have already alluded several times to the fact that transistor circuits are not linear and that this can be a problem in designing linear amplifier. In your third lab, you will use a MOSFET transistor to build an amplifier to drive the speaker for music. We will examine MOSFETs more toward the middle of the course but for the purpose of this problem all you need to know is that some of them have a current-voltage relation that is quadratic. Analogous to the vacuum tube grid to filament voltage controlling the plate current, the "drain" current of a MOSFET may be proportional to the square of the "gate" to "source" voltage.

In the circuit below, the device labeled M1 is the MOSFET transistor. If the potential $v_{DS} \geq 0$, then the drain current is roughly $i_D = K_N v_{GS}^2$ where K_N is a constant. The "gate" terminal is the one connected to the common node of RG and C. The gate draws no AC or DC current. Suppose further that V_{GG} = 0.7 volts, K_N = 500 μA/volt^2, RG = 22K and C = 0.68 μfd.

7.1) What kind of filter is the linear part of this circuit for the signal v_{in} to v_{GS}? What is its cutoff frequency?

7.2) Suppose $v_{in} = 0.25\left(\sin\left(\omega_1 t\right) + \sin\left(\omega_2 t\right)\right)$ where $f_1 = 1$ KHz and $f_2 = 1.5$ KHz. (This would be a simple test of intermodulation distortion.) Write an expression for v_{GS}, the voltage on the gate. You may use a simple approximation if you tell me what it is. Use numeric coefficients.

7.3) The output of the circuit is the current i_D. Because the non-linearity of the MOSFET is a simple function, it is possible to write the output as a sum of simple sinusoids. Find an expression for the current expressed that way.

7.4) What are the peak currents of the linear part of the output, that is, the terms with the same frequencies as the original input signal v_{in}?

7.4) What are the peak voltages and frequencies of all the output terms that are not in the original signal? What percent of the 1 KHz output term is the amplitude term at 2 KHz? (This is called the second harmonic distortion and a good amplifier has less than 0.5 %.) If you used the output current to drive a speaker, would this be a good amplifier for a music signal applied at v_{in}?