## Engineering 1620 -- Spring 2020

## Homework Assignment 3: Diode Applications and Bipolar Transistors

1.) Many systems require very stable sources of voltage that are independent of the power mains, room temperature, etc. One way to do this is the circuit shown below. The zener diode, D2, has model parameters: $\mathrm{I}_{\mathrm{ZT}}=10 \mathrm{ma}, \mathrm{V}_{\mathrm{ZT}}=7.5$ volts, and $\mathrm{R}_{\mathrm{ZT}}=12 \mathrm{ohms}$. Its temperature coefficient of voltage is $+.027 \%$ per degree centigrade. (Its $\mathrm{V}_{\mathrm{ZT}}$ increases by that percentage for each degree increase of temperature. This number is known to be fairly stable for silicon devices of this breakdown voltage regardless of the details of manufacture.) The regular diode, D1, has a forward drop of .65 volts at 15 ma , an ideality factor of 1.8 , and a temperature coefficient of -2.2 mv per deg centigrade. The amplifier, U1, is a power operational amplifier in a circuit that has a voltage gain of +1 .
1.1) What value of R should be used to set the circuit up for operation at 15 ma in D1? Find Vout and vout in that condition. (Hint: you will need a small signal model for the forward biased diode to get $\mathrm{V}_{\text {out. }}$ )
1.2) For a 10 deg increase in temperature, how much will Vout change? (Express your answer in both volts and percent. Hint: the primary effect of temperature is the change in the model DC sources.)
1.3) What function does the amplifier serve? (Consider the effect of a load current in RL if the circuit did not have an amplifier and RL were for example 30 ohms.)

2.) The growth, processing, distribution, and preparation of food is one of the largest single industries in the United States. Personally my favorite products of this industry are baked goods, sugar being my favorite food group. The circuit below is a small piece of a production mixing system for cookie dough. It (in my dreams) is a vibrator that shakes the sugar chute to keep the flow of sugar constant as part of the first continuous casting cookie factory. A flow sensor
changes the input signal under closed-loop fuzzy-logic computer control. Your job is to select the biasing resistors R1 and R2 to ensure that the circuit will always be able to shake the chute enough. Determine a set of resistor values and show by direct calculation that the circuit will maintain between $80 \%$ and $125 \%$ of the optimum $I_{C Q}$ over transistor variations due to manufacture and temperature. (The plant is one of several to be built around the world, and each plant uses several of the shakers. No one wants to tweak each one!) The frequency of the input signal is always 60 Hz . Pick a value of $\mathrm{C}_{\text {IN }}$ such that the loss in the input coupling circuit is less than 1 DB for nominal $\mathrm{h}_{\mathrm{fe}}=4000$.

The two transistors in the dashed box are called a Darlington pair and may be purchased as a single device. They act together like a single transistor with the effective terminals marked C, E, and B . The current gain, $\mathrm{h}_{\mathrm{FE}}$ is 750 min and $20,000 \mathrm{max}$. Because there are two base-emitter junctions in series between the base and emitter terminals of the compound device, the effective $\mathrm{V}_{\text {BEQ }}$ is higher than for a single transistor. You may assume that the measured $\mathrm{V}_{\text {BEQ }}$ of 1.35 volts at $45^{\circ} \mathrm{C}$ is reliable for all devices. However, remember that each of the internal transistors is temperature sensitive and each of the internal $\mathrm{V}_{\text {beq's }}$ changes by -2.2 mv per degree C . The circuit must work over a range of temperature from about $10^{\circ} \mathrm{C}$ in winter on startup to $80^{\circ} \mathrm{C}$ in summer operation. Because of the internal compound structure, the saturation voltage of the Darlington pair is high -- about 1.1 volts. Although this circuit is an example of reactive load used over a restricted frequency range, the appropriate formula for the optimum quiescent current in circuits of this type is the same as the one I derived in class for resistive loads even though the load is highly reactive in this case. What is the phase angle of the shaker voltage with respect to the shaker current?

3.) In class I "designed" by hand an example NPN CE stage with gain $30, I_{C E Q}=6.6 \mathrm{~mA}$ and -3 DB cutoff at 50 Hz . Here with no adjustment for the properties of a 2N3906 PNP transistor, I redrew the circuit to operate upside down from a negative supply. (Not necessarily a good idea.) Using SPICE, determine how well this circuit matches what I was trying for, that is, find the $I_{C E Q}$, in-band gain, and -3DB point.

4.) Here is a this-and-that problem, the sort of thing one puts together to point out ideas rather than practical applications. You might find it useful to be reminded of a result I cited in class and is in your textbook, namely that the output impedance of a common collector stage is $Z_{E} \|\left(r_{e}+Z_{S} /\left(1+h_{f e}\right)\right)$ where $Z_{S}$ was the input signal's source impedance. You may use $\mathrm{V}_{\mathrm{BE}}=$ 0.7 volts and $h_{f e}=100$ for all transistors.
4.1) What is the stage type for each of Q1 and Q2 (CE, CB, CC)?
4.2) There are two undetermined resistors in the circuit. Choose one to make $I_{C l}=5 \mathrm{~mA}$.
4.3) Select the other to make the circuit output impedance 18 ohms.
4.4) What is the midband gain of the circuit?
4.5) This circuit probably works best at frequencies around 1 MHz . Select $\mathrm{C}_{\text {IN }}$ to give it a low frequency cutoff of 10 kHz .
4.6) What is the function or purpose of $\mathrm{C}_{\mathrm{IN}}$ ?

5.) The circuit below is an example of one that might have been used in a long-ago battery-powered consumer product. The transistor, Q 1 , has a nominal gain of $h_{F E}=120, \mathrm{~V}_{\text {CESAT }}=0.2$ volts and $\mathrm{I}_{\mathrm{SE}}$, the Shockley coefficient for the base-emitter junction, is nominally $1.2 \cdot 10^{-14} \mathrm{amps}$. A reasonable value of Vout is 5 volts. For the components shown, what are the actual values of $I_{C}$ and the node voltages of the base and collector terminals? (WARNING: I have deliberately made a mistake in setting up the problem.) Without changing the gain of the circuit or lowering the input impedance, change one component to set the circuit to the 5 volt output quiescent condition. What do you change and what value do you suggest? Finally, for that corrected configuration, what is the range of Vout for a range of transistor gain from 80 to 160 ?

6.) The collector current for a properly biased BJT ought to depend on the base-emitter voltage, $v_{B E}$, as $i_{C}=I_{S C E} \exp \left(\frac{q v_{B E}}{n k T}\right)$ where $I_{S C E}$ is a temperature dependent constant of the device and $n$ is the ideality factor, a constant very near unity. I hope that your measurements in lab 2 confirmed this equation, at least for currents that are not too large. (One person in the class gave me lab 2 in pdf and I reduced the data myself, getting $n=1.05$.) The circuit below is something we will not get to for another month but its analysis is well within your capability. A constant current source, $I_{B I A S}$, sets the collector current of Q1 while the base-emitter voltage of Q1 sets the current in Q2. The two transistors are identical so they have the same values of n and $\mathrm{I}_{\mathrm{SCE}}$. The current gains of the two transistors are $\beta=100$. To keep the problem easy, neglect the Early effect in Q2.
6.1) Find an equation for $\mathrm{I}_{\mathrm{C} 1}$, the collector current of Q 1 by neglecting the base current of Q 2 but not of Q1.
6.2) Find a transcendental equation for the collector current of Q 2 by using Kirchoff's voltage law. I would like the equation in terms of device parameters and the resistor RE. Again you can neglect the base current of Q2.
6.3) Suppose the current source is 1 mA and you wish the collector current of Q 2 to be 50 microamperes. What value of RE should you use?

7.) In class I included the Early effect in all models of the bipolar transistor because the effect is quite visible in measurements and is important to their use in integrated circuits. (The Early effect is the tilt of $I_{C} v s V_{C E}$ curves such that if their flat sections are extrapolated to negative values of $\mathrm{V}_{\mathrm{CE}}$, these lines all intersect the X -axis at approximately the same point, the Early voltage.) However, when I calculated gains and output impedances for our first circuits, I omitted the ro component that the Early effect adds to the model. Here is an example of a problem with no discrete resistors so that the only effective resistance is from the Early effect. The circuit on the left is sometimes used for IC biasing and the circuit on the right represents transistor Q2 as biased by Q1 in the first circuit. The analysis requires using small signal models.
7.1) The circuit is completely symmetric except for the collector voltage so the two transistors have approximately the same current. Find an expression for that current in terms of the transistor current gain, $h_{f e}$, and the input bias current. If $h_{f e}=100$ and $I_{\text {BIAS }}=500 \mathrm{uA}$, what is IC
7.2) The bias current is constant so together with $Q 1$ and one of the $R_{E}$ 's it forms a Thevenin equivalent voltage source shown on the right as $\mathrm{V}_{B B}$ and $\mathrm{R}_{B B}$. We do not need the value of $\mathrm{V}_{B B}$ for this problem because you already found the DC currents in part 4.1. Find the resistance of R $_{\text {BB }}$. [Hint: try the tee-model for Q1.] For the same current and current gain in Q1, what is the value of this impedance if $R_{E}=700$ ohms.
7.3) The output of this circuit is a constant or almost constant current. Because there is no such thing as a perfect current source, what is important to a designer is the output resistance of the
circuit. To find this, you have to make a small signal model of the right hand circuit and connect the output to a signal source. The easiest way to do the problem is to use either a transconductance model (hybrid-pi) or h-model. Drive the output with a small current source, $i_{\text {test }}$, and calculate the resultant voltage, $v_{\text {test }}$. Be very careful about the sign of the base current - the point of the problem is how the controlled source raises the output impedance beyond $r_{o}+R_{E}$.
7.4) Evaluate the output resistance from part 4.3 assuming that $\mathrm{V}_{\mathrm{A}}=80 \mathrm{~V}$ and $\mathrm{V}_{\mathrm{CE}}=3 \mathrm{~V}$.

8. Razavi textbook problem 5.18 reproduced below.
9. Razavi textbook problem 5.19 reproduced below.
10. Razavi textbook problem 5.20 reproduced below.
5.18. In the circuit depicted in Fig. 5.11 $I_{S 1}=I_{S 2}=4 \times 10^{-16} \mathrm{~A}, \beta_{1}=\beta_{2}=100, \mathrm{a}$ $V_{A}=\infty$.
(a) Determine the operating point of transistor.
(b) Draw the small-signal equivalent circ.


Figure 5.117
*5.19. Consider the circuit shown in Fig. 5.118, where $I_{S 1}=2 I_{S 2}=5 \times 10^{-16} \mathrm{~A}, \beta_{1}=\beta_{2}=$ 100 , and $V_{A}=\infty$.
(a) Determine the collector currents of $Q_{1}$ and $Q_{2}$.
(b) Construct the small-signal equivalent circuit.


Figure 5.118

## Section 5.2.4 Self-Biased Stage

5.20. The circuit of Fig. 5.119 must be biased with a collector current of 1 mA . Compute the required value of $R_{B}$ if $I_{S}=3 \times 10^{-16} \mathrm{~A}$, $\beta=100$, and $V_{A}=\infty$.


Figure 5.119

